

UNIONIZATION IN A DYNAMIC OLIGOPOLISTIC MODEL OF INTERNATIONAL TRADE

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Resumen: El estudio del comportamiento estratégico dinámico en situaciones de comercio internacional con mercados imperfectos de factores (sindicatos) da implicaciones de política muy diferentes al del caso estático. Encontramos que al contrario de éste, el equilibrio de nuestro modelo es a prueba de renegociaciones; la implementación unilateral de subsidios puede tener efectos negativos en el bienestar nacional; y los instrumentos de política comercial no son útiles para reacomodar rentas.

Abstract: The study of dynamic strategic behavior in international trade environments with imperfect factor markets (unions) yields significantly different policy implications compared to those that obtain under static settings. We find that contrary to static equilibria, the equilibrium of our model exhibits renegotiation-proofness; unilateral implementation of cost subsidies may yield negative domestic welfare effects; and trade policy tools are not useful in pursuing rent-shifting objectives.

1. Introduction

Unionized labor markets are a fact of life in most modern market economies. Hence, increasingly broad attention has been devoted in academic and policy

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circles to the connection between labor market structures and international trade patterns, particularly in the presence of imperfect competition (also a characteristic feature of modern economies, Helpman and Krugman, 1989). This connection has not been explored in the literature except for a limited number of papers. Brecher (1974) and Feenstra (1980) address implications of labor market imperfections on international trade policy in the absence of imperfect competition and union bargaining power. Prior to the innovative paper by Brander and Spencer (1988) the literature was devoid of theoretical work addressing policy and positive issues within an international setting exhibiting imperfect competition and imperfectly competitive labor markets.

The main predecessor to this paper is Brander and Spencer (1988). Brander and Spencer (1988) is a simple yet powerful one-stage (static) strategic model of international trade. Their model for an imperfectly competitive output market may be briefly summarized as follows. The international economy consists of two countries, each of which is populated by a firm and a union. There is only one input of production (labor) and technology is the same for both countries. Both firms produce a homogeneous good and compete for profits with each other in the integrated world economy where goods are freely tradeable but inputs are not mobile. Likewise, each firm individually faces a (domestic) union against which the firm bargains the actual wage rate paid to its workers. Equilibrium in their strategic model entails Cournot-Nash equilibrium of the firm-firm game and Generalized Nash Bargaining solution of the firm-union game in each country.

The main results obtained in their paper include: (1) trade policies (tariffs, quotas, subsidies) can be used to the national advantage as rent-shifting mechanisms, (2) the introduction of a union in one country causes output in the industry to fall and reduces profit for the unionized firm.

Ours is a multi-period strategic model of international trade with unionized labor markets. Labor is the only variable input used in the production of two non-storable consumption goods. Good n is produced in a competitive industry in the presence of competitive labor markets, while good m is produced in an imperfectly competitive industry with imperfect labor markets. Labor is immobile internationally but mobile domestically. The description of the game for the good m market is as follows: Each period, firms and unions in each country bargain with one another over the wage rate. The bargaining process is assumed to evolve according to Rubinstein's (1982) alternating offers scheme¹. Upon agreement, firms engage in production activities, facing competition from rival firms abroad. Production levels determine the distribution of labor input allocations across sectors in each country.

¹ The analysis necessitated the extension of Rubinstein's model to endogenous surplus size settings, studied in Asilis (1992).

Equilibrium of the dynamic game is characterized by the following features: *a*) in the presence of international inter-union and inter-firm side payments, only the firm located in the country where the strongest union is based produces output. More interestingly, in the absence of international side payments, firms (across countries) alternate over time in production. Such non-side payments environments exhibit the highly attractive feature that the dynamic equilibrium is not only subgame perfect but also renegotiation-proof;² *b*) contrary to the static model, uniqueness of (symmetric) equilibrium in the dynamic model is robust to the admission of a larger class of preferences and technology than that studied in the literature thus far.

Important policy implications emerge from this analysis. In particular, we show that, in sharp contrast with static settings, trade policies may not be used to the national advantage as rent-shifting mechanisms. Specifically, we show, for standard assumptions on international trade schemes, the non-existence of renegotiation-proof (or the larger class of subgame perfect) equilibria in which trade policies are chosen as rent-shifting mechanisms. Likewise, we also find that unilateral implementation of labor cost subsidies on domestic production may yield negative effects on domestic welfare.

The reversal of many of the results obtained in the static model arises as a result of the dynamic nature of the strategic interactions among firms and unions in the model. Formally, the incorporation of a multi-period set-up (specifically, as an infinite horizon game) yields as implication that the appropriate class of non-cooperative strategies made available to each agent should not be restricted to that associated with one shot Cournot-Nash outcomes (as preceding papers have done) but rather should also admit the richer class of collusive non-cooperative subgame perfect (and renegotiation-proof) equilibrium strategies. In other words, rather than rule out cooperation by adopting the static Cournot-Nash equilibrium solution, we should allow players (firms, unions and governments) the choice of strategies yielding tacitly collusive payoffs. The motivations for doing so are based in the well known result that non-cooperative games may yield cooperative equilibrium outcomes. Aumann (1959, 1961) proved that core outcomes are attainable as non-cooperative equilibrium points in repeated games. The insight behind this argument is well known and can be summed up as follows: if a market relationship is repeated *ad-infinitum* the members of such industry may obtain

² A subgame perfect equilibrium strategy profile is said to be renegotiation-proof if for no continuation subgame, equilibrium behavior dictates pareto dominated payoffs to all players. In practice, the renegotiation-proofness refinement imposes the condition that when deviations occur the punisher can do no better by discontinuing the punishment and "cooperating" again with the deviant player (Farrell and Maskin, 1989; Abreu and Pearce, 1989; and Asilis-Kahn-Mookherjee, 1991 for a unified treatment)

collusive outcomes even if they are not colluding in an explicit manner. The enforcement of the implicit agreement to collusive actions is supported by losses experienced by a deviant player in the future. The losses to the deviant player take place through the adoption of retaliatory actions taken by the remaining members of the industry vis-a-vis the deviant member.

