PUBLIC DEBT AND DEFICIT IN MEXICO: A COMMENT *

John H. Welch
Federal Reserve Bank of Dallas

Resumen: Este comentario muestra que el balance presupuestario intertemporal de México fue mantenido durante el periodo de 1981 a 1988. Este resultado se opone al obtenido por Feliz y Torres (1991).

Abstract: This comment shows that the Mexican intertemporal budget balance was maintained for the period 1981 to 1988. This result contrasts with the one obtained by Feliz and Torres (1991).

The question of government solvency has received a lot of attention in recent years especially with the large U.S. fiscal deficits and the fiscal distress suffered by Latin American governments since the onset of the "debt crisis" in the beginning of the 1980s. Feliz and Torres (1991) tests whether the dynamic government budget constraint of Mexico is binding for the period 1981 to 1988. They find that intertemporal budget balance was violated over this period. The methodology is based upon the stochastic characteristics of the components of the budget constraint. Briefly, if we assume interest rate parity and that, on average, the exchange rate follows the domestic rate of inflation, the dynamic government budget constraint is [1]

$$\frac{D_{t+1}}{P_{t+1}}=\frac{(G_t-T_t)}{P_{t+1}}+\frac{(1+\rho_t)(1+\pi_t^e)}{(1+\pi_t^e)}\frac{D_t}{P_t}-\frac{\Delta M_{t+1}}{P_{t+1}} \tag{1}$$

* I would like to thank Shengyi Guo for careful research assistance and Raúl A. Feliz for his comments and for providing me with his data. Of course, any remaining errors and omissions are the responsibility of the author. The views expressed in this article are those of the author and should not be attributed to the Federal Reserve Bank of Dallas or the Federal Reserve System.

[1] The model follows the discussion of Welch, Primo Braga and André (1987) and Trehan and Walsh (1988).

where D_t is (domestic plus foreign) government debt, M_t is monetary base, G_t is the totality of government spending, T_t is the totality of government non-borrowed revenues, π_t^e is the expected rate of inflation, and ρ_t is the average real rate of interest on government debt.

Let the real level of debt be d_t, the real primary government deficit be δ_t, and the real value of seignorage be $\sigma_{t+1} = \Delta M_{t+1} / P_{t+1}$. Equation (1) can be rewritten as

$$d_{t+1} = \delta_t + (1 + \rho_t) d_t - \sigma_{t+1} \quad . \tag{2}$$

Suppose the time series vector $X_t = [\delta_t, d_t, \sigma_{t+1}]$ is first difference stationary. By the Wold decomposition theorem, X_t can be represented

$$(1 - L)X_t = \lambda + C(L)\nu_t \, , \tag{3}$$

where $C(L)$ is a 3x3 matrix in the lag operator, λ is a drift term, and ν_t is a vector white noise process with $\nu_t = [\nu_{1,t}, \nu_{2,t}, \nu_{3,t}]$. We can form the inclusive of debt interest government deficit by multiplying X_t by the cointegrating vector $\beta' = [1, \rho, -1]$. This yields the following expression

$$(1 - L)\beta' X_t = \beta'\lambda + \beta' C(L)\nu_t \, . \tag{4}$$

One can use equation (4) to rationally forecast the value of future government debt. Substituting equation (4) into equation (2) and iterating forward, one finds the solution to the value of d_t. As Trehan and Walsh (1991) show, equation (4) implies that if intertemporal budgets are satisfied (no bubbles), real government debt will follow the following process [2]

$$(1 - L)d_{t+1} = \delta_t + \rho_t d_t - \sigma_{t+1} = \frac{\beta'\lambda}{\rho} + D(L)\nu_t \, , \tag{5}$$

where $D(L)\nu_t$ is stationary. Equation (5) implies that for dynamic budget balance to obtain, the primary deficit, the stock of internal debt, the stock of foreign debt, and seignorage are cointegrated with cointegrating vector $\beta' = [1, \rho_t, -1]$. Feliz and Torres (1991) choose to test for government insolvency by testing whether such cointegration exists. Unfortunately, this aproach suffers from two severe limitations. The first is that one must assume that the real interest rate on government debt is constant. Secondly, the test

[2] This particular solution assumes a constant real interest rate. The test below are based upon an extension by Trehan and Walsh (1991) to variable real interest rates.

necessitates the measurement of real seignorage which is problematic at best. I will discuss this issue further below.

