REGULATION THROUGH REFERENCE PRICES

REGULACIÓN A TRAVÉS DE PRECIOS DE REFERENCIA

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Resumen: Analizamos el papel de los precios de referencia sobre la competencia y el bienestar en un modelo de ciudad circular donde pagar precios por encima de una referencia afecta negativamente la utilidad del consumidor. Los agentes juegan en tres etapas. Primero, un hacedor de políticas elige un precio de referencia; segundo, las empresas toman su decisión de entrada; finalmente, las empresas compiten en precios y los consumidores deciden su consumo. Se encuentra que, en equilibrio, el precio de mercado y el precio de referencia óptimo siempre coinciden, reduciendo los precios al consumidor e implicando una ganancia neta de bienestar.

Abstract: We analyze the role of reference prices on competition and welfare in a circular city model where paying prices above a reference negatively affects consumers' utility. Agents interact in a three-stage game. First, a policymaker chooses a reference price; second, firms make their entry decision; finally, firms compete in prices while consumers make their consumption decisions. We find that the market price and the optimal reference price always coincide in equilibrium, reducing consumer prices and implying a net welfare gain.

Clasificación JEL/JEL Classification: C7, D4, D9, L1, L2, L5

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1. Introduction

Reference prices have been analyzed in the literature as a regulatory tool. In general, implementing this type of policy seeks to increase consumer welfare and, under certain conditions, social welfare by reducing market prices paid by consumers and the firms' market power. The related literature has analyzed different rules to determine reference prices, such as an international price, the average price of a bundle of goods offered in the market, or an average of prices observed in the past. However, the possibility of determining the reference price directly as the result of maximizing consumer welfare has not been analyzed. This analysis is interesting because it allows for endogenously determining an optimal reference price as a function of the parameters of interest of the model, to study its relationship with market equilibrium prices, and to establish some comparative statics exercises concerning those parameters. Thus, this paper adds to the existing literature by analyzing the case in which the reference price is a policy decision, such that a policymaker chooses reference prices to maximize consumer surplus. To this end, we consider an extension of a static circular city model (Salop, 1979) with reference prices. In this simple framework it is possible to analyze the effects of reference prices on market prices and welfare in an environment of free entry of firms since, in the equilibrium of a simple Salop model, consumer welfare and social welfare coincide. Furthermore, it is well known that one of the main results of the Salop model is that, in equilibrium, firms have too many incentives to enter, given the possibility of stealing the business of other firms. Our model also allows for analyzing the role of reference prices in creating incentives for firms to enter a free-entry market.

Several theoretical and empirical papers have studied the role of price perception in consumer demand models (Koschate-Fischer and Wüllner, 2017; Putler, 1992). These models have been developed in both dynamic and static settings. In both cases, a consumer is assumed to buy a product whenever its current price level is below a reference price. In a dynamic framework, it is usually assumed that consumers can use past prices and other relevant setting variables to form a subjective reference price to make a consumption decision (Chenavaz, 2016; Popescu and Wu, 2007). In a static setting, the effect of a reference price has been analyzed in the context of pharmaceutical markets where the reference price can be modeled as a price of an international product, or as a price of a generic, or as a price of a bundle of several products (Brekke *et al.*, 2016; Brekke *et al.*, 2009; Brekke *et al.*, 2011; Kaiser *et al.*, 2014). In a more general static setting, Zhou (2011) examines the impact of consumer reference dependence on the market competition when consumers take some actual product in the market as the reference point. He shows that the prominent firm whose product is more likely to be taken as the reference point has incentives to randomize between a high and a low price. Hence, reference dependence can cause price variation in the market.

The literature on the effect of reference prices has also analyzed different issues related to implementing reference prices as a tool for regulation. For instance, Brekke *et al.* (2009) compare the effects of regulating pharmaceutical prices through either reference prices or price caps. They find that reference prices seem more effective than price caps for lowering consumer drug prices. This result suggests the possibility of attaining consumer welfare gains by implementing reference prices as a tool for regulators. In a similar paper, Brekke *et al.* (2011) also show that reference prices result in significantly lower brand-name market shares. In a setting that considers the role of the external sector, Kaiser *et al.* (2014) study the result of a change from international to local reference pricing in Denmark in 2005 and find that this policy change yielded substantial reductions in retail prices, reference prices, and patient co-payments as well as a decrease in overall producer revenues and health care expenditures. In a recent paper, Brekke *et al.* (2016) develop a model where a brand-name producer competes in prices with several generic producers. They find that reference prices discourage generic producers from entering the market, whereas the net effect of reference pricing on drug prices is ambiguous, implying that it can be counterproductive in reducing expenditures in some settings. They also show that reference pricing may be welfare-improving when accounting for brand preferences instead of its effects on entry and prices.

Other significant contributions to the reference price regulation literature include Miraldo (2009), who analyzes different reference price rules: 1) reference price as the minimum of the observed prices in the market, 2) reference price as a linear combination of firm prices. Her results show that firms cannot coordinate on higher prices under the minimum policy, while a linear policy implicitly functions as a coordination device. With quality differentiation, both the "minimum" and "linear" policies unambiguously lead to higher prices. Birg (2015) studies the effect of two regulatory instruments (a price cap and a reference price system), a mandatory substitution rule, and the combination of both on generic competition in a Salop-type model in a pharmaceutical industry. She shows that the two regulatory instruments reduce the brand-name drug price, among other results. In addition, the reference price system reduces generic prices and the price cap only if applied in combination with the mandatory substitution rule. On the downside, both regulatory instruments reduce the generic market share and the number of generic competitors. This suggests that there may be a conflict between price reductions and generic competition.

In this paper, we analyze an environment where the reference price is a policy decision of a policymaker. We consider a simple environment where consumers are uniformly distributed along a unit circle and pay a transportation cost to buy one unit of a homogeneous good. Firms enter the market at a fixed cost and pay a constant marginal cost of production (Salop, 1979). Based on previous literature, we introduce reference price effects on consumer preferences as a given parameter such that a consumer experiments a disutility from paying market prices above the reference price. However, unlike the previous literature where reference prices implemented by the policymaker are determined based on different exogenous rules (for instance, as the price of a product that is already offered in the market -national or international- as a linear combination of the prices of various products, or a linear combination of past prices, among others), in our setting reference prices are optimally chosen by a policymaker who seeks to maximize consumer welfare. Although this scheme is interesting and differs from previous settings because it allows the optimal reference prices to be determined endogenously as a function of the fundamental parameters of preferences and technology, it is restrictive for several reasons. For example, the reference price effect parameter on consumer preferences is taken as given; it is also assumed that the policymaker can perform the necessary calculations to determine the optimal reference prices; it is also considered that the policymaker knows the consumer preferences and firms' technology.

In this setting, we analyze the effect of introducing reference prices on equilibrium market prices, competition, and welfare. Three types of agents interact: policymakers, firms, and consumers. The policymaker takes consumer preferences and firms' technology as given. Under these conditions, agents play a game in three stages. In the first stage, a policymaker chooses a reference price to maximize the consumer surplus. In the second stage, firms freely choose whether to enter or not into the market and where to locate. In the third and final stage, firms compete in prices, taking the reference price and consumer demand as given, and consumers make consumption decisions. We characterize the equilibrium of the reference price game, which is determined by an optimal reference price and equilibrium market prices. Our main result is that the optimal reference price and the equilibrium market prices always coincide in the equilibrium. Intuitively, reference prices are a focal point agents use to coordinate their strategic choices. In addition, we also show that the optimal reference price depends negatively on the intensity of the reference price effect on consumer preferences and positively on the marginal cost, on the extent of product differentiation determined by transportation costs, and on the cost of entry into the market.

Our characterization of the equilibrium allows for performing welfare comparisons between a setting without references to prices and the setting of our model. Compared with a standard Salop model, we show that market prices are lower in equilibrium due to reference pricing effects. This is because the reduction in market prices leads to a reduction in firms' revenue, leading to lower profits that ultimately translate into a lower entry of firms into the market. In the face of a smaller number of firms, consumers are forced to pay higher transportation costs to satisfy their demand. These two results have opposite effects on consumer welfare. On the one hand, the price reduction increases their utility, while the increase in transportation costs reduces it. Therefore, a natural question is related to the sign of the net effect of the reference price effects on consumer welfare (which, in equilibrium, coincides with social welfare). Our results indicate that positive welfare gains associated with the reference price effects are achieved in equilibrium. This implies that the increase in transportation costs is compensated by the decrease in prices, leading to a net positive effect on consumer utility. In addition to the previous analytical result, numerical solutions of the model further suggest that those welfare gains could be even greater in markets that are naturally less competitive, i.e., in a context with either high entry costs or high transportation costs. Conversely, the welfare gains would be smaller in markets with high inefficiencies in production, i.e., markets with high marginal costs.

Although our model is limited by its simplicity, our theoretical results demonstrate that, in a context where a policymaker seeks to strengthen competition through free entry, a regulatory policy through reference prices seems to be effective at improving social welfare whenever high enough reference price effects characterize consumer preferences. Understanding what other markets would be suitable for implementing regulation with reference prices, as proposed in our analysis, remains an interesting question not directly addressed in this paper.

As mentioned, related literature has analyzed the use of reference prices as a regulatory tool in pharmaceutical markets, providing evidence on the effects of introducing reference prices on market prices, market shares, firm entry, and consumer welfare. In general, as in the case of our paper, the results of those contributions suggest that the introduction of reference prices is welfare-improving for consumers since, in general, it generates a reduction in consumer prices, a decrease in overall producer revenue, and lower market shares (Birg, 2015; Brekke *et al.*, 2016; Brekke *et al.*, 2009; Brekke *et al.*, 2011; Kaiser *et al.*, 2014; Miraldo, 2009).

Our framework suggests that reference prices could be helpful in markets with free entry and relatively homogeneous goods. Hence, certain retail energy markets where market liberalization has been an important issue in recent years, such as gasoline and diesel markets, gas for domestic consumption, and charging stations for electric cars, can be candidates for implementing a policy of reference prices. In this kind of markets, goods are basically homogeneous and location, brand name, and service facilities are elements of product differentiation that should impact pricing as in the case of the classical Salop framework.