There is a point of more general interest to note. Our results (including Appendix B) in effect extend Rubinstein's perfect equilibrium bargaining model to a class of multi-stage games with endogenous surplus size³ (where surplus size is linear in the surplus shares accruing to the agents firms and unions). Three main hypotheses of Rubinstein's (1982) complete information dynamic strategic alternating offers bargaining model are: (1) the exogenous size of the pie (surplus or value) over which bargaining takes place; (2) risk neutrality of preferences; (3) heterogeneity of agents according to the negotiating order to follow (that is to say, who has the first turn in making offers). The equilibrium implications of the second and third hypotheses are as follows. It may be argued, for example, that the third hypothesis is equivalent to the arbitrary imposition of a first mover advantage on one of the players. Any such advantage, however, becomes negligible as the length of the bargaining round goes to zero (i.e., continuous bargaining). The implications of the second hypothesis are, however, a little more subtle. If, as is the case in Rubinstein's model, the equilibrium strategies are pure, then a change in a player's risk aversion that preserves its preferences over certain outcomes has no effect on its opponent's payoff (Osborne, chapter 9 in Roth, 1985a). However, changes in risk aversion as those considered in Roth (1985b) do not preserve a player's preferences over certain outcomes and would thus have non-trivial (equilibrium) implications. The results of this paper contribute to the analysis of multi-stage bargaining games in which different players interact in succeeding stages of the game yielding surplus size endogenous. We find that for such games the equilibrium value share accruing to players positioned in earlier stages is a fraction of the standard Rubinstein solution.

The organization of the paper is as follows. Section 2 introduces the model. Section 3 describes the static game G . Section 4 is a description of the dynamic game G^∞ . Section 5.1 solves for the equilibrium of the static game while section 5.2 does the same for the dynamic game G^∞ . Section 6 studies some trade policy implications for the case of government strategic behavior in each country. Section 7 concludes the paper.

³ Asilis (1992) contains a formal extension of Rubinstein's model to endogenous surplus size settings.

2. The Model

The world economy consists of two domestic economies, each of which is populated by four types of agents: households, firms, a union and a government. Preferences, information, production and trading technologies are assumed to satisfy conditions made precise below.

2.1. Notation and Conventions

- i = Indexes countries from $\{1, 2\}$.
- β_i = Discount factor for country i 's consumers.
- β_f, β_u = Discount factors (for firms and unions, respectively).
- τ_f, τ_u = Rates of time preference (for firms and unions, respectively).
- $Q \equiv (Q^1, Q^2)$ where $Q \in \{\beta_f, \beta_u\}$.
- T = Proportional tax rate.
- n = Homogeneous, non-storable, tradeable consumption good produced by a perfectly competitive industry.
- m = Homogeneous, non-storable, tradeable consumption good produced by an international duopoly (one firm in each country).
- l = Labor input, sole variable factor in the production of both goods n and m .
- U = Per period utility function defined over n and m .
- W = Wage income per household per period.
- $\bar{\pi}$ = Profit income per household per period.
- P = Relative price of good m in terms of good n .
- $\bar{N}$ = Number of union members.

Convention: Superscripts are used for country indexes (except for the wage rate bargaining share g_i and the consumer's discount factor β_i) and subscripts for time and consumption goods indexes. For example, T_{mt}^i denotes the proportional excise tax imposed by country i 's government on good m consumption in period t .

2.2. Household Sector

Both economies' household sectors are assumed to be populated by a representative household endowed with preferences over consumption streams of two non-storable goods m and n . Thus, the behavior of the representative household corresponds to the choice of sequences of functions (n_t^i, m_t^i) so that $\{(n, m)_t^i\}_{t=0}^\infty$ is chosen so as to maximize the following standard additively separable utility functional:

$$\bar{U}^i[\{(n, m)_t^i\}_{t=0}^\infty] = \sum_{t=0}^{\infty} \beta_t^i U(n_t^i, m_t^i) \quad i = 1, 2 \quad (1)$$

where: *i*) $\beta_i \in (0, 1)$, $n \in \mathbf{R}_+$, $m \in \mathbf{R}_+$, $U: \mathbf{R}_+^2 \rightarrow \mathbf{R}$ strictly increasing and twice continuously differentiable in both arguments and strictly concave; *ii*) the constraint associated with the maximization of (1) corresponds to the restriction to sequences $\{(n, m)_t^i\}_{t=0}^\infty$ that satisfy, for a parametrically given sequence $\{(P, W, \bar{\pi}, T_m, T_n, T_w, T_\pi)_t^i\}_{t=0}^\infty$, a sequence of one period budget constraints to the household, namely,

$$P_t^i m_t^i (1 + T_{mt}^i) + n_t^i (1 + T_{nt}^i) = W_t^i (1 - T_{wt}^i) + \bar{\pi}_t^i (1 - T_{\pi t}^i), \quad i = 1, 2 \quad \text{all } t \in \mathbf{N}, \quad (2)$$

where each household is assumed to offer inelastically one unit of labor each period. Since the focus of this paper is the effect of unionization on international trade under the presence of an international oligopoly, both the inelastic labor supply assumption and the assumption on the absence of capital markets for the household, implicit in (2), are made purely for analytical simplicity.

Sufficient first order conditions for the household's maximization problem are, therefore, (2) above and

$$\frac{U_m(n, m)}{U_n(n, m)} = \frac{P(1 + T_m)}{(1 + T_n)}.$$

These two conditions yield the indirect utility function for the consumer $v(P, T_m, T_n, T_w, T_\pi, W, \bar{\pi})$. By Roy's identity and aggregating across the two countries we obtain the inverse demand function for good *m* which we denote by $P(m; \cdot)$.

2.3. Firms Sector

Two non-storable consumption goods are produced in the world economy; each country produces both goods. Furthermore, both goods are tradeable internationally. Labor is taken to be the sole variable factor used in production and is assumed to be immobile internationally. Furthermore, firms are hypothesized to choose actions so as to maximize their discounted sum of profits.

2.4. Market for Good *N*

Production of a homogeneous good *n* is assumed to exhibit constant returns to scale (CRS) where the marginal product of labor corresponds to $C^i > 0$. The market structure posited to this sector is that of perfect competition. As a result, the wage corresponds to

$$W_{nt}^i = C^i \quad i = 1, 2 \quad \text{all } t \quad (3)$$

2.5. Market for Good M

World demand for a homogeneous good m is met by the production of two firms (one in each country) with access to identical production technologies.

The incorporation of a multi-period set-up (specifically, an infinite horizon game) yields as implication that the appropriate class of non-cooperative strategies made available to each player should not be restricted to that associated with one-shot Cournot-Nash equilibrium outcomes but should also admit the richer class of collusive non-cooperative subgame perfect (and renegotiation-proof) equilibrium strategies.

A satisfactory description of the dynamic model must proceed, necessarily, in two parts. First, we need to describe the static game, and, secondly, the repeated game.

3. Static Game G

The static game entails two phases of strategic interaction: firm-union and firm-firm.⁴ First, a firm-union wage rate bargaining game takes place in each country, and secondly, a Cournot-Nash duopoly game is played internationally between two profit maximizing firms (one located in each country). Only after an agreement between each firm and its union is arrived at do the firms engage in Cournot-Nash competition.