A better approach is to perform an equivalent test which looks at the stationarity of the first difference of the real government debt which, from equation (5) is equal to the cointegration relationship between the primary deficit, the level of real debt, and real seignorage or, in other words, the deficit inclusive of interest payments.[3] If the first difference of real debt is stationary, budget balance holds. Table 1 shows the results of augmented Dickey-Fuller and Phillips-Perron tests of stationarity using Feliz and Torres' data on real government debt in Mexico. All tests significantly reject the null hypothesis of non-stationary first differences of real debt. Hence, the evidence strongly indicates that intertemporal budget balance was maintained over this period.

Table 1
Mexico: Tests of a Unit Root and Time Trend 1986:3-1990:2
Real Internal Government Debt

a) Null Hypothesis: Variable has a Unit Root (with time trend)		
Variable	*Phillips-Perron Test T-ratio*	*Augmented Dickey-Fuller Test [(a)] T-ratio*
Δ real government debt [(b)]	-14.03***	-13.73***
b) Null Hypothesis: Variable has Unit Root (with no time trend)		
Variable	*Phillips-Perron Test T-ratio*	*Augmented Dickey-Fuller Test [(a)] T-ratio*
Δ real government debt [(b)]	-13.73***	-13.73***

(a) One lag was used in these tests of stacionarity. The lag structure was chosen by adding lags until the Q(22) statistic did not reject the null hypothesis of autocorrelated residuals.

(b) Variable significantly violates normality assumption either because of skewness or kurtosis using the tests developed in Jarque and Bera (1980)

* signifies significance at the $\alpha = 0.10$ level, ** signifies significance at the $\alpha = 0.05$ level, and *** signifies significance at the $\alpha = 0.01$ level.

[3] Trehan and Walsh (1991) extend their results of (1988) to the case where the real interest rate on government debts is not constant, as in this case. Their results shows that if p_t is a stochastic process bound strictly below by $\lambda > 0$ in expected value and $(1-L)\,d_t$ is a stationary process, then intertemporal budget balance is satisfied. Real interest rates on internal government debt were positive in both Argentina and Brazil over the period.

What accounts for the discrepancy of between these findings and those of Feliz and Torres (1990)? As discussed in Welch, Primo Braga, and André (1987) and Cukierman (1988), simple discrete measurement of seignorage generates a biased measure of the real resource flow from money creation as money growth is more or less a continuous process. To see this, consider the continuous time amount of seignorage collected at time t

$$\hat{\sigma}_t = \frac{\dot{M}_t}{P_t} \tag{6}$$

where variables are defined as above and the dot represents an instantaneous time derivative. Note that

$$M_t = M_0 e^{\mu t} \tag{7}$$

where μ now represents the instantaneous rate of nominal money growth.[4] Hence

$$\dot{M}_t = \mu M_0 e^{\mu t}. \tag{8}$$

Equation 6 now becomes

$$\hat{\sigma}_t = \mu \frac{M_0}{P_0} e^{(\mu - \pi)t} \tag{9}$$

where π is the instantaneous rate of inflation.[5]

Integrating equation (9) from t to $t + 1$ yields

$$\sigma^*_{t+1} = \int_t^{t+1} \hat{\sigma}_t dt = \frac{\mu}{(\mu - \pi)} \frac{M_0}{P_0} e^{(\mu - \pi)t} [e^{(\mu - \pi)} - 1]$$

$$= \frac{M_t}{P_t} \left[\frac{\mu}{\mu - \pi} (e^{\mu - \pi} - 1) \right] \tag{10}$$

[4] The instantaneous rate of growth of money, μ, can be aproximated by $\ln(1+\mu^*)$ where μ^* is the discrete time rate of growth. Note that for the period of time selected, money growth is assumed constant.