Even when this kind of application for real-world markets seems interesting, it is important to note that our model is limited by its simplicity and is not designed to establish policy recommendations. This is because providing an appropriate regulatory policy in a complex framework is not straightforward and requires deep empirical analysis to understand the functioning of these markets. Our model is built on strict assumptions. For instance, we assume that the policymaker has complete information, i.e., knows consumer preferences, the degree of competition, and the firm's technology. Furthermore, consumer preferences depend on reference price, which is a strong assumption about consumer utility functions. In addition, we consider particular policymaker preferences that aim to maximize consumer welfare (which coincides with social welfare only in equilibrium). Hence, a proper understanding of markets where a reference pricing policy can be applied requires an in-depth analysis of consumer preferences. In addition, other potential issues of real-world markets must be considered. For example, there could be barriers to entry at each link of the production chain. Entry could depend on the allocation of permits, concessions, or any other property rights that would hinder the free entry of firms into the market. It is also important to consider the degree of dependence on foreign markets. For example, in the gas and gasoline markets, movements in retail prices strongly depend on international prices due to the usually high share of imports in total consumption. The presence of taxes and subsidies can also distort market prices. Finally, regulation through reference prices could generate externalities on the whole production chain depending on the link where it would be implemented.

The rest of the paper is organized as follows. In section 2, we introduce the model and main definitions. In section 3, we characterize the equilibrium of the reference price game played by the policymaker, firms, and consumers. In section 4, we carry out several comparative static exercises to understand the effect of changes in crucial parameters of the model on the equilibrium reference price. Section 5 analyzes the welfare implications of reference prices and the effect of changes in the model's parameters on welfare. In section 6, we offer some general conclusions.

2. The model

There are three types of agents: 1) a policymaker who chooses a reference price to maximize consumer welfare; 2) a set of consumers who take as given the reference and market prices and seek to maximize their utility; and 3) firms that compete in prices to maximize profits taking the reference price and consumer preferences as given.

Consumers are uniformly distributed on the unit circle. A consumer demands one unit of an indivisible good, resulting in a gross utility $U > 0$. To buy a unit of the good from firm i, a consumer must pay the price p_i and a transportation cost $t\,|d|$ where $|d|$ is the linear distance between the consumer and firm i and t is the transportation cost per unit of distance. Unlike the classical Salop (1979) model, we include a reference price effect in consumer preferences. In particular, we assume that consumers suffer a utility loss from paying prices above a reference price published by a policymaker. In line with the price effect literature, we assume that such an effect is asymmetric for consumers, i.e., given a reference price r, we assume that whenever $p_i > r$ consumers incur into a disutility proportional to a margin above the reference price. Otherwise, consumers do not suffer a utility loss (Putler, 1992). Generally, a consumer who buys one unit of the good from firm i obtains a net utility given by the following expression:

$$U - \theta max\left\{\frac{p_i - r}{r}, 0\right\} - p_i - t\,|d| \tag{1}$$

Where $\frac{p_i - r}{r}$ is the margin over the reference price that is observed by consumers and $\theta > 0$ is a parameter that measures the intensity of reference price effect in consumer preferences. Note that, if $\theta = 0$, the model collapses to the classical Salop (1979) model. Firms pay a fixed cost of entry and a variable cost with constant marginal cost given by:

$$C(q) = cq + F \tag{2}$$

There is free entry, i.e., firms enter the market until their profits equal zero. Agents interact in an extensive form game that runs in three stages as following:

1. The policymaker chooses a reference price r^{***} to maximize the consumer surplus.
2. Given the optimal reference price r^{***} and consumer preferences, firms freely choose whether to enter the market and where to locate.
3. Firms that decide to enter the market compete in prices to maximize their profits.

The equilibrium of the model is characterized by backward induction. In the third stage of the game, $N > 0$ firms compete in prices taking the reference price chosen by the policymaker and consumer preferences as given. In this stage, equilibrium market prices are characterized as a function of the reference price r, the parameters of consumer preferences, t and θ; the marginal cost c and the fixed number of firms $N > 0$. In the game's second stage, firms make their entry decisions, and a zero-profit condition determines the number of firms that enter the market. In this stage, equilibrium market prices and the number of firms are determined as functions of the reference price r and the relevant parameters of the model c, t, F, and θ. In the first stage, the policymaker chooses an optimal reference price r^{***} taking the optimal behavior of firms and consumers in the next two stages as given. Basically, the policymaker calculates the consumer surplus, which coincides with the social surplus of the market because of firms' free entry and chooses a reference price that maximizes the consumer welfare. Hence, the optimal reference price r^{***} must depend on the parameters c, t, F, and θ.

3. Characterization of equilibrium

3.1 *Price competition*

In the last stage of the reference price game, $N \geq 0$ firms compete in prices by taking as given the references price already chosen by the policymaker and consumer preferences. Recall that a consumer, who has bought one unit of the homogeneous good from firm i, obtains a net utility given by the equation (1). Without loss of generality, we assume that firms are located along the unit circle market according to the principle of maximal differentiation, i.e., N firms will be equidistantly located along the unit circle so that the distance between any two firms will be equal to $\frac{1}{N}$. We characterize a symmetric equilibrium of the game played by N firms, in which all firms choose the same equilibrium price, i.e., $p^* = p_1 = p_2 = \ldots = p_N$. In order to characterize this equilibrium, we consider the optimization problem of one firm i that takes as given the symmetric strategy followed by the rest of its competitors, i.e., the firm i will choose a price p_i by considering that the rest of firms play the same strategy p. In this situation, an indifferent consumer at position x must obtain the same net utility from consuming one unit of the good from firm i or the closest alternative firm. Formally,

$$U - \theta max \left\{ \frac{p_i - r}{r}, 0 \right\} - p_i - tx = U - \theta max \left\{ \frac{p - r}{r}, 0 \right\} - p - t \left(\frac{1}{N} - x \right) \tag{3}$$

By solving the previous equation for x, we find that for a consumer to be indifferent the following condition must hold:

$$x = \frac{1}{2N} + \frac{p - p_i}{2t} + \frac{\theta}{2tr} \left[max \left\{ \frac{p - r}{r}, 0 \right\} - max \left\{ \frac{p_i - r}{r}, 0 \right\} \right] \tag{4}$$

Since firms are equidistant from each other and all other firms but i are following a symmetric strategy, we know that the demand of firm i is equal to $d_i = 2x$. As expected, including a reference price in the circular city model changes the intercept and slope of the demand curve faced by any firm. The following program gives the optimization problem of any typical firm i,

$$max_{p_i} \, (p_i - c) \left(\frac{1}{N} + \frac{p - p_i}{t} + \frac{\theta}{tr} \left[max \left\{ p - r, 0 \right\} - max \left\{ p_i - r, 0 \right\} \right] \right) \tag{5}$$

Before establishing our first result, we introduce the following Lemma, which will be very useful to characterize the equilibrium market price of the model.

Lemma 1. The function $g(r) = c + \frac{tr}{N(r+\theta)}$ has a unique fixed point r^* such that $g(r^*) = r^*$ and $c < r^* < c + \frac{t}{N}$.

Note that the profit function of firm i given by the equation (5) is continuous in p_i. The derivative of this profit function, however, may have a discontinuity at $p_i = r$. Hence, the first order condition (FOC) of the maximization problem of firm i are helpful to characterize a symmetric equilibrium only when the equilibrium price satisfies either $p^* > r$ or $p^* < r$, otherwise the best response function of firm i is not well-defined. Given this observation, we can establish our first result.

Proposition 1. The price strategy given by the function:

$$p^*(r) = \begin{cases} c + \frac{tr}{N(r+\theta)} & if \quad 0 \leq r < r^* \\ r & if \quad r^* \leq r \leq c + \frac{t}{N} \\ c + \frac{t}{N} & if \quad c + \frac{t}{N} < r \end{cases} \tag{7}$$

is a symmetric equilibrium of the price game for any given number of firms $N > 0$, where $r^* > 0$ is the unique reference price that satisfies the condition: $r^* = c + \frac{tr^*}{N(r^*+\theta)}$.

The equilibrium market price of the model with reference pricing is an increasing and continuous function of the reference price of the market. Intuitively, consumers may delay their consumption plans in the presence of high market prices relative to the reference price as they would incur a utility cost when paying a market price above the reference price provided by the policymaker. Hence, a high reference price allows firms to charge higher market prices, avoiding consumers' reactions to market demand since they also observe high reference prices. Another interesting implication of Proposition 1 is that, in the presence of reference price effects, consumers will generally pay a lower market price than in a situation with no reference prices. As we mentioned before, in the presence of reference prices, consumers may delay their consumption plans since a reference price allows consumers to calculate whether they are overpaying for consuming a good. To avoid a lower demand that may significantly reduce profits, firms react by charging lower prices than in a market without reference prices.

3.2 *Free entry of firms*

The equilibrium price characterized in the previous section depends on a fixed number of firms $N > 0$. In this section, we analyze the second part of the game by considering that in equilibrium firms enter the market until a zero-profit condition is satisfied. Profits depend on the demand and equilibrium prices faced by firms, i.e., $\Pi(r) = (p^*(r) - c)\frac{1}{N} - F$, since, in a symmetric equilibrium with $N > 0$ firms, every firm will have a demand of $\frac{1}{N}$. Hence, in a symmetric equilibrium, firms' profits must satisfy the following equation, whose characterization depends on the equilibrium market price stated in Proposition 1,

$$\Pi^*(r) = \begin{cases} \frac{tr}{N^2(r+\theta)} & if \quad 0 \leq r < r^* \\ \frac{r-c}{N} - F & if \quad r^* \leq r \leq c + \frac{t}{N} \\ \frac{t}{N^2} - F & if \quad c + \frac{t}{N} < r \end{cases} \tag{8}$$

The zero-profit condition implies that the number of firms in the market will be a function of the model's reference price and other parameters. Throughout the analysis, we assume that the number of firms is a continuous function instead of an integer. This assumption is useful to characterize the model's equilibrium prices and perform comparative statics exercises. To characterize the number of firms endogenously determined in equilibrium, let us consider the first part of the firm's profit function, which is defined for reference prices between 0 and r^*. In this case, N is equal to $\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{t}{F}}$ because of the zero-profit condition. By substituting the expression for N in the first line of the equilibrium price equation (7), we obtain $\hat{g}(r) = c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$. As in the case of Lemma 1, it is easy to show that the function $\hat{g}(r)$ is continuous and strictly increasing in the interval $[0, \infty)$, and its derivative satisfies $\widehat{g'}(r) = \frac{\theta}{(r+\theta)^2}\sqrt{tF} > 0$ for all $r \geq 0$. In addition, $\hat{g}(0) = c$ and $\lim \hat{g}(r) = c + \sqrt{tF}$ as $r \to \infty$. Therefore, it is easy to show that the function $\hat{g}(r)$ must have a unique fixed point r^{**} such that $r^{**} = c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$ and $c < r^{**} < c + \sqrt{tF}$. According to the characterization of the function $\hat{g}(r)$ and the application of the zero-profit condition, the endogenous number of the firm in equilibrium satisfies the following general form:

$$N(r) = \begin{cases} \sqrt{\frac{r}{r+\theta}}\sqrt{\frac{t}{F}} & if \quad 0 \leq r < r^{**} \\ \frac{r-c}{F} & if \quad r^{**} \leq r \leq c + \sqrt{tF} \\ \sqrt{\frac{t}{F}} & if \quad c + \sqrt{tF} < r \end{cases} \tag{9}$$