Denoting by $\bar{N}$ the number of union members, the actions of the union in the wage rate bargaining game are taken so as to maximize total union income (taking into account the effect of their actions on employment in the continuation output game). Labor is mobile domestically across the n and m -good market sectors. Total union income is the sum of union wage income in the m -good industry $Wm(W)$ and union wage income in the n -good industry $C(\bar{N} - m(W))$ where, for simplicity, we assume the production technology is linear, namely, $m = f(L) = L$.

Firm i 's objective in the output game corresponds to the maximization of total profits

$$\{P(m^i(W^1, W^2) + m^{-i}) - W^i\}m^i(W^1, W^2)$$

taking parametrically the bargaining equilibrium wage rate pair (W^1, W^2) and the output level m^{-i} of the rival firm.

Formally, the description of the component game G is as follows: (1) The period begins and each firm offers a (wage rate) amount $W_f \in \mathbf{R}$ to the union

⁴ For purposes of clarity we postpone the introduction of the government as a strategic player to Section 6.

in compensation for services to be rendered. (2) The labor union chooses an action from {accept, reject} in response to the wage offer made by the firm. (3) If the union chooses to accept then the description of the game proceeds as stated in (4)-(6) below; otherwise, the union makes a counteroffer of W_u to the firm in the next subperiod and so on until an agreement is arrived at. That is to say, if we define a set $H_0 = \bigcup_{s \in \mathbb{N}} \{\{\text{accept, reject}\} \times \mathbf{R}\}^s$, where s indexes the period's bargaining rounds, the firm's strategy is given by a mapping $\Sigma : H_0 \rightarrow \{\text{accept, reject} \times \mathbf{R}\}$. Likewise, the strategy for the union corresponds to the mapping $\nu : H_0 \rightarrow \{\text{accept, reject} \times \mathbf{R}\}$. Thus, the alternation of offers and counteroffers corresponds to the bargaining scheme introduced in Rubinstein (1982). (4) Once an agreement is reached between the union and the firm, the firm chooses an amount $l_m \in \mathbf{R}$ of labor needed in production taking the wage rate W^i parametrically. Production, in order to take place, requires the attainment of wage rate agreement in both countries (so that if and when production occurs it does simultaneously across countries).⁵ (5) The world price of good m is given by the inverse demand function $P(m; \cdot)$ where m denotes world output of good m . (6) Payoffs to the firms and unions obtain.⁶

The payoffs to the players are given by the following expressions:

$$\begin{array}{ll}
 P(m^1 + m^2; \cdot)m^1 - W^1m^1 & \text{Country 1's firm} \\
 P(m^1 + m^2; \cdot)m^2 - W^2m^2 & \text{Country 2's firm} \\
 W^1m^1 + C^1(\bar{N} - m^1) & \text{Country 1's union} \\
 W^2m^2 + C^2(\bar{N} - m^2) & \text{Country 2's union}
 \end{array} \quad (4)$$

4. The Repeated Game $G^\infty(\beta_f, \beta_u)$

The infinitely repeated game consists of the infinite repetition of the static game G together with discount factors $\beta_f \equiv (\beta_f^1, \beta_f^2)$, $\beta_u \equiv (\beta_u^1, \beta_u^2)$. A complete description is included in Appendix A.

⁵ That is to say, production is non-existent in both countries along subgames for which wage agreement is reached at most in one country.

⁶ Note that given the deterministic nature of production, and the full-information, full-enforcement environment within which actions take place, once an agreement is reached between the union and the firm, moral hazard and incentive issues do not emerge.

5. Equilibrium

In this section our objective is twofold. First, as a benchmark case, we compute the equilibrium of the static game G . Secondly, we study the equilibrium of the dynamic game and compare its policy implications with those arising in the static game.

5.1. Game G Equilibrium

In computing the equilibrium of the static game one first notes that the sequence of actions chosen by firms and unions is such that production takes place only after an agreement is reached between unions and their firms as to the wage rates W^1 and W^2 . Therefore, computation of equilibrium proceeds in two stages. First, for a given wage rate pair (W^1, W^2) , agreed upon by the respective countries' unions and firms, the strategy corresponding to each country's m -good firm is given by the choice of m^i , given an m^{-i} chosen by its rival, so as to maximize $\pi(m^i, m^{-i}, W^i) = [P(m^1 + m^2; \cdot) - W^i]m^i$. Thus the solution concept used herein is that of Cournot-Nash equilibrium. The first order conditions for this problem correspond to the following expressions, where subscripts denote partial derivatives,

$$\pi_{m^i} = m^i P' + P - W^i = 0 \quad i = 1, 2 \quad (5)$$

where the second order conditions, in turn, correspond to:

$$\pi_{m^i m^i} = 2P' + m^i P'' < 0 \quad i = 1, 2 \quad (6)$$

As in Brander and Spencer (1988) we assume that a firm's own marginal revenue decreases as its rival's output increases, given by the following condition assumed to hold globally:

$$\pi_{m^i m^{-i}} = P' + m^i P'' < 0 \quad i = 1, 2 \quad (7)$$

Furthermore, we make the assumption that the Gale-Nikaido condition, as expressed below, holds globally, thus ensuring uniqueness of the output game equilibrium.

$$D = \pi_{m^1 m^1} \pi_{m^2 m^2} - \pi_{m^1 m^2} \pi_{m^2 m^1} > 0 \quad (8)$$

From (5) we can express the output m^i of each firm as a function of the wage rates W^i , W^{-i} , namely,

$$m^i = m^i(W^i, W^{-i}) \quad i = 1, 2 \quad (9)$$

Comparative static effects of changes in W^i , $i = 1, 2$, yield the results outlined below (as in Brander and Spencer, 1988):

$$\begin{aligned} m_{w^i}^i &= \frac{\pi_{m^{-i}m^{-i}}}{D} < 0 \\ m_{w^i}^{-i} &= -\frac{\pi_{m^{-i}m^i}}{D} > 0 \end{aligned} \quad i = 1, 2 \quad (10)$$

Thus, given an agreement wage rate pair (W^1, W^2) and standard assumptions on preferences, a unique Cournot-Nash equilibrium obtains. Thus, we are in a position to examine the bargaining equilibrium determination of the wage rate agreement pair. In doing so, we must begin by spelling out explicitly the object (surplus, value) bargained over by firms and unions.