[5] The instantaneous inflation rate can be approximated in a similar fashion as described in footnote 3.

On the other hand, discrete time measurement of seignorage gives

$$\sigma_{t+1} = \frac{M_t}{P_t}\left[\frac{M_{t+1}}{M_t} - 1\right] = \frac{M_t}{P_t}\left[\frac{M_0 e^{\mu(t+1)}}{M_0 e^{\mu t}} - 1\right] = \frac{M_t}{P_t}[e^{\mu} - 1]. \tag{11}$$

Subtracting equation (10) from (11) yields an expression for the bias inherent in Feliz and Torres' measure of seigniorage

$$\sigma_{t+1} - \sigma^*_{t+1} = \frac{M_t}{P_t}\left[\frac{\mu}{\mu - \pi}(e^{\mu - \pi} - 1) - e^{\mu} + 1\right]. \tag{12}$$

This non-linear bias becomes larger the larger the money growth rate and the larger the divergence of inflation from the money growth rate. A Taylor series expansion also shows that the bias can reverse sign depending upon the relative difference between inflation and money growth.

In summary, the conclusion of this comment is that somehow seignorage (and the other parts of the interest inclusive deficit) behaves to keep dynamic budget balance. This conclusion also holds for Argentina and Brazil, countries which have an historical inflation rate much higher than Mexico's and consequently have a smaller monetary base (Welch, 1991). The major problem in Latin American countries in a long term framework, however, remains to shift from monetary financed deficits to tax financed deficits. Fortunately, Mexico seems to have crossed this hurdle.

References

Cukierman, A. (1988). "Rapid Inflation - Deliberate Policy or Miscalculation?", *Carnegie-Rochester Conference Series on Public Policy*, vol. 29, pp. 11-75.

Dickey, D. A., and W. A. Fuller (1979). "Distribution of the Estimators of Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, vol. 74, pp. 427-431.

Engle, R. F., and C.W.J. Granger (1987). "Co-Integration and Error Correction: Representation, Estimation, and Testing", *Econometrica*, vol. 55, pp. 251-276.

Feliz, R.A., and F.A. Torres (1991). "Deuda y déficit público en México", *Estudios Económicos*, vol. 6, pp. 91-109.

Fuller, W.A. (1976). *Introduction to Statistical Time Series*, New York, John Wiley and Sons.

Granger, C.W.J., and P. Newbold (1986). *Forecasting Economic Time Series*, New York, Academic Press.

Hakkio, C., and M. Rush (1991). "Is the Budget Deficit 'Too Large'?", *Economic Inquiry*, vol. 29, pp. 429-445.

Jarque, C. M., and A. K. Bera (1980). "Efficient Tests For Normality, Homoscedasticity, and Serial Dependence of Regression Residuals", *Ecomonics Letters*, vol. 6, pp. 255-259.

Phillips, P.C.B., and P. Perron (1988). "Testing for a Unit Root in Time Series Regression", *Biometrica* vol. 75, pp. 335-346.

Trehan, B., and C.E. Walsh (1988). "Common Trends, the Government's Budget Constraint, and Revenue Smoothing", *Journal of Economic Dynamics and Control*, vol. 12, pp. 425-444.

______ (1991). "Testing Intertemporal Budget Constraints: Theory and Applications to U.S. Budget and Current Account Deficits", *Journal of Money, Credit, and Banking*, vol. 23, pp. 206-223.

Welch, J. H. (1991). "Rational Inflation and Real Internal Debt Bubbles in Argentina and Brazil?", Federal Reserve Bank of Dallas, Working Paper no. 9113 (August).

______, C.A. Primo Braga, and P.T.A. André (1987). "Brazilian Public Sector Disequilibrium", *World Development*, vol. 15, pp. 1045-1052.