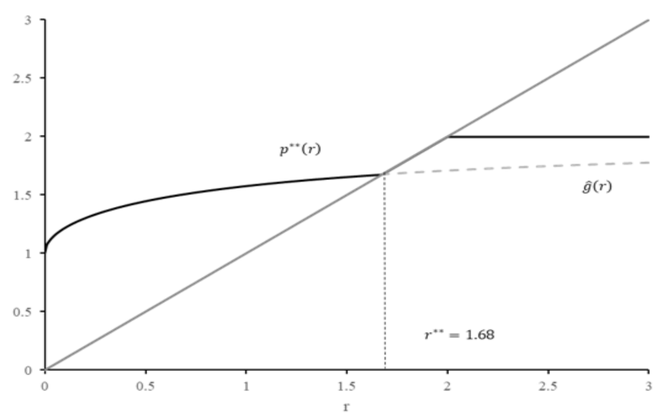
Hence, by substituting the number of firms that satisfy the zero-profit condition into the equilibrium market price function, we have that the following holds,

$$N(r) = \begin{cases} c + \sqrt{\frac{r}{r+\theta}}\sqrt{tF} & if \quad 0 \leq r < r^{**} \\ r & if \quad r^{**} \leq r \leq c + \sqrt{tF} \\ c + \sqrt{tF} & if \quad c + \sqrt{tF} < r \end{cases} \tag{10}$$

The previous result shows that in equilibrium, both the number of firms that enter the market and equilibrium market prices are increasing in the reference price of the market. Intuitively, higher reference prices allow firms to charge higher market prices, and this positive relation implies that, in the presence of higher market prices, more firms are willing to enter the market.

As shown in figure 1, our characterization of the equilibrium market price allows for distinguishing three different areas depending on the value of the reference price. For reference prices below r^{**}, the market price is greater than the reference price and lies between the marginal cost c and r^{**}. For reference prices between r^{**} and $c+\sqrt{tF}$, the latter being the equilibrium price in the model without reference prices,[1] the equilibrium market price is equal to the reference price. Finally, for reference prices above $c+\sqrt{tF}$., the market price is constant and equal to $c+\sqrt{tF}$. According to the previous argument, when reference prices are used as a policy instrument, they cannot induce inconsistent market prices because we will never observe market prices below the marginal cost nor above the equilibrium price without reference price effects. So, if they exist, optimal reference prices must lie between those well-defined limits.

Figure 1
Equilibrium market price



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1, and θ=2.

In the next section, we will show that a unique optimal reference price exists and maximizes consumer surplus. This result implies that there is a well-defined optimal policy that the policymaker can implement. Furthermore, we will demonstrate that this optimal reference price has several interesting properties and is specifically related to equilibrium market prices.

[1] It is easy to argue that our model reduces to the Salop model when the parameter θ of consumer preferences is equal to zero. In particular, according to our simple framework, without reference prices, the equilibrium market price of the model would be equal to $c+\sqrt{tF}$, whereas the endogenous number of firms that is determined by the zero-profit condition is equal to $\sqrt{\frac{t}{F}}$. Furthermore, social welfare, that in equilibrium would be equal to consumer surplus, is equal to $U-c-\frac{5}{4}\sqrt{tF}$.

3.3 *Optimal reference price*

A policymaker chooses a reference price r to maximize consumer surplus. Note that, in the symmetric equilibrium with free entry, consumer surplus is equivalent to the social welfare of the model since all firms make zero profits.

The consumer surplus for a given number of firms is equal to:

$$CS = 2N \int_{0}^{\frac{1}{2N}} \left(U - \theta max \left\{ \frac{p-r}{r}, 0 \right\} - p - tx \right) dx \tag{11}$$

which is equivalent to:

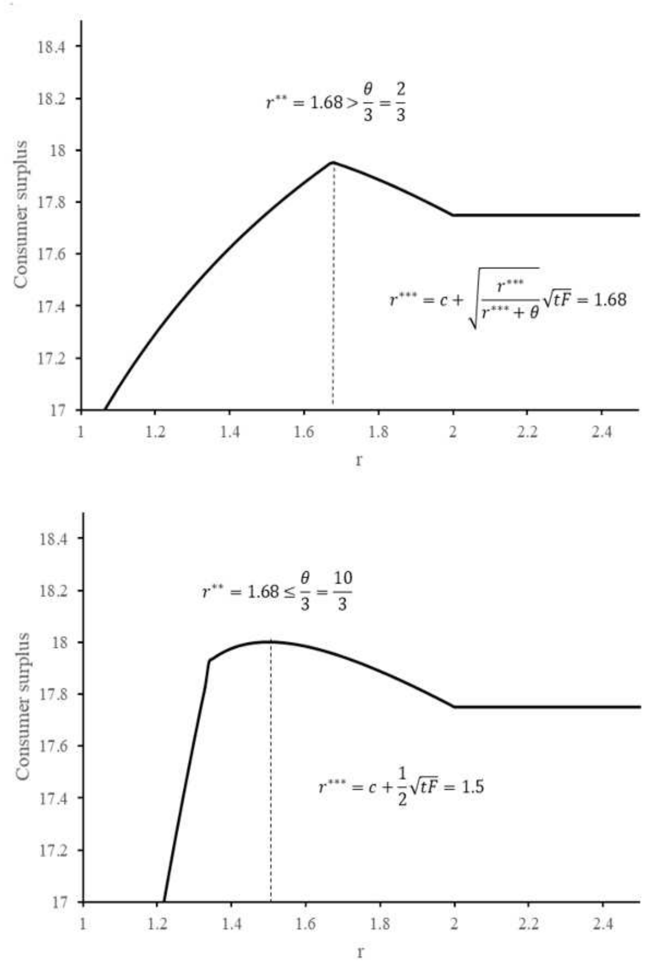
$$CS = U - \theta max \left\{ \frac{p-r}{r}, 0 \right\} - p - \frac{t}{4N} \tag{12}$$

By substituting the number of firms consistent with the free-entry condition and the equilibrium market price that is consistent with this number of firms, we can show that the consumer surplus satisfies the following expression:

$$CS(r) = \begin{cases} U - \theta max \left\{ \frac{c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}-r}{r}, 0 \right\} - c - \sqrt{\frac{r}{r+\theta}}\sqrt{tF} & \\ \quad -\frac{1}{4}\sqrt{\frac{r+\theta}{r}}\sqrt{tF} \quad if \quad 0 \le r < r^{**} & \\ U - -r - \frac{1}{4}\left(\frac{tF}{r-c}\right) \quad if \quad r^{**} \le r \le c + \sqrt{tF} & \\ U - c - \sqrt{tF} - \frac{1}{4}\sqrt{tF} \quad if \quad c + \sqrt{tF} < r & \end{cases} \tag{13}$$

The function $CS(r)$ is well-defined, continuous, and differentiable almost everywhere. Furthermore, note that, for a sufficiently large r, reference price effects become negligible for consumer surplus (or social welfare), leading to the same consumer welfare as in the model without reference price effects. This is consistent with the observation that market prices increase in r and lie between the limits c and $c + \sqrt{tF}$.

Figure 2
Consumer surplus function



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: 1. Left: U=10, c=1, F=1, t=1 and θ=2. 2. Right: U=10, c=1, F=1, t=1, and θ=10.

The policymaker's problem consists of maximizing the consumer surplus function $CS(r)$ with respect to r. Note that the existence of an argmax of $CS(r)$ is non-trivial since $CS(r)$ is not differentiable at several points of its domain and is not a strictly concave function. Figure 2 illustrates the issues involved in the policymaker maximization problem. Nonetheless, the following result shows that a well-defined and unique optimal reference price globally maximizes the consumer surplus.

Proposition 2. The reference price function:

$$r^{***} = \begin{cases} r^{**} & if \quad r^{**} > \frac{\theta}{3} \\ c + \frac{1}{2}\sqrt{tF} & if \quad r^{**} \leq \frac{\theta}{3} \end{cases} \tag{14}$$

is an optimal reference price for the policymaker, where r^{**} is the unique reference price that satisfies $r^{**} = c + \sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$.

Note that the optimal reference price chosen by the policymaker crucially depends on r^{**}, which is the unique fixed point of the function $\hat{g}(r)$. Hence, r^{**} must be greater than c and lower than $c+\sqrt{tF}$. In addition, from Proposition 2, we already know that $r^{**} = c + \frac{1}{2}\sqrt{tF}$ when $r^{**} \leq \frac{\theta}{3}$, so that the optimal reference price must satisfy the condition $r^{**} \leq r^{***} < c + \sqrt{tF}$ for any value of the parameters of the model, including θ. According to the previous argument and the characterization of equilibrium market prices of the model (see Equation 10), it is easy to show that, in equilibrium, the optimal reference price r^{***} and the symmetric equilibrium market price $p^{**}(r^{***})$ always coincide. Formally, we establish the following Corollary of Proposition 2.

Corollary 1. The equilibrium market price of the model satisfies that $p^{**}(r^{***}) = r^{***}$ for any value of the parameters of the model, including θ.

4. Comparative statics

In equilibrium, the optimal reference price chosen by the policymaker and the equilibrium market price attained through firm competition always coincide. This implies that comparative statics analyses of the optimal reference price are sufficient to understand how changes in the model's underlying parameters affect the equilibrium market price and welfare. There are four parameters of interest in the model that affect the equilibrium price:

1. Marginal cost, c.
2. Transportation cost, t.
3. Entry cost, F.
4. Reference price effects on consumer preferences, θ.

Given the characterization of equilibrium presented in the previous section, we can establish the following result.

Proposition 3. The optimal reference price r^{***} satisfies the following properties:

1. $\frac{\partial r^{***}}{\partial c} > 0$.
2. $\frac{\partial r^{***}}{\partial t} > 0$.
3. $\frac{\partial r^{***}}{\partial f} > 0$.
4. $\frac{\partial r^{***}}{\partial \theta} < 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\frac{\partial r^{***}}{\partial \theta} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$.