The object over which the union-firm bargaining process is assumed to take place corresponds to the firm's gross profits π_c^i (same as revenue in this single variable factor setting) since that is the measure of rents generated from the interaction of both parties in the game. Specifically,

$$\pi_c^i = P [m^i(W^i, W^{-i}) + m^{-i}(W^i, W^{-i})] m^i(W^i, W^{-i}) \quad i = 1, 2 \quad (11)$$

In what follows, we denote by g_i the i th firm's offer to the union it faces as a fraction of π_c^i , where $g_i \in [0, 1]$, $i = 1, 2$ (so that $g_i \pi_c^i$ corresponds to the total wage bill offered by the firm). g_i' denotes the fraction of gross profits accruing to the union when it is its turn to make a counteroffer to the firm, so that in a perfect equilibrium of the period t game the following conditions must hold⁷ at every bargaining stage (or subperiod) s , (where Z denotes the length of the bargaining round). For the firm in each country, for $i = 1, 2$,

$$\begin{aligned} &[1 - g_i(t + sZ)] \pi_c^i(g_i(t + sZ)) \\ &= \frac{[1 - g_i'(t + (s + 1)Z)] \pi_c^i(g_i'(t + (s + 1)Z))}{1 + \tau_f^i Z} \quad \text{for } t, s \in \mathbf{N} \end{aligned} \quad (12)$$

For the union in each country for $t, s \in \mathbf{N}$,

⁷ This is so provided that the largest and smallest equilibrium payoffs coincide (see e.g. Sutton, 1986). The reader may verify that the approximate linearity assumption on gross profits used in our model is sufficient to ensure the latter mentioned equivalence.

$$\begin{aligned}
& g_i'(t + (s + 1)Z)]\pi_c^i(g_i'(t + (s + 1)Z)) \\
& = \max\left\{ \frac{g_i(t + (s + 2)Z)]\pi_c^i(g_i(t + (s + 2)Z))}{1 + \tau_u^i Z}, C^i \right\} \quad (13)
\end{aligned}$$

(12) states that in a perfect equilibrium, the i th firm is indifferent between making an offer in (the bargaining) period $t + sZ$ (translating into a payoff of $[1 - g_i(t + sZ)]\pi_c^i(g_i(t + sZ))$ to itself) and waiting one bargaining period to be made an offer by the i th union (in period $t + (s + 1)Z$) translating into a discounted payoff of

$$\frac{1 - g_i'(t + (s + 1)Z)]\pi_c^i(g_i'(t + (s + 1)Z))}{1 + \tau_f^i Z}$$

to the i th firm. The unique equilibrium to the period t bargaining process is given by the stationary equilibrium of the system of difference equations (12) and (13) above. We show in Appendix B that in equilibrium the following holds for scalars $K_1, K_2, \dots$

$$g_i^* = \frac{(1 + \tau_u^i Z)[\tau_f^i - K_1 Z - K_2 Z^2 - K_3 Z^3 - \dots]}{(\tau_f^i + \tau_u^i + \tau_f^i \tau_u^i Z)} \quad i = 1, 2 \quad (14)$$

Therefore, agreement is reached every period, without delay. Furthermore, as $Z \rightarrow 0^+$ (continuous bargaining), (14) becomes

$$g_i^* = \frac{\tau_f^i}{2(\tau_f^i + \tau_u^i)} \quad i = 1, 2 \quad (15)$$

By definition of π_c^i we have that union-firm agreement is reached for wage rates $W^i = g_i^*P$ which correspond to the following expressions

$$\left(\frac{1}{2}\right) \left(\frac{\tau_f^1}{\tau_f^1 + \tau_u^1}\right) P(m^1(W^1, W^2), m^2(W^1, W^2); \cdot) = W^1 \quad (16)$$

$$\left(\frac{1}{2}\right) \left(\frac{\tau_f^2}{\tau_f^2 + \tau_u^2}\right) P(m^1(W^1, W^2), m^2(W^1, W^2); \cdot) = W^2 \quad (17)$$

Combining (16) and (17) above, we have that equilibrium of the static game G is characterized by wage rates W^i satisfying the following proportionality relationship:

$$\frac{W^1}{W^2} = \left(\frac{\tau_f^1}{\tau_f^2} \right) \left(\frac{\tau_f^2 + \tau_u^2}{\tau_f^1 + \tau_u^1} \right) = \frac{1 + \tau_u^2/\tau_f^2}{1 + \tau_u^1/\tau_f^1}. \quad (18)$$

The above result is summarized in the following proposition:

PROPOSITION 1. *The equilibrium of the static game⁸ G is characterized by wage rates W^1 and W^2 exhibiting a proportionality relationship between them, where the constant of proportionality is such that W^1 is greater (less) than W^2 if τ_u^2/τ_f^2 is greater (less) than τ_u^1/τ_f^1 , respectively.*

Thus far, we have examined the firm-union interaction within each country and the firm-firm Cournot-Nash output game played internationally. We conclude this section by determining the unique equilibrium wage rate pair (W^1, W^2) , through the restatement of the strategic behavior of each firm in terms of equilibrium wage rates W^i imposing the equilibrium condition for firm-union bargaining, namely that the wage rates correspond to a fraction g^i of the output price P .

Formally, the problem is given by expression (19) below, for $i = 1, 2$,

$$\Psi(W^{-i}) = (1 - g^i)P[m^i(W^1, W^2) + m^{-i}(W^1, W^2)]m^i(W^1, W^2). \quad (19)$$

Noting that $W = gP$, sufficient first order conditions for this problem, where subscripts denote partial derivatives, become

$$\Psi_{w^i}(W^{-i}) = P'(\cdot)[m_{w^i}^i(\cdot) + m_{w^i}^{-i}(\cdot)]m^i(\cdot) - m^i(\cdot) + [P(\cdot) - W^i]m_{w^i}^i(\cdot) = 0. \quad (20)$$

From (20) we obtain the reaction functions $W^i(W^{-i})$ for $i = 1, 2$.

Furthermore, the following conditions are assumed to hold globally so as to ensure uniqueness of equilibrium

$$\Psi_{w^i w^i}(W^{-i}) < 0 \quad i = 1, 2. \quad (21)$$

At this stage, we summarize the previous results in the following proposition:

PROPOSITION 2. *The static game G has a unique equilibrium characterized by an 8-tuple $(W^1, W^2, \tau_u^1, \tau_u^2, \tau_f^1, \tau_f^2, m^1(W^1, W^2), m^2(W^1, W^2)) \in \mathbf{R}^2 \times [0, \infty]^4 \times (\mathbf{R}^2 \rightarrow \mathbf{R})^2$ satisfying the conditions given below:*

⁸ Whose proof of existence and uniqueness is yet to be completed.

i) (W^{1*}, W^{2*}) uniquely solves the pair of equations given in (20).

ii) $g_i^* = \frac{\tau_f^i}{2(\tau_f^i + \tau_u^i)}$, $i = 1, 2$, define the unique subgame perfect equilibrium of the bargaining game played between the union and the m -good producing firm in each country as specified in equations (12) and (13).

iii) For a given wage rate pair (W^1, W^2) , $m^1(W^1, W^2)$ and $m^2(W^1, W^2)$ uniquely define the output levels for each m -good producing industry as specified in equation (5) where the equilibrium pair denoted above defines the equilibrium payoffs to the players in the following manner:

Firm i 's payoff: $\pi^i(m^i(W^{i*}, W^{-i*}), m^{-i}(W^{i*}, W^{-i*}))$,

i union's payoff: $W^{i*}m^i(W^{i*}, W^{-i*}) + C^i[\bar{N} - m^i(W^{i*}, W^{-i*})]$.