Figure 3
Comparative statics of the optimal reference price

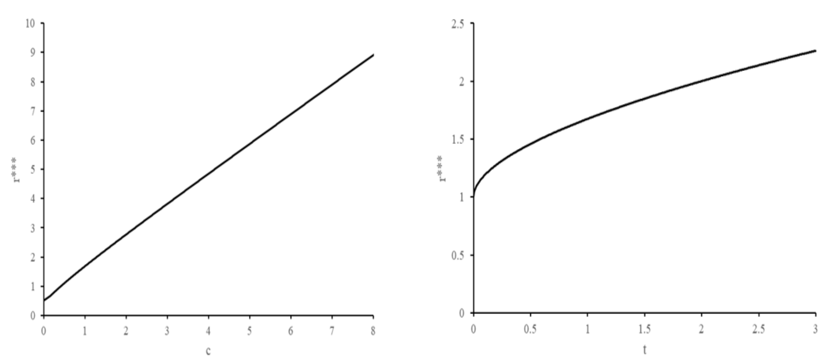
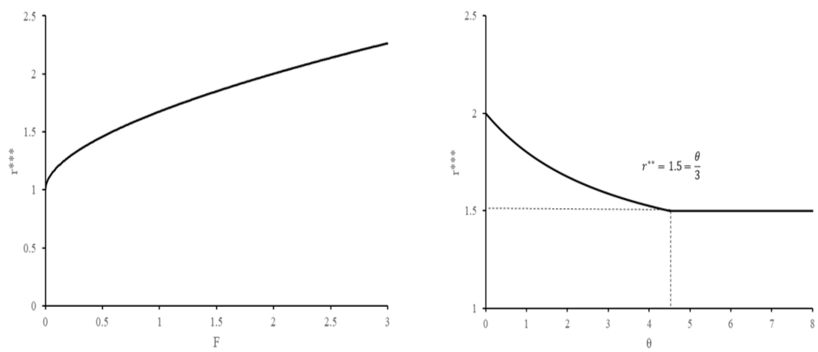


Figure 3
(Continued)



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1 and θ=2.

Figure 3 shows that markets characterized by higher production costs, greater product differentiation, or higher barriers to entry are also characterized by higher reference prices. On the one hand, higher marginal costs lead to higher reference prices. On the other hand, higher entry costs or greater product differentiation through transportation costs lead to increase market power of firms, due to which reference prices are higher. In contrast, when reference price effects are high, reference prices decrease. Intuitively, this greater sensitivity of consumers to reference prices reduces firms' market power, contributing to lower optimal reference prices.

Given that the equilibrium market price satisfies the condition that $p^{**}\left(r^{***}\right) = r^{***}$, we know that all comparative static properties of the previous result extend directly as a corollary to the comparative statics analysis of the equilibrium market price. In addition, it is also possible to use the previous result to analyze the effect of movements in the underlying parameters of the model on the equilibrium number of firms that enter the market and on the total transportation cost paid by consumers. On the one hand, we can consider that the optimal number of firms would satisfy the following equation $N\left(r^{***}\right) = \frac{r^{***}-c}{F}$, since the optimal reference price will satisfy $r^{**} \leq r^{***} < c + \sqrt{tF}$. On the other hand, total transportation cost paid by consumers can be calculated as the result of the integral $TC = 2N \int_{0}^{\frac{1}{2N}} (tx)\, dx$, which is equal to $TC = \frac{t}{4N}$.

After evaluating the total transportation cost at the optimal number of firms, we obtain the equation $TC\left(r^{***}\right) = \frac{1}{4}\left(\frac{tF}{r^{***}-c}\right)$ for $r^{**} \leq r^{***} < c + \sqrt{tF}$. Furthermore, by considering our characterization of the optimal reference price (see Proposition 2), it is easy to show that the optimal number of firms satisfies $N\left(r^{***}\right) = \frac{r^{***}-c}{F}$ if $r^{**} > \frac{\theta}{3}$ and $N\left(r^{***}\right) = \frac{1}{2}\sqrt{\frac{t}{F}}$ if $r^{**} \leq \frac{\theta}{3}$. In a similar way, total transportation cost satisfies $TC\left(r^{***}\right) = \frac{1}{4}\left(\frac{tF}{r^{**}-c}\right)$ if $r^{**} > \frac{\theta}{3}$ and $TC\left(r^{***}\right) = \frac{1}{2}\sqrt{tF}$ if $r^{**} \leq \frac{\theta}{3}$. Hence, we can establish the following results.

Proposition 4. The optimal number of firms $N(r^{***})$ satisfies the following properties at the equilibrium market price $p^{**}(r^{***}) = r^{***}$:

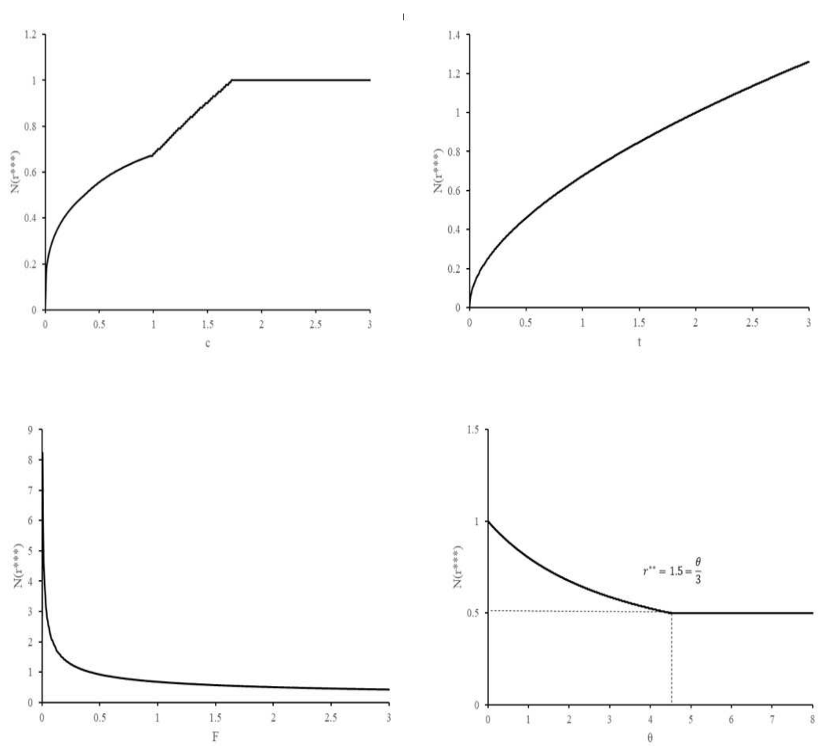
1. $\frac{\partial N(r^{***})}{\partial c} > 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\frac{\partial N(r^{***})}{\partial c} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$.
2. $\frac{\partial N(r^{***})}{\partial t} > 0$.
3. $\frac{\partial N(r^{***})}{\partial F} < 0$.
4. $\frac{\partial N(r^{***})}{\partial \theta} < 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\frac{\partial N(r^{***})}{\partial \theta} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$.

Proposition 5. The optimal transportation cost $TC(r^{***})$ satisfies the following properties at the optimal reference price $p^{**}(r^{***}) = r^{***}$:

1. $\frac{\partial TC(r^{***})}{\partial c} < 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\frac{\partial TC(r^{***})}{\partial c} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$.
2. $\frac{\partial TC(r^{***})}{\partial t} > 0$.
3. $\frac{\partial TC(r^{***})}{\partial F} > 0$.
4. $\frac{\partial TC(r^{***})}{\partial \theta} > 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\frac{\partial TC(r^{***})}{\partial \theta} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$.

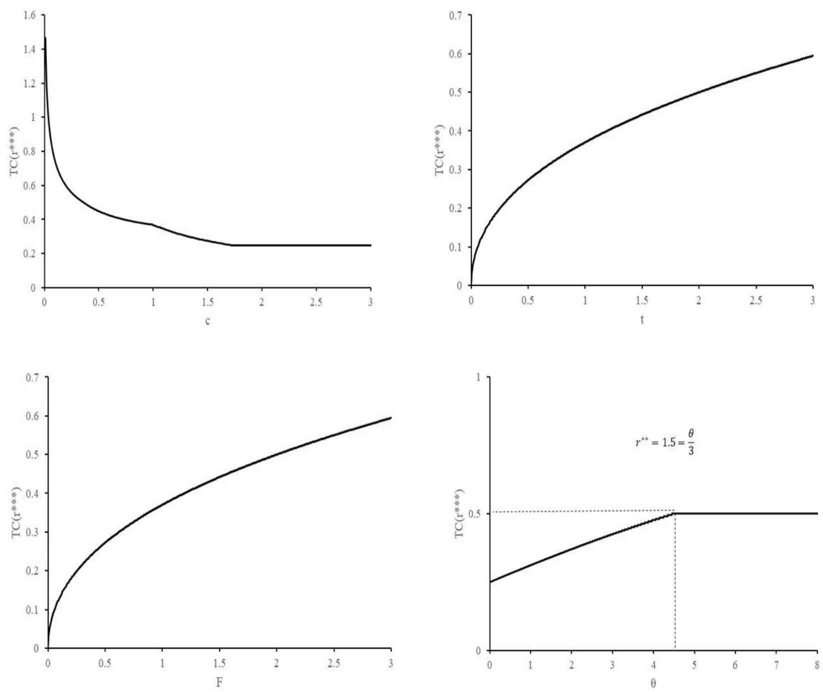
Figures 4 and 5 illustrate an interesting effect of reference price effects on the optimal number of firms and the transportation costs consumers pay in equilibrium. In fact, in an equilibrium with a higher parameter of reference price effects, the optimal number of firms decreases while transportation costs increase, which implies that the degree of differentiation of products in the market decreases. This result leads to a reduction in the utility obtained from consumers. In turn, we also show that an increase in the same parameter reduces consumer market prices. These two results have opposite effects on consumer preferences and leave open the question of the net's effect sign of the reference price effects on consumer welfare. In the next section, we explore this question more in depth and show that reference price effects always lead to a positive utility gain for consumers.

Figure 4
Comparative statics of the equilibrium number of firms



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1 and θ=2.

Figure 5
Comparative statics of the equilibrium transportation cost



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1 and θ=2.

5. Welfare comparisons

In the previous section, we showed that the optimal reference price increases as the parameters c, t, and F increase. Since the equilibrium market price and the optimal reference price coincide, this result is consistent with a model with no reference prices. That is, the equilibrium market price must increase due to greater inefficiencies in production (associated with a high marginal cost), higher horizontal differentiation (associated with a high transportation cost), and higher entry cost for firms (high F).[2] We also analyzed the case of an

[2] It is easy to observe these results in a model with no reference prices since, as we argued before, in this framework the equilibrium market price is equal to $c + \sqrt{tF}$.

increase in the parameter θ, which is associated with a greater sensitivity of consumers to the difference between the market price and the reference price. In this case, we also showed that an increase in the parameter θ is associated with a decrease in the equilibrium price whenever $r^{**} > \frac{\theta}{3}$, otherwise an increase in the parameter θ has no effect on the equilibrium price. Intuitively, the fixed point r^{**} defines a limit that fully characterizes the effect of an increase in the parameter θ of consumer preferences on the market's equilibrium price. In this way, we say that θ is low enough when a variation in this parameter has a non-zero effect on equilibrium prices, i.e., whenever $r^{**} > \frac{\theta}{3}$.

In addition, we also show a comparative static analysis of the equilibrium number of firms and the total transportation cost paid by consumers. The results of this analysis in the previous section have several implications for understanding the impact of an increase in the underlying parameters of the model on social welfare (i.e., consumer surplus in equilibrium). In contrast, it is more challenging to establish similarly clear conclusions regarding consumer surplus in equilibrium with a comparative static analysis. For instance, we showed that an increase in the parameter θ decreases the equilibrium market price and increases the equilibrium total transportation cost. Hence, in principle, the effect on equilibrium consumer surplus may be ambiguous since a price decrease must increase consumer welfare, while an increase in total transportation costs must decrease it.

In addition, we know that for a low enough value of the parameter θ, i.e., whenever $r^{**} > \frac{\theta}{3}$, the equilibrium consumer surplus function is not differentiable at the argmax of the policymaker problem, and a direct application of the envelope theorem is not feasible in this case. Fortunately, right-hand and left-hand side derivatives of the equilibrium consumer surplus exist at the argmax of this function, making it possible to determine the upper and lower bounds of the variation of the value function of the policymaker. In this way, it is possible to determine the effects of a parameter variation over the equilibrium welfare of the model. According to the previous argument, we establish the following result.

Proposition 6. If $r^{**} > \frac{\theta}{3}$, then the social welfare function $CS(r)$ satisfies the following properties at r^{***}:

1. $\frac{\partial CS(r^{***},c,t,F)}{\partial c} \in \left[-\frac{r^{**}+\theta}{r^{**}}, -\frac{r^{**}+\theta}{4r^{**}}\right]$.
2. $\frac{\partial CS(r^{***},c,t,F)}{\partial t} \in \left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$.

3. $\frac{\partial CS(r^{***},c,t,F)}{\partial F} \in \left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right].$
4. $\frac{\partial CS(r^{***},c,t,F)}{\partial \theta} \in \left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right)\right].$

Otherwise, $CS(r)$ satisfies:

1. $\frac{\partial CS(r^{***},c,t,F)}{\partial c} = -1.$
2. $\frac{\partial CS(r^{***},c,t,F)}{\partial t} = -\frac{1}{2}\sqrt{\frac{F}{t}}.$
3. $\frac{\partial CS(r^{***},c,t,F)}{\partial F} - \frac{1}{2}\sqrt{\frac{t}{F}}.$
4. $\frac{\partial CS(r^{***},c,t,F)}{\partial \theta} = 0.$

Based on the previous result, we can expect a decrease in social welfare in the face of greater inefficiencies in production (an increase in marginal cost c), high entry cost (high F) that reduces the number of firms in the market, or greater horizontal differentiation (higher t) that increases transportation cost. Our model also shows that a greater reference price effect (high θ) can be associated with an increase in consumer surplus, which implies that the decrease in equilibrium market prices compensates for the increase in transportation cost associated with a lower number of firms in the market (figure 6).

Figure 6
Comparative statics of the equilibrium social welfare

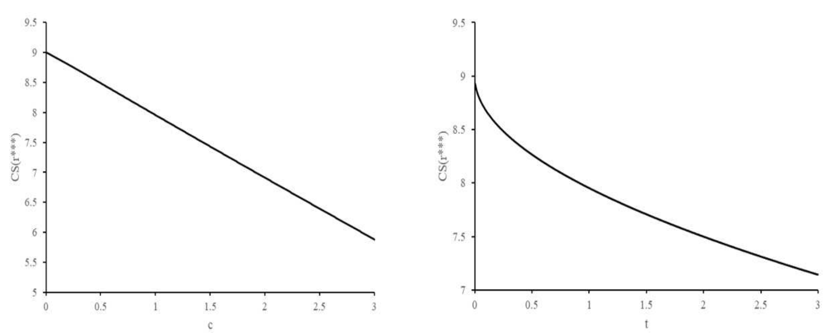
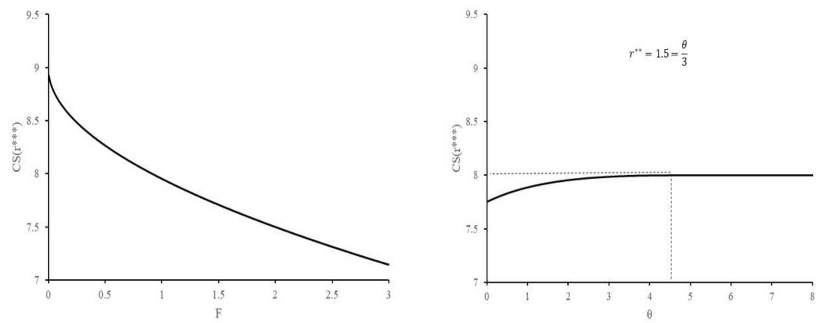


Figure 6
(Continued)



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1 and θ=2.

In addition to the comparative static analysis of the equilibrium social welfare, a natural question arises regarding the comparison between the social welfare of both models, with and without reference price effects, to understand whether reference prices are an effective policy tool for obtaining welfare gains compared to a standard Salop model. Let us define the function $\Delta CS\left(r,c,t,F,\theta\right) = CS\left(r,c,t,F,\theta\right) - \left(U - c - \sqrt{tF} - \frac{1}{4}\sqrt{tF}\right)$ as the welfare difference between the equilibrium of the model with and without reference price effects. Then, the following result may be established.

Proposition 7. The difference in social welfare $\Delta CS\left(r,c,t,F,\theta\right)$ is always positive at the optimal reference price r^{***}.

Intuitively, reference prices have three different effects that result in greater social welfare compared to a base model without reference prices. First, market prices are always lower in equilibrium than those of a model without reference prices. Second, the equilibrium number of firms is also lower, which makes the model closer to the one with an optimal number of firms, i.e., a model in which the sum of production and transportation costs is minimized. Third, even when a lower number of firms implies a higher transportation cost, it is possible to show that the reduction in market prices in equilibrium compensates for the increase in transportation cost, leading to higher social welfare in the presence of reference prices.

A second interesting question concerns the comparative statics analysis of the difference in social welfare $\Delta CS\left(r,c,t,F,\theta\right)$. The previous result shows that reference prices generate welfare gains in markets with horizontal differentiation where firms enter freely into the market and compete in prices. However, it would be interesting to draw some conclusions about the size of welfare gains in the presence of reference prices in a context with greater inefficiencies, either high entry costs or high horizontal differentiation, and greater consumer sensitivity to reference prices. The following result addresses this issue, but it is inconclusive since the sign of the variation of the difference in social welfare is ambiguous for sufficiently low values of the parameter θ. Nonetheless, it is possible to establish several conclusions in the case of a sufficiently high value of the parameter θ. Formally, a Corollary of Proposition 6 is the following.

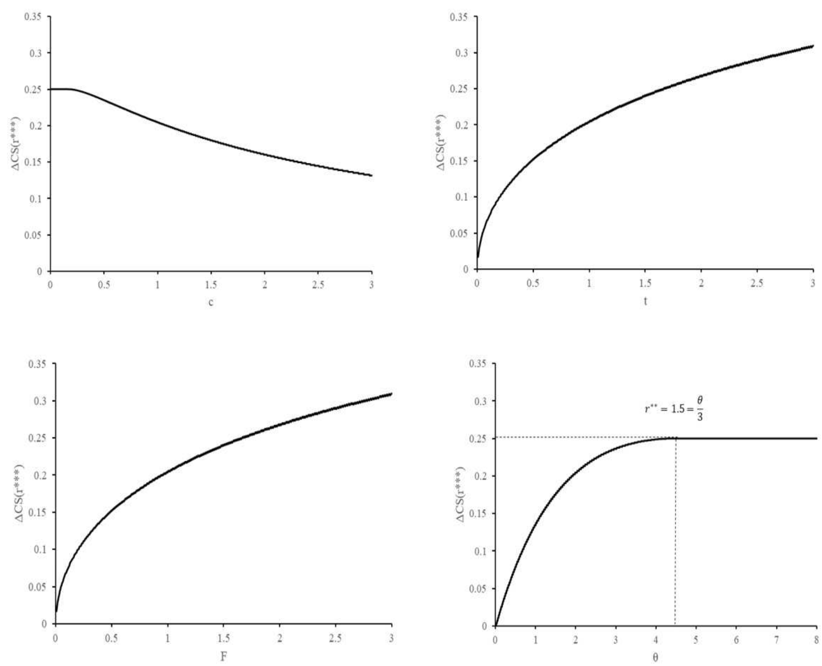
Corollary 2. The difference in social welfare $\Delta CS\left(r, c, t, F, \theta\right)$ satisfies the following properties at r^{***}:

1. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial c} \in \left[-\frac{\theta}{r^{**}}, 1-\frac{r^{**}+\theta}{4r^{**}}\right]$ if $r^{**} > \frac{\theta}{3}$, otherwise $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial c} = 0$.
2. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial t} \in \left[\frac{5}{8}\left(1-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{F}{t}}, \frac{1}{4}\left(\frac{5}{2}-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{F}{t}}\,\right]$ if $r^{**} > \frac{\theta}{3}$, otherwise $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial t} = \frac{1}{8}\sqrt{\frac{F}{t}}$.
3. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F} \in \left[\frac{5}{8}\left(1-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{t}{F}}, \frac{1}{4}\left(\frac{5}{2}-\sqrt{\frac{r^{**}+\theta}{r^{**}}}\right)\sqrt{\frac{t}{F}}\,\right]$ if $r^{**} > \frac{\theta}{3}$, otherwise $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F} = \frac{1}{8}\sqrt{\frac{t}{F}}$.
4. $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial \theta} \in \left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)\sqrt{tF}\right]$ if $r^{**} > \frac{\theta}{3}$, otherwise $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial \theta} = 0$.

The previous result shows that the variation of welfare gains from the use of reference prices as a regulatory instrument is ambiguous in the face of an increase in underlying parameters of the model since those welfare gains may be either positive or negative when either c, t, or F increase. Only in the case of the reference price effect parameter θ we can conclude that an increase of this parameter leads to unambiguously higher welfare gains. Only for sufficiently large values of θ the variation of welfare gains is not ambiguous, having positive variation for either t or F, and no variation for either c or θ. Numerical solutions for the equilibrium of the model at different parameter specifications show that, generally, we can expect a negative variation of welfare gains in the face of an increase in marginal cost. Conversely, in the case of an increase of either horizontal differentiation t or entry cost F, it seems reasonable to expect an increase in welfare gains (see figure 7).

Figure 7

Comparative statics of the difference in equilibrium social welfare



Source: Own elaboration based on numerical solutions for the equilibrium market price with parameters: U=10, c=1, F=1, t=1 and θ=2.