5.2. Game $G^\infty(\beta_f, \beta_u)$ Equilibrium

This section studies equilibria of the full-information infinitely repeated game with discounting defined as the infinite repetition of the static game analyzed above.

A principal objective is to establish if any policy implication of the static game equilibrium (to be simultaneously derived with the dynamic policy results) is altered in a meaningful way once one allows for dynamic strategic behavior.

By far the best known class of subgame perfect non-cooperative equilibria of oligopolistic supergames is that supported by trigger strategies (Friedman, 1971). This class of games, however, is plagued by a multiplicity feature, oftentimes exhibiting a continuum of them. Furthermore, collusive subgame perfect trigger strategy equilibria do not always correspond to extremal equilibria yielding maximal degree of collusive behavior outcomes (Abreu, 1983, 1986, 1988). The reason for this latter result hinges on the concept of optimal punishments. In general, the Cournot-Nash reversionary punishment strategies supporting Friedman's trigger strategy equilibrium payoffs are not optimal in the sense that there may exist alternative punishment profiles yielding lower payoffs to possible deviants, and thus supporting more collusive outcomes.

Avoidance of extrinsic asymmetric features to the model leads us to restrict our attention to games associated with symmetric equilibria. As a result, we focus our analysis on subgame perfect symmetric⁹ equilibria (and

⁹ Asymmetric equilibrium extensions of the model would yield outcomes differing from those of symmetric equilibria only in that the players' payoff weights would no longer be equal (one-half each) but rather would be a function of the players' rates of time preference.

also renegotiation-proof equilibria) of G^∞ . The analysis yields the desired features of uniqueness and optimality (i.e., maximal collusive outcomes) of equilibrium. In what follows, we denote by σ^{c*} the reward strategy profile (carrot) prescribed by tacitly collusive behavior while σ^{p*} denotes the punishment strategy profile (stick) prescribed by occurrences of unilateral deviation. Specifically, σ^{c*} entails the play of perfect joint (world) monopoly output levels (yielding a symmetric period payoff of $\pi(\bar{m}^*)$ to each firm) while σ^{p*} entails the play of Cournot-Nash output levels (yielding a period payoff of $\pi(\sigma^{N*})$ to each firm).¹⁰

Lemma 1 provides sufficient conditions to ensure that the multiplicity of equilibria feature of the static model resulting from the admission of a larger class of preferences and technology than that studied thus far does not extend to the dynamic case where uniqueness of optimal symmetric equilibrium obtains.

LEMMA 1. *Given $(\pi(\bar{m}^*), \pi(\sigma^{N*}))$, there exists a $\delta^* \in (0, 1)$ such that for $\delta^i \in (\delta^*, 1)$, $\delta^i \in \{\beta_u^i, \beta_f^i\}$, $i = 1, 2$, $(\sigma^{c*}, \sigma^{p*})$ supports maximal collusive subgame perfect equilibrium payoffs*

$$\Lambda^i(\sigma^{c*}) = \frac{\pi(\bar{m}^*)}{1 - \delta^i}$$

through reversions to Cournot-Nash play σ^{N} yielding continuation payoffs*

$$\Lambda^i(\sigma^{p*}) = \sum_{\tau=t}^{\infty} (\delta^i)^{(\tau-t)} \pi(\sigma^{N*}(\tau)) = \frac{\pi(\sigma^{N*})}{1 - \delta^i} \quad t \in \mathbb{N}$$

for a given sequence $\{\sigma^{N}(\tau)\}_{\tau=t}^{\infty}$ of static game G Nash equilibria (where $\sigma^{N*}(\tau) = \sigma^{N*}$ for all t by results in Section 5.1.).*

Lemma 1 leads to Proposition 3.

PROPOSITION 3. *Given that:*

i) $2\pi(\bar{m}^*)$ corresponds to the net payoffs extracted by a "multi-plant" type (world) monopolist facing two, generally distinct, cost structures g^i indexed by $i \in \{1, 2\}$;

ii) Lemma 1 shows that for $\delta^i \in (\delta^*, 1)$, $i \in \{1, 2\}$, the strategy profile $(\sigma^{c*}, \sigma^{p*})$ supports maximal collusive subgame perfect equilibrium payoffs in which each firm receives net payoffs amounting to $\pi(\bar{m}^*)$, it follows that:

¹⁰ From Section 5.1., $\pi(\sigma^{N*})$ is unique.

- a) the maximal collusive subgame perfect equilibrium supported by (σ^c, σ^p) entails side payments flowing from the low g -type firm to the high g -type firm;
 b) $\bar{m}^* = (\bar{m}^1, \bar{m}^2)$ solves

$$\max_{(m^1, m^2)} P(m^1 + m^2; \cdot) (m^1 + m^2) - W^1 m^1 - W^2 m^2 \quad (22)$$

where W^1 and W^2 correspond to the agreed upon amounts between the union and firm in country one and country two, respectively, determined before the employment decision $m \equiv (m^1, m^2)$ takes place.

The solution to (22) prescribes the following taxonomy of actions:

$$m^i = \begin{cases} \bar{m}^* & \text{if } W^i < W^{-i} \\ [0, \bar{m}^*] & \text{if } W^i = W^{-i} \\ 0 & \text{if } W^i > W^{-i} \end{cases} \quad i = 1, 2$$

where $\bar{m}^*$ solves

$$\max_m P(m; \cdot) m - \min\{W^1, W^2\} m. \quad (23)$$

The analysis leads to Propositions 4 and 5.

PROPOSITION 4. *If union behavior is hypothesized to follow Bertrand strategies then the maximal collusive subgame perfect equilibrium supported by the strategy profile (σ^c, σ^p) entails period payoffs $g^i m^i P(\bar{m}^*)$ to the unions, where:*

- i) the union endowed with the lower reservation wage C^i (perhaps reflecting a less developed sector, region or country) captures the totality of labor services demanded by the m -good world industry;
- ii) the surplus or rent extracted by the low reservation wage union i corresponds to the difference $|C^1 - C^2|$.

Since the structure of the game is common knowledge to all players, the unions know the output market game decision problem of the industry is prescribed by the solution to (22). A natural question to ask becomes: can both unions, by tacitly colluding, attain strictly preferred net payoffs to the pair $(0, |C^1 - C^2| \bar{m}^*)$? If the answer to the previous question is affirmative, the immediate question becomes: Is the division of payoffs across unions uniquely determined? The next proposition addresses those two issues.

PROPOSITION 5. *If union behavior is hypothesized to admit the class of collusive non-cooperative strategies and $\beta_u \in (\beta_u^*, 1)$, then the maximal collusive subgame*

perfect equilibrium (with side payments) supported by the strategy profile (σ^*, σ^{p*}) for $\delta^i \in (\delta^*, 1)$ entails period payoffs $\frac{P(\bar{m}^*)\bar{m}^* \max\{g^1, g^2\}}{2}$ to the unions in both countries, where:

i) $\frac{P(\bar{m}^*)\bar{m}^* \max\{g^1, g^2\}}{2}$ corresponds to the maximum symmetric (period) payoff supported by reversion to $(0, |C^1 - C^2|\bar{m}^*)$ provided $\beta_u \in (\beta_u^*, 1)$.

ii) Subgame perfect equilibrium prescribes that only the high g -type union provides labor services to the industry.

iii) From ii), equilibrium behavior prescribes production only by the firm located in the country where the high g -type union resides. Consequently, such firm becomes a (world) monopoly.

iv) Equilibrium behavior entails side payments amounting to one half of total revenues flowing from the high g -type union to the low g -type union.

PROOF. The proof follows from a straightforward application of Lemma 1 where:

a) $\pi(\bar{m}^*)$ corresponds to $\frac{g^* P(\bar{m}^*)\bar{m}^*}{2}$ where $g^* = \max\{g^1, g^2\}$;

b) $\pi(\sigma^{N*})$ becomes $|C^1 - C^2|\bar{m}^*$. It is assumed here that C^1 and C^2 are such that $|C^1 - C^2| < \frac{g^* P(\bar{m}^*)}{2}$. As before (see Lemma 1), we restrict $\Omega \in (\Omega^*, 1)$ where $\Omega^* \equiv \max\{\Omega^1(\alpha), \Omega^2(\alpha)\}$, $\alpha = \frac{g^* P(\bar{m}^*)\bar{m}^*}{2} - |C^1 - C^2|\bar{m}^*$ and

$$\Omega^i(\alpha) = \frac{\frac{g^* P(\bar{m}^*)\bar{m}^*}{2}}{\frac{g^* P(\bar{m}^*)\bar{m}^*}{2} + \alpha} . \quad \blacksquare$$

6. Trade Policy with Government Strategic Behavior

The incorporation of government strategic behavior in each country implies the modification of the static game G and the dynamic game G^∞ . In particular, we assume:

i) The strategy space available to governments corresponds to the class of proportional expenditure and income taxes, *ad-valorem* and specific tariffs, quotas and proportional cost subsidies;

ii) The timing of the government's actions within the static game is taken to be the following: a) the announcement and collection of tax revenues take place simultaneously (across countries) and at the end of the period; b) the announcement of trade policies is naturally assumed to take place before the production activity (and the determination of the wage).

The adjusted static game G induces, in a straightforward manner, corresponding redefinitions of histories and strategies in the repeated game.

Proposition 6 demonstrates that the adoption of rent-shifting trade policies does not occur in the dynamic equilibrium. This result hinges on the fact that upon the adoption of rent-shifting policies by any one country's government, the countries' behavior is credibly switched to the perennial punishment phase (Nash behavior forever after) regardless of the trade policy tool used (quota or tariff). Proposition 6 shows this formally.

PROPOSITION 6. *The extended (quantity) strategy profile $(\sigma^*, \sigma^*)^s$ supports maximal collusive subgame perfect equilibrium payoffs $\Lambda^i(\sigma^*, \sigma^*) = \Lambda^i(\sigma^*)$. That is to say, rent-shifting oriented government actions (such as, for example, tariffs and quotas) do not occur in equilibrium.*

PROOF. Since $(\sigma^*, \sigma^*)^s$ is defined as (σ^*, σ^*) plus the prescription of firm-union reversion to σ^* if their rival's government adopts rent-shifting trade policy tools, it follows that, for each country:

$$\text{Joint firm-union surplus} = \begin{cases} \frac{P(\bar{m}^*)\bar{m}^*}{2} \left[\frac{g_i^*}{1 - \beta_u^i} + \frac{1 - g_i^*}{1 - \beta_f^i} \right] & \text{if no government pursues} \\ & \text{rent-shifting objectives} \\ \pi(\sigma^{N*}) \left[\frac{g_i^*}{1 - \beta_u^i} + \frac{1 - g_i^*}{1 - \beta_f^i} \right] + \max_{\{m^i\}} \frac{P(\bar{m}^* - i^* + m^i)m^i}{2} & \text{otherwise} \end{cases}$$

Since $\pi(\bar{m}^*) > \pi(\sigma^{N*})$, the result follows. ■

Perennial reversions to Nash equilibrium is obviously not renegotiation proof as these are inferior or Pareto dominated strategies from a collective rationality perspective. In other words, following a trade policy action by any one government, unions and firms may make themselves better off by letting "by gones be by gones" and switching to a profile different from Nash forever. Proposition 7 describes a simple renegotiation proof equilibrium strategy profile for the game. In particular, following a unilateral government deviation, the strategy profile entails the one period reversion¹¹ to a profile prescribing production to be carried out only by the firm located in the country with the non deviant government.

¹¹ In general, reversions may be multi period depending on the relative discount factors of unions and firms.

PROPOSITION 7. *The strategy profile*

$$(m^1, m^2) = \begin{cases} (\bar{m}^1, \bar{m}^2) & \text{at } t \text{ if at } t-1: (\bar{m}^1, \bar{m}^2); \text{ or if } (\bar{m}^1, 0) \text{ or } (0, \bar{m}^2) \text{ and} \\ & (m^i \neq \bar{m}^i \text{ and } m^{-i} = \bar{m}^{-i}) \text{ for } i = 2 \text{ (1, respectively)} \\ & \text{at } t-2. \\ (\bar{m}^1, 0) \text{ or } (0, \bar{m}^2) & \text{at } t \text{ if at } t-1: m^2 \neq \bar{m}^2 \text{ and } m^1 = \bar{m}^1 \\ & (m^1 \neq \bar{m}^1 \text{ and } m^2 = \bar{m}^2, \text{ respectively}). \\ (\bar{m}^1, \bar{m}^2) & \text{otherwise (such as histories with multilateral deviations).} \end{cases}$$

is renegotiation proof.