Finally, our results show that using reference prices as a regulatory tool leads to positive welfare gains compared to a market with no reference prices. These gains are even higher in less competitive markets with high entry or transportation costs. Likewise, it seems reasonable to expect small welfare gains in markets with a high marginal cost of production. Furthermore, even when a high reference price effect (higher values of θ) leads to greater welfare gains, we show that this effect is bounded above by the first best solution of the circular city model. For high values of θ, equilibrium market prices stop declining in the face of greater sensitivity of consumers to reference prices.

6. Conclusions

We analyze a circular city model with free entry and reference price effects, i.e., consumers suffer a utility loss from paying market prices above a reference price. Unlike the previous literature, we consider a reference price model where the reference price is explicitly modeled as a decision variable of a policymaker that seeks to maximize the consumer welfare of the market. In this setting, agents play a game in three stages. First, a policymaker chooses a reference price to maximize consumer surplus. Second, firms freely choose whether to enter the market and where to locate. Third, firms compete in prices, taking the reference price chosen by the policymaker and consumer preferences as given. We characterize the equilibrium of the reference price game of the model by backward induction. An optimal reference price and an equilibrium market price determine the equilibrium of this model.

Our main result shows that the optimal reference price and the equilibrium market price always coincide in equilibrium. This result implies that the strategic interaction of the policymaker, firms, and consumers is internally consistent. A comparative static analysis of the model shows that the optimal reference price (equilibrium market price) depends negatively on the intensity of the reference price effect of consumer preferences and positively on the marginal cost, the extent of product differentiation, and the cost of market entry.

Our characterization of the equilibrium demonstrates that in our model, market prices reduce in equilibrium compared to a standard Salop model with no reference price effects. Intuitively, this reduction in market prices leads to lower profits that ultimately translate into a lower entry of firms. Consumers pay higher transportation costs with a smaller number of firms. Our results also show that positive welfare gains associated with the reference price effects are achieved in equilibrium. This implies that the decrease in prices more than compensates for the increase in transportation costs. Furthermore, numerical solutions of the model provide evidence that those welfare gains are even higher in markets that are less competitive with either higher entry or transportation costs. Likewise, it seems reasonable to expect small welfare gains in markets with higher inefficiencies in production.

Finally, our results present some evidence about how a policy-maker whose preferences depend on consumer welfare can benefit consumers through regulatory tools, such as reference prices, without directly determining market prices and taking advantage of their knowledge about preferences and production technology. Furthermore, our results suggest that this type of policy could be used without directly affecting market entry, that is, without directly intervening in the state of market competition. In this sense, our results also contribute to the understanding of policy tools that are useful for reducing the incentives of firms' over-entry in the context of Salop-type models associated with the possibility of stealing the business of other firms. In this way, the reference prices, as modeled in this paper, are effective tools to bring the market closer to its efficient result. As we argued above, our results could support the analysis of regulatory tools in general markets with free entry and homogeneous goods. We believe that it could be beneficial in analyzing energy markets in which the policy maker intends to send a signal to producers and consumers of what would be a correct price for the market given preferences, production technology and level of competition in the market.

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Appendix: Proofs

Proof of Lemma 1

Proof. The function $g(r)$ is continuous and strictly increasing in the interval $[0,\infty)$, its derivative satisfies $g'(r) = \frac{t\theta}{N(r+\theta)^2} > 0$ for all $r \geq 0$. In addition, it is easy to show that $g(0) = c$ and $\lim g(r) = c + \frac{t}{N}$ as $r \to \infty$. Then, there must exist a unique r^* such that $g(r^*) = r^*$ that satisfies $c < r^* < c + \frac{t}{N}$. This completes the proof.

Proof of Proposition 1

Proof. For a symmetric equilibrium of the model, there are three relevant cases for equilibrium prices for any given reference price r, i.e., $p^* > r$, $p^* < r$, and $p^* = r$. The FOC of this maximization problem satisfies the following expression:

$$(p_i - c)\frac{\partial d_i}{\partial p_i} + \left(\frac{1}{N} + \frac{p-p_i}{t} + \frac{\theta}{tr}\left[max\{p-r,0\} - max\{p_i - r, 0\}\right]\right) = 0 \tag{15}$$

where $\frac{\partial d_i}{\partial p_i} = -\left[\frac{1}{t} + \frac{\theta}{tr}\left(\frac{1+sign(p_i-r)}{2}\right)\right]$ and:

$$sign\left(x\right) = \begin{cases} 1 \quad if \quad x > 0 \\ -1 \quad if \quad x < 1 \end{cases}$$

Then the best response function of firm i satisfies the following equation:

$$BR_i\left(p\right) = \begin{cases} \frac{c}{2} + \frac{tr}{2N(r+\theta)} + \frac{pr}{2(r+\theta)} + \frac{\theta}{2(r+\theta)}\left(max\left\{p-r,0\right\}+r\right) \\ if \quad p_i > r \\ \frac{c}{2} + \frac{t}{2N} + \frac{p}{2} + \frac{\theta}{2r} max\left\{p-r,0\right\} if \quad p_i < r \end{cases} \tag{16}$$

Hence, the FOC is useful to characterize the first two relevant cases for the equilibrium price. We know that, in a symmetric equilibrium, $p_i = p = p^*$. Based on the best response function of firms, it is easy to show that in the first relevant case of the proof, the equilibrium price must satisfy $p^* = c + \frac{tr}{N(r+\theta)}$ whenever $p^* > r$. According to Lemma 1, this condition holds whenever $r^* > r$, where r^* is the unique reference price that satisfies $r^* = c + \frac{tr^*}{N(r^*+\theta)}$. For the second case, the equilibrium price satisfies $p^* = c + \frac{t}{N}$ whenever $c + \frac{t}{N} > r$.

For the third relevant case, i.e., $p^* = r$, we must implement a direct proof since the best response function of firm i is not well-defined at this point. In general, we have to prove that playing r is the best response of the firm i whenever all other firms play symmetrically $p = r$. Suppose that all firms, except for the firm i, play a price $p = r$ that satisfies $r^* \leq r \leq c + \frac{t}{N}$. There are two cases to be analyzed: $p_i > r$ and $p_i < r$, for a proper $\varepsilon > 0$, respectively. Consider the first case, it is clear that profits of firm i can be written in the following way:

$$\left(r+\varepsilon-c\right)\left(\frac{1}{N}-\frac{\varepsilon}{t}-\frac{\theta\varepsilon}{tr}\right) = \left(r-c\right)\frac{1}{N}$$
$$+\left[1-\left(r-c\right)\left(\frac{\left(r+\theta\right)N}{tr}\right)\right]\frac{\varepsilon}{N}-\left(\frac{r+\theta}{tr}\right)\varepsilon^2 \tag{17}$$

where $(r-c)\frac{1}{N}$ is the profit that is obtained by following the strategy $p_i = r$. Since $r^* \leq r \leq c + \frac{t}{N}$, there must exist $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$. By substituting in equation (17) and by noting that $\frac{tr}{N(r+\theta)}$ is a continuous and strictly increasing function in r, we find that:

$$(r-c)\frac{1}{N} + \left[1 - \frac{\frac{t\bar{r}}{N(\bar{r}+\theta)}}{\frac{tr}{N(r+\theta)}}\right]\frac{\varepsilon}{N} - \left(\frac{r+\theta}{tr}\right)\varepsilon^2 < (r-c)\frac{1}{N} \qquad (18)$$

Following a similar argument, in the case where $p_i < r$ profits of firm i can be written as:

$$(r-\varepsilon-c)\left(\frac{1}{N}+\frac{\varepsilon}{t}\right) = (r-c)\frac{1}{N} + \left[(r-c)\frac{N}{t} - 1\right]\frac{\varepsilon}{N} - \frac{\varepsilon^2}{t} \quad (19)$$

As before, by substituting $\bar{r} \geq r$ such that $r = c + \frac{t\bar{r}}{N(\bar{r}+\theta)}$ in equation (19) we find that:

$$(r-c)\frac{1}{N} + \left[\frac{\bar{r}}{\bar{r}+\theta} - 1\right]\frac{\varepsilon}{N} - \frac{\varepsilon^2}{t} < (r-c)\frac{1}{N} \qquad (20)$$

Hence, playing $p_i = r$ is a best response for firm i whenever all other firms are playing $p = r$. Then, the equilibrium market price is given by the function:

$$p^*(r) = \begin{cases} c + \frac{tr}{N(r+\theta)} & if \quad 0 \leq r < r^* \\ r & if \quad r^* \leq r \leq c + \frac{t}{N} \\ c + \frac{t}{N} & if \quad c + \frac{t}{N} < r \end{cases} \qquad (21)$$

This completes the proof.

Proof of Proposition 2

Proof. Assume that $0 < r < r^{**}$, then the derivative of $CS(r)$ satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=-\frac{\theta}{r^{2}}\left[\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta r}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}-c-\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right]$$

$$-\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\left(\frac{\theta}{\left(r+\theta\right)^{2}}\right)\sqrt{tF}+\frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r^{2}}\right)\sqrt{tF} \quad (22)$$

By rearranging the previous expression, it is possible to show that equation (22) satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{\theta}{r+\theta}-2\right)\sqrt{tF}$$

$$-\frac{c\theta}{2r^{2}}\sqrt{\frac{r}{r+\theta}}\left(\frac{r}{r+\theta}-\frac{1}{4}\right)\sqrt{tF} \quad (23)$$

which reduces to the following expression:

$$\frac{\partial CS\left(r\right)}{\partial r}=\frac{c\theta}{r^{2}}\left(1+\frac{5}{8}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\right)>0 \quad (24)$$

Then, the consumer surplus is always strictly increasing for $0<r<r^{**}$. Now consider the case where $r^{**}\leq r\leq c+\sqrt{tF}$, in this case the derivative of consumer surplus satisfies:

$$\frac{\partial CS\left(r\right)}{\partial r}=-1+\frac{tF}{4\left(r-c\right)^{2}} \quad (25)$$

Since $r^{**}>c$, this derivative is well defined and the $CS(r)$ attains a maximum at the reference price $\hat{r}=c+\frac{1}{2}\sqrt{tF}$ and $CS\left(\hat{r}\right)=U-c-\sqrt{tF}>U-c-\frac{5}{4}\sqrt{tF}$. In addition, note that there are two real roots $r1>0$ and $r2>0$ that satisfy the condition $U-r-\frac{1}{4}\left(\frac{tF}{r-c}\right)=U-c-\frac{5}{4}\sqrt{tF}$. Given that, it is possible to show that $r1=c+\frac{1}{4}\sqrt{tF}$ and $r2=c+\sqrt{tF}$. Since $CS(r)$ is a continuous function at the point r^{**}, the following condition is satisfied:

$$U-\theta max\left\{\frac{c+\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}-r^{**}}{r^{**}},0\right\}-c-\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}$$

$$-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}=U-r^{**}-\frac{1}{4}\left(\frac{tF}{r^{**}-c}\right) \tag{26}$$

This implies that $CS\left(r^{**}\right)>U-c-\frac{5}{4}\sqrt{tF}$ for all all $r1<r^{**}<r2$. Hence, there are two candidates for being the global argmax of the function $CS(r)$, depending on the values of parameters. Accordingly, $\hat{r}$ would be the argmax of $CS(r)$ only if it is on the right side of r^{**}, otherwise the global argmax will be r^{**}. Hence, $\hat{r}=c+\frac{1}{2}\sqrt{tF}\geq c+\sqrt{\frac{r^{**}}{r^{**}+\theta}}\sqrt{tF}=r^{**}$ whenever $\frac{1}{4}\geq\frac{r^{**}}{r^{**}+\theta}$, which is equivalent to $r^{**}\leq\frac{\theta}{3}$. This completes the proof.

Proof of Proposition 3

Proof. For the case of $r^{**}\leq\frac{\theta}{3}$, the previously mentioned properties are trivially satisfied, given that r^{***} takes the closed form solution of $c+\frac{1}{2}\sqrt{tF}$. Since the optimal reference price has no closed form solution when $r^{**}>\frac{\theta}{3}$, we can calculate the implicit derivatives from the function $r=c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ concerning each parameter of interest. In order to simplify, with some abuse of notation, we use r instead of r^{**} to indicate the optimal reference price in this case.

Case 1: By implicitly differentiating $r=c+\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ with respect to c, we attain the following expression:

$$\frac{\partial r}{\partial c}=1+\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial c}-r\frac{\partial r}{\partial c}}{(r+\theta)^{2}}\right) \tag{27}$$

By rearranging equation (27), we obtain:

$$\frac{\partial r}{\partial c}=1+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{tF}\left(\frac{1}{r+\theta}\right)\left(\frac{\theta}{r}\right)\frac{\partial r}{\partial c} \tag{28}$$

Since, by definition, $r-c=\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$, equation (28) can be reduced to:

$$\frac{\partial r}{\partial c}=\frac{1}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)}>0 \tag{29}$$

Case 2: Similarly, by implicitly differentiating the reference price function with respect to t, we obtain:

$$\frac{\partial r}{\partial t}=\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial t}-r\frac{\partial r}{\partial t}}{(r+\theta)^2}\right)+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} \tag{30}$$

By rearranging this equation and substituting $r-c=\sqrt{\frac{r}{r+\theta}}\sqrt{tF}$ in equation (30), we have that:

$$\frac{\partial r}{\partial t}=\frac{\frac{1}{2}\left(\frac{r-c}{t}\right)}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)}>0 \tag{31}$$

Case 3: Basically, the same procedure as for t. Simply exchange t by F in the previous implicit derivative.

Case 4: By implicitly differentiating the reference price function with respect to θ, we obtain:

$$\frac{\partial r}{\partial \theta}=\frac{1}{2}\sqrt{\frac{r+\theta}{r}}\sqrt{tF}\left(\frac{(r+\theta)\frac{\partial r}{\partial \theta}-r\left(\frac{\partial r}{\partial \theta}+1\right)}{(r+\theta)^2}\right) \tag{32}$$

By rearranging equation (31), we can express equation (32) as follows:

$$\frac{\partial r}{\partial \theta}=-\frac{\frac{1}{2}\left(\frac{r-c}{r+\theta}\right)}{1-\frac{1}{2}\left(\frac{r-c}{r}\right)\left(\frac{\theta}{r+\theta}\right)}<0 \tag{33}$$

This completes the proof.

Proof of Proposition 4

Proof. Let us consider the function that characterizes the number of firms given by the expression $N\left(r\right)=\frac{r-c}{F}$ for $r^{**}\leq r<c+\sqrt{tF}$.

Case 1: By directly differentiating $N(r)$ with respect to c, we obtain:

$$\frac{\partial N\left(r\right)}{\partial c}=\frac{1}{F}\left(\frac{\partial r^{***}}{\partial c}-1\right) \tag{34}$$

For the case of $r^{**}>\frac{\theta}{3}$, equation (34) is equivalent to the following:

$$\frac{\partial N\left(r^{***}\right)}{\partial c}=\frac{1}{F}\left(\frac{\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{35}$$

This equation is positive, since $1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$. For the case of $r^{**}\leq\frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c}=1$.

Case 2: In a similar way, by differentiating $N(r)$ with respect to t, we have that:

$$\frac{\partial N\left(r^{***}\right)}{\partial t}=\frac{1}{F}\frac{\partial r^{***}}{\partial t} \tag{36}$$

Hence, $\frac{\partial N\left(r^{***}\right)}{\partial t}>0$, since $\frac{\partial r^{***}}{\partial t}>0$.

Case 3: By directly differentiating $N(r)$ with respect to F, we obtain:

$$\frac{\partial N\left(r\right)}{\partial F}=\frac{1}{F}\left(\frac{\partial r^{***}}{\partial F}-\frac{r^{***}-c}{F}\right) \tag{37}$$

We know that $\frac{\partial r^{***}}{\partial F}=\frac{\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{F}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for the case of $r^{**}>\frac{\theta}{3}$. Hence, equation (37) is equivalent to the following:

$$\frac{\partial N\left(r^{***}\right)}{\partial F}=-\frac{1}{F}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right) \tag{38}$$

It is clear that $1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>\frac{1}{2}$, then $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$, which implies that $\frac{\partial N(r^{***})}{\partial F}<0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$, which implies that $\frac{\partial r^{***}}{\partial F}=\frac{1}{4}\sqrt{\frac{t}{F}}$ and $\frac{r^{***}-c}{F}=\frac{1}{2}\sqrt{\frac{t}{F}}$, hence $\frac{\partial N(r^{***})}{\partial F}<0$.

Case 4: For the last case, by differentiating $N(r)$ with respect to θ, we have the expression:

$$\frac{\partial N\left(r^{***}\right)}{\partial\theta}=\frac{1}{F}\frac{\partial r^{***}}{\partial\theta} \tag{39}$$

which directly implies the result, since we know that $\frac{\partial(r^{***})}{\partial\theta}<0$ whenever $r^{**}>\frac{\theta}{3}$ and $\frac{\partial r^{***}}{\partial\theta}=0$ whenever $r^{**}\leq\frac{\theta}{3}$. This completes the proof.

Proof of Proposition 5

Proof. Let us consider the transportation cost function that is relevant for the analysis given by $TC\left(r\right)=\frac{1}{4}\left(\frac{tF}{r-c}\right)$ for $r^{**}\leq r<c+\sqrt{tF}$.

Case 1: By directly differentiating $TC(r)$ with respect to c, we obtain:

$$\frac{\partial TC\left(r^{***}\right)}{\partial c}=-\frac{tF}{4(r^{***}-c)^{2}}\left(\frac{\partial r^{***}}{\partial c}-1\right) \tag{40}$$

Since for the case of $r^{**}>\frac{\theta}{3}$ we know that $\frac{\partial r^{***}}{\partial c}>1$, this implies that $\frac{\partial TC(r^{***})}{\partial c}<1$. For the case of $r^{**}\leq\frac{\theta}{3}$, we know that $\frac{\partial r^{***}}{\partial c}=1$, hence $\frac{\partial TC(r^{***})}{\partial c}=0$.

Case 2: In a similar way, by differentiating $TC(r)$ with respect to t, we have the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{\left(r^{***}-c\right)F-tF\frac{\partial r^{***}}{\partial t}}{4(r^{***}-c)^{2}} \tag{41}$$

After some manipulation, equation (41) reduces to the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{tF}{4(r^{***}-c)^2}\left(\frac{r^{***}-c}{t}-\frac{\partial r^{***}}{\partial t}\right) \tag{42}$$

Given that $\frac{\partial r^{***}}{\partial t}=\frac{\frac{1}{2}\left(\frac{r^{***}-c}{t}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}$ for $r^{**}>\frac{\theta}{3}$, , we know that equation (42) reduces to the following:

$$\frac{\partial TC\left(r^{***}\right)}{\partial t}=\frac{F}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{43}$$

This equation is positive, since $\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)>0$. When $r^{**}\leq\frac{\theta}{3}$, the optimal reference price satisfies $r^{***}=c+\frac{1}{2}\sqrt{tF}$, which implies that $\frac{\partial r^{***}}{\partial t}=\frac{1}{4}\sqrt{\frac{F}{t}}$ and $\frac{r^{***}-c}{t}=\frac{1}{2}\sqrt{\frac{F}{t}}$, hence $\frac{\partial N(r^{***})}{\partial F}>0$.

Case 3: Basically, the same case as for t, by substituting t with F and vice versa. Hence, when $r^{**}>\frac{\theta}{3}$ the following condition is satisfied:

$$\frac{\partial TC\left(r^{***}\right)}{\partial F}=\frac{t}{4\left(r^{***}-c\right)}\left(\frac{\frac{1}{2}-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}{1-\frac{1}{2}\left(\frac{r^{***}-c}{r^{***}}\right)\left(\frac{\theta}{r^{***}+\theta}\right)}\right)>0 \tag{44}$$

and whenever $r^{**}\leq\frac{\theta}{3}$ we have:

$$\frac{\partial N\left(r^{***}\right)}{\partial F}=\frac{tF}{4(r^{***}-c)^2}\left(\frac{r^{***}-c}{F}-\frac{\partial r^{***}}{\partial F}\right)>0$$

Case 4: For the last case, by differentiating $TC(r)$ with respect to θ, we have the following expression:

$$\frac{\partial TC\left(r^{***}\right)}{\partial\theta}=-\frac{tF}{4(r^{***}-c)^2}\frac{\partial r^{***}}{\partial\theta} \tag{45}$$

which directly implies the result, since we know that $\frac{\partial r^{***}}{\partial \theta} < 0$ whenever $r^{**} > \frac{\theta}{3}$, and $\frac{\partial r^{***}}{\partial \theta} = 0$ whenever $r^{**} \leq \frac{\theta}{3}$. This completes the proof.

Proof of Proposition 6

Proof. For the case of $r^{**} > \frac{\theta}{3}$, the social welfare function is not differentiable at r^{**}, which is the argmax of the policymaker problem. In this case, it is only possible to determine the lower and upper bounds of the variation of the value function through the right-hand and the left-hand side partial derivatives of the objective function evaluated at the optimal reference price. Otherwise, whenever $r^{**} \leq \frac{\theta}{3}$, the objective function is differentiable at the optimal reference price r^{**}, and a regular envelope theorem can be applied.