PROOF. Suppose a (unilateral) deviation takes place whereby country 1's government adopts a trade policy tool where $m^1 \neq \bar{m}^1$ while $m^2 = \bar{m}^2$. The strategy profile given above implies country 1's union firm joint continuation discounted sum of payoffs corresponds to

$$\bar{\Lambda}^1 = \frac{P(\bar{m}^*)\bar{m}^*}{2} \left[\frac{g_1^* \beta_u^1}{1 - \beta_u^1} + \frac{(1 - g_1^*) \beta_f^1}{1 - \beta_f^1} \right]$$

while country 2's union firm continuation discounted sum of payoffs corresponds to

$$\bar{\Lambda}^2 = P(\bar{m}^*)\bar{m}^* + \frac{P(\bar{m}^*)\bar{m}^*}{2} \left[\frac{g_2^* \beta_u^2}{1 - \beta_u^2} + \frac{(1 - g_2^*) \beta_f^2}{1 - \beta_f^2} \right].$$

Country 1's union and firm are made worse off by their government's actions as

$$\pi(\text{deviating}) - \frac{P(\bar{m}^*)\bar{m}^*}{2} < \frac{P(\bar{m}^*)\bar{m}^*}{2} [g_1^* \beta_u^1 + (1 - g_1^*) \beta_f^1]$$

as $\pi(\text{deviating})$ is bounded from above by $P(\bar{m}^*)\bar{m}^*$. Moreover,

$$\bar{\Lambda}^2 > \frac{P(\bar{m}^*)\bar{m}^*}{2} \left[\frac{g_2^*}{1 - \beta_u^2} + \frac{(1 - g_2^*)}{1 - \beta_f^2} \right].$$

Therefore, the profile is individually rational for country 1 and country 2, respectively. Moreover, it is renegotiation-proof as the continuation values $\bar{\Lambda}^1 + \bar{\Lambda}^2$ lie on the Pareto frontier of the reduced game. ■

The $(\sigma^c, \sigma^p)^g$ profile of Proposition 6 does not admit interactions between the government and its union or firm. If we allow for government-

(union or firm) interactions, Proposition 8 shows that $(\sigma^*, \sigma^{p*})^g$ continues to be the (subgame perfect) equilibrium of the general game. That is to say,

PROPOSITION 8. *There exists no subgame perfect equilibrium strategy profile exhibiting government (union or firm) interactions which can credibly break the international non cooperative collusive behavior across countries among firms and unions (Proposition 6).*

PROOF. By contradiction. Suppose there in fact exists a subgame perfect equilibrium strategy profile exhibiting government (union or firm) interaction which credibly breaks the $(\sigma^*, \sigma^{p*})^g$ profile (Proposition 6) of international non cooperative collusive behavior among firms and unions. The latter implies that once that government engages in trade policy actions, it does so credibly. That is to say, the local firm and/or union breaks its interaction with its foreign counterparts. As a result, were the government to discontinue its cooperation (through protective trade policy tools), the firm would face Nash behavior for all subsequent periods. However, when the future arrives, such discontinuation of cooperation is preferred by the government to continued tariffs or quota protection. As a result, the alternative profile is not subgame perfect. Contradiction. ■

Corollary 1 offers an even stronger support to the result that in equilibrium unilateral trade policy actions do not occur. Thus, Propositions 8 and Corollary 1 afford theoretical basis for the current wave of international cooperative economic schemes.

COROLLARY 1. *There exists no renegotiation-proof equilibrium strategy profile involving government-(union or firm) interactions which can credibly break the international non-cooperative collusive behavior among firms and unions (Proposition 7).*

PROOF. Noting that renegotiation-proofness is a refinement of subgame perfection, Proposition 8 delivers the result. ■

Finally, we examine the welfare implications arising from the implementation of non-negative cost subsidies to home production. The static game welfare objective posited to the government corresponds to that introduced in the previous description of the modified static game G . The reader may check that by assuming the existence of lump sum taxes t^i and normalizing the social marginal utility of income to one, the differential of domestic welfare ω can be expressed by (where m^d denotes m -good demand)

$$d\omega^i = -m^{di}dP^i + d\pi^i - dt^i + d(\text{wage income})^i. \quad (24)$$

Since the dynamic welfare objective of the government is posited to take an additively separable form (standard in economic analysis), namely,

$$D\omega^i = \sum_{t=0}^{\infty} \beta_t^i \omega^i[v^i(\cdot)_t]$$

it follows that for stationary dynamic equilibrium

$$d(D\omega^i) = \left(\frac{1}{1 - \beta_i} \right) d\omega^i \quad (25)$$

Thus, the sign of $d(D\omega^i)$ is the same as the sign of $d\omega^i$. Since $t^i = S^i m^i$, it follows that $dt^i = S^i dm^i + m^i dS^i$. By preceding Propositions, it follows that $d\pi^i + d(\text{wage income})^i < dt^i$ as any change in payoffs to firms and unions is "shared" internationally. Note that in the static game, this latter mentioned implication does not obtain. Moreover, as t^i (lump sum) is a lower bound for the cost of a subsidy, we have, therefore, that $d(D\omega^i) / dS^i < 0$.

The previous results are summarized in Proposition 10 below.

PROPOSITION 10. *The optimal cost subsidy in the maximal collusive subgame perfect equilibrium is zero.*

The above result differs rather sharply from those obtained in static settings, most notably Brander and Spencer (1988) where the optimal cost subsidy is positive.

7. Conclusion

The construction of a dynamic strategic model of international trade with unionized labor markets is shown to yield significantly different equilibrium outcomes and policy implications compared to those that obtain under static settings.

The equilibrium of the dynamic game is characterized by the following features: *a)* in the presence of international side payments among unions and among firms, only the firm located in the country where the strongest union is based produces output. More interestingly, in the absence of international side payments, firms (across countries) alternate over time in production. Such non-side payments settings exhibit the highly attractive feature that the dynamic equilibrium is not only subgame perfect but is also renegotiation-proof; *b)* contrary to the static model, uniqueness of (symmetric) equilibrium in the dynamic model is robust to the admission of a larger class of preferences and technology than that studied in the literature thus far.