Case 1: By differentiating the right-hand side of the social welfare function $CS(r, c, t, F, \theta)$ with respect to c, we obtain:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{tF}{4(r-c)^2} \tag{46}$$

Since $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**} > \frac{\theta}{3}$, after evaluating $\frac{\partial CS(\cdot)}{\partial c}$, we have that:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{r^{**}+\theta}{4r^{**}} \tag{47}$$

Similarly, by differentiating the left-hand side of the social welfare function and evaluating at $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$, we obtain the following:

$$\frac{\partial CS\left(\cdot\right)}{\partial c} = -\frac{r^{**}+\theta}{r^{**}} \tag{48}$$

Hence, the variation of the social welfare function in the face of an increase in the marginal cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial c}$ must be in the interval $\left[-\frac{r^{**}+\theta}{r^{**}}, -\frac{r^{**}+\theta}{4r^{**}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the partial derivative of the value function satisfies the equation (34). Evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS\left(r^{***}, c, t, F, \theta\right)}{\partial c} = -1 \tag{49}$$

Case 2: By differentiating the right-hand side of the social welfare function $CS(r, c, t, F, \theta)$ with respect to t, we obtain

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{F}{4(r-c)} \tag{50}$$

Since $r^{**} = c + \sqrt{\frac{tFr^{**}}{r^{**}+\theta}}$ for $r^{**} > \frac{\theta}{3}$, after evaluating $\frac{\partial CS(\backslash cdotp)}{\partial t}$ atr^{**}, we have that:

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{51}$$

Similarly, by differentiating the left-hand side of the social welfare function, we obtain:

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} - \frac{1}{2}\sqrt{\frac{r}{r+\theta}}\sqrt{\frac{F}{t}} - \frac{1}{8}\sqrt{\frac{r+\theta}{r}}\sqrt{\frac{F}{t}} \tag{52}$$

After simplifying and evaluating at r^{**}, equation (52) reduces to the following:

$$\frac{\partial CS(\cdot)}{\partial t} = -\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{53}$$

Hence, the variation of the social welfare function in the face of an increase in the transportation cost $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial t}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the partial derivative of the value function satisfies equation (50). Evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS(r^{***}, c, t, F, \theta)}{\partial t} = -\frac{1}{2}\sqrt{\frac{F}{t}} \tag{54}$$

Case 3: The case of F is identical to the one of t. It suffices to exchange t with F and vice versa. Hence, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F}$ must be in the interval $\left[-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}, -\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right]$ whenever $r^{**} > \frac{\theta}{3}$, and $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial F} = -\frac{1}{2}\sqrt{\frac{t}{F}}$ whenever $r^{**} \leq \frac{\theta}{3}$.

Case 4: By right-hand side differentiating the social welfare function $CS(r, c, t, F, \theta)$ with respect to θ, we obtain:

$$\frac{\partial CS(\cdot)}{\partial \theta} = 0 \tag{55}$$

Similarly, by differentiating the left-hand side of the social welfare function with respect to θ, we obtain the following:

$$\frac{\partial CS(\cdot)}{\partial \theta} = \frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right) - \left(\frac{c+\sqrt{\frac{tFr}{r+\theta}}-r}{r}\right)$$

$$+\frac{1}{2}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r+\theta}\right) - \frac{1}{8}\sqrt{\frac{r}{r+\theta}}\left(\frac{\sqrt{tF}}{r}\right) \tag{56}$$

After simplifying and evaluating at r^{**}, equation (56) reduces to the following:

$$\frac{\partial CS(\cdot)}{\partial \theta} = \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right) > 0 \tag{57}$$

Hence, the variation of the social welfare function in the face of an increase in the parameter θ, $\frac{\partial CS(r^{***},c,t,F,\theta)}{\partial \theta}$ must be in the interval $\left[0, \frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta} - \frac{1}{4r^{**}}\right)\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the partial derivative of the value function satisfies equation (55). Evaluating this expression at the optimal reference price $r^{***} = c + \frac{1}{2}\sqrt{tF}$ implies that:

$$\frac{\partial CS(r^{***}, c, t, F, \theta)}{\partial \theta} = 0 \tag{58}$$

This completes the proof.

Proof of Corollary 2

Proof. The proof of this Corollary is essentially based on the proof of Proposition 6. Note that the difference in consumer welfare is defined as $\Delta CS\left(r,c,t,F,\theta\right) = CS\left(r,c,t,F,\theta\right) - \left(U - c - \frac{5}{4}\sqrt{tF}\right)$. Hence, the second part of this expression is simply the consumer welfare of a model with no reference prices, so that the function $\widehat{CS} = U - c - \frac{5}{4}\sqrt{tF}$ is independent of reference prices and their right-hand and the left-hand side partial derivatives coincide and are equal to their corresponding partial derivatives. Following the previous argument, it is easy to show that partial derivatives of the function $\widehat{CS}$ are equal to $\frac{\partial \widehat{CS}(\cdot)}{\partial c} = -1$, $\frac{\partial \widehat{CS}(\cdot)}{\partial t} = -\frac{5}{8}\sqrt{\frac{F}{t}}$, $\frac{\partial \widehat{CS}(\cdot)}{\partial F} = -\frac{5}{8}\sqrt{\frac{t}{F}}$ and $\frac{\partial \widehat{CS}(\cdot)}{\partial \theta} = 0$. Following the proof of Proposition 6 and the previous observation about partial derivatives of $\widehat{CS}$, we can establish the following cases.

Case 1: Assume that $r^{**} > \frac{\theta}{3}$, by differentiating the right-hand side of the difference in social welfare function $\Delta CS\left(r,c,t,F,\theta\right)$ with respect to c and evaluating it at r^{**}, we obtain:

$$\frac{\partial \Delta CS\left(\cdot\right)}{\partial c} = 1 - \frac{r^{**}+\theta}{4r^{**}} \tag{59}$$

Similarly, by the left-hand side differentiating the difference in social welfare function and evaluating it at r^{**}, we obtain the following:

$$\frac{\partial \Delta CS\left(\cdot\right)}{\partial c} = 1 - \frac{r^{**}+\theta}{r^{**}} \tag{60}$$

Hence, the variation of the difference in social welfare function in the face of an increase in the marginal cost must be in the interval $\left[1 - \frac{r^{**}+\theta}{r^{**}}, 1 - \frac{r^{**}+\theta}{4r^{**}}\right]$ whenever $r^{**} > \frac{\theta}{3}$. For the case in which $r^{**} \leq \frac{\theta}{3}$, the variation of $\Delta CS\left(r,c,t,F,\theta\right)$ is equal to:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial c} = 0 \tag{61}$$

Case 2: By differentiating the right-hand side of the function ΔCS $\left(r,c,t,F,\theta\right)$ with respect to t and evaluating it at r^{**}, we obtain the following:

$$\frac{\partial \Delta CS\left(\cdot\right)}{\partial t} = \frac{5}{8}\sqrt{\frac{F}{t}} - \frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{62}$$

Similarly, by differentiating the left-hand side of the social welfare function, we obtain:

$$\frac{\partial \Delta CS\left(\cdot\right)}{\partial t}=\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}} \tag{63}$$

Hence, the variation of ΔCS function in the face of an increase in transportation cost must be in the interval:

$$\left[\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}},\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{F}{t}}\right]$$

whenever $r^{**}>\frac{\theta}{3}$. For the case in which $r^{**}\leq\frac{\theta}{3}$ the partial derivative of ΔCS function is equal to the following:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial t}=\frac{5}{8}\sqrt{\frac{F}{t}}-\frac{1}{2}\sqrt{\frac{F}{t}} \tag{64}$$

Case 3: The case of F is very similar to the one of t. It is enough to interchange t with F and vice versa. Hence:

$$\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F}\in\left[\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{5}{8}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}},\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{1}{4}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{\frac{t}{F}}\right] \tag{65}$$

Otherwise, $\frac{\partial \Delta CS(r^{***},c,t,F,\theta)}{\partial F}=\frac{5}{8}\sqrt{\frac{t}{F}}-\frac{1}{2}\sqrt{\frac{t}{F}}$ whenever $r^{**}\leq\frac{\theta}{3}$.

Case 4: Since $\frac{\partial \widehat{CS}(\cdot)}{\partial\theta}=0$, it is clear that whenever $r^{**}>\frac{\theta}{3}$, the following holds:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial \theta}\in\left[0,\frac{1}{2}\sqrt{\frac{r^{**}+\theta}{r^{**}}}\sqrt{tF}\left(\frac{1}{r^{**}+\theta}-\frac{1}{4r^{**}}\right)\right] \tag{66}$$

For the case in which $r^{**}\leq\frac{\theta}{3}$, it is easy to show that:

$$\frac{\partial \Delta CS\left(r^{***},c,t,F,\theta\right)}{\partial \theta}=0 \tag{67}$$

This completes the proof.

Proof of Proposition 7

Proof. We know that at the optimal reference price r^{***} the difference in social welfare function satisfies the expression:

$$\Delta CS\left(r^{***}, c, t, F, \theta\right) = U - r^{***} - \tfrac{1}{4}\left(\tfrac{tF}{r^{***}-c}\right) - \left(U - c - \sqrt{tF} - \tfrac{1}{4}\sqrt{tF}\right) \tag{68}$$

We also know that the optimal reference price satisfies $r^{***} = r^{**}$ whenever $r^{**} > \frac{\theta}{3}$, otherwise $r^{***} = c + \frac{1}{2}\sqrt{tF}$. Hence, for the case when $r^{**} \leq \frac{\theta}{3}$ equation (68) reduces to $\Delta CS\left(r^{***}, c, t, F, \theta\right) = \frac{1}{4}\sqrt{tF} > 0$. When $r^{**} > \frac{\theta}{3}$, the optimal reference price satisfies $r^{***} = c + \sqrt{\frac{r^{***}}{r^{***}+\theta}}\sqrt{tF}$; hence, equation (68) reduces to $\Delta CS\left(r^{***}, c, t, F, \theta\right) = \left(\frac{5}{4} - \sqrt{\frac{r^{***}}{r^{***}+\theta}} - \frac{1}{4}\sqrt{\frac{r^{***}+\theta}{r^{***}}}\right)\sqrt{tF}$. It easy to show that the derivative of the function $f\left(r\right) = \sqrt{\frac{r}{r+\theta}} + \frac{1}{4}\sqrt{\frac{r+\theta}{r}}$ satisfies the following:

$$f'\left(r\right) = \frac{\theta}{2r}\sqrt{\frac{r}{r+\theta}}\left(\frac{1}{r+\theta} - \frac{1}{r}\right) \tag{69}$$

It is clear that $f'\left(r\right) > 0$ whenever $r^{**} > \frac{\theta}{3}$ and $\lim f\left(r\right) = \frac{5}{4}$ as $r \to \infty$, hence $\Delta CS\left(r^{***}, c, t, F, \theta\right) > 0$ whenever $r^{**} > \frac{\theta}{3}$. This completes the proof.