At first reading, the features of the dynamic equilibrium (for example, the alternation in production in the renegotiation-proof equilibrium) would seem not to conform well with observed behavior. That is to say, rarely does one observe firms producing zero output in some periods and positive output levels in others. A proper reading of the equilibrium results requires the calibration of the model's simplifying assumptions to realistic characteristics of international trade. As a simplifying assumption, firms are assumed in the model to produce a single homogeneous product. Extending the model to more realistic environments in which firms enter into product lines production activities would yield (alternating production) renegotiation-proof equilibria in which *all* firms produce in *all* periods (as the alternation schemes would apply to elements of the firms' product lines).

Important policy implications emerge from this analysis for full information environments. In particular, we show that, in sharp contrast with static environments, trade policies may not be used to the national advantage as rent-shifting mechanisms. Specifically, for standard assumptions on international trade schemes, we show the non-existence of subgame perfect (and, consequently, of renegotiation-proof) equilibria in which trade policies are chosen as rent-shifting mechanisms. Likewise, we also find that unilateral implementation of labor cost subsidies on domestic production may yield negative effects on domestic welfare.

In summary, we have constructed a dynamic full information strategic model of international trade yielding a unique optimal subgame perfect and renegotiation-proof equilibrium. In doing so we showed the significant sensitivity of results obtained in models of static full information environments with respect to the avoidance of dynamic considerations. As a result, this paper suggests that full information strategic trade models are inadequate to study rent-shifting considerations of trade policy. Finally, the applicability of the framework transcends the specific issues addressed in this paper and may prove useful in other analyses of international policy matters in dynamic contexts.

Appendix A

Description of the Repeated Game G^∞

First, we describe the strategies for the firms, and, secondly, the strategies for the labor unions.

Let T_t^{*i} denote period t 's bargaining round in which wage bargaining agreement is reached between country i 's union and firm. Furthermore, let us denote the history of actions associated with period t 's bargaining game by $a_t \equiv (a_t^1, a_t^2)$, where

$$a_t^i = \{(l_{m,(t,s)}^i, W_{f,(t,s)}^i, W_{u,(t,s+1)}^i)\}_{s=0}^{T-t} \quad i = 1, 2 \quad (\text{A.1})$$

A history to period t is therefore a sequence of actions $\{a_T\}_{T=0}^t$. Let H denote the set of all such histories, namely,

$$H = \bigcup_{t \in \mathbb{N}} [(\mathbb{R}^3)^{2(N-|\infty|)}]^t \quad (\text{A.2})$$

A strategy Σ^i for country i 's firm specifies either acceptance or rejection and counteroffer made to the union together with a choice of labor units, all as a function of the history of actions $\{a_T\}_{T=0}^t$. That is to say,

$$\Sigma^i: H \rightarrow \{\text{accept, reject} \times \mathbb{R}\} \times \mathbb{R} \quad (\text{A.3})$$

Country i 's labor union also expresses its actions as a function of its information. Specifically, a strategy for country i 's labor union is given by

$$v^i: H \rightarrow \{\text{accept, reject} \times \mathbb{R}\} \quad (\text{A.4})$$

A strategy profile for the game is denote by pairs (Σ, v) where $\Sigma \equiv (\Sigma^1, \Sigma^2)$ and $v \equiv (v^1, v^2)$. Finally, we define the discounted payoffs of the firms and union members. Given the discount factors β_u and β_f , and the strategy profile (Σ, v) , the i th firm's discounted sum of payoffs as of period t is given by

$$\sum_{T=t}^{\infty} (\beta_f^i)^{T-t} \pi^i(\Sigma(T), v(T)) \quad i = 1, 2 \quad (\text{A.5})$$

Analogously, the i th union member's discounted sum of payoffs as of period t corresponds to

$$\sum_{T=t}^{\infty} (\beta_u^i)^{T-t} R^i(\Sigma(T), v(T)) \quad i = 1, 2 \quad (\text{A.6})$$

Appendix B

Computation of Bargaining Equilibrium¹²

In this appendix we derive the solution to the dynamic bargaining game between the firm and the union in each country. Omitting country indices for

¹² Asilis (1992) provides a formal analysis of the extended Rubinstein model.

simplicity, we have that equations (B.1) and (B.2) below denote the perfect equilibrium conditions of the bargaining game. Namely, for the firm

$$[1 - g]\pi(g) = \frac{[1 - g']}{1 + \tau_f Z} \pi(g') \quad (\text{B.1})$$

while, for the union,

$$g'\pi(g') = \frac{g\pi(g)}{1 + \tau_u Z} \quad (\text{B.2})$$

By definition of gross profits $\pi(\bar{g})$ for $\bar{g} = g, g'$, we have

$$\pi(\bar{g}) = P(m(W(\bar{g})))m(W(\bar{g})) \quad (\text{B.3})$$

$$\frac{d\pi(\bar{g})}{d\bar{g}} = P \frac{d\bar{m}}{d\bar{g}} + m \frac{dP}{d\bar{g}} \quad (\text{B.4})$$

where

$$\frac{dm}{d\bar{g}} = \frac{dm}{dW} \frac{dW}{d\bar{g}}$$

$$\frac{dP}{d\bar{g}} = \frac{dP}{dm} \left[\frac{dm^1}{dW^1} + \frac{dm^2}{dW^2} \right] \frac{dW}{d\bar{g}} = \left[\frac{dP}{dm} \right]^2 \frac{dW}{d\bar{g}}.$$

Since the i th union's payoff is given by $W^i m^i = \bar{g} P m^i$, we have that

$$\begin{aligned} \frac{dW^i}{d\bar{g}} &= \frac{P}{1 - \bar{g} \frac{dP}{dm^i} \left[\frac{dm^i}{dW^i} + \frac{dm^{-i}}{dW^i} \right]} \\ &= \frac{P}{1 - \bar{g} \frac{P'}{P''} \left[\frac{3P' + 2m^{-i}P''}{3P' + P''(m^i + m^{-i})} \right]} \end{aligned}$$

from (10) and substituting from (5), (6) and (8). Thus,

$$\frac{dW^i}{d\bar{g}} = \frac{P}{1 - \bar{g} \left[\frac{3P' + 2m^{-i}P''}{3P' + P''(m^i + m^{-i})} \right]} > 0$$

for demand functions satisfying standard conditions of not being too concave (i.e., $|P''|$ small) or for approximately symmetric environments (as is also the case here, so that $m^i + m^{-i}$ be close to $2m^{-i}$).

(B.4) can be rewritten as

$$\begin{aligned}\frac{d\pi(\bar{g})}{d\bar{g}} &= P \frac{d\bar{m}}{d\bar{g}} \left[1 + \frac{m}{P} \frac{dP}{dm} \right] + m^i \frac{dP}{dm} \frac{dm^{-i}}{dW^i} \frac{dW^i}{d\bar{g}} \\ &= P \frac{d\bar{m}}{d\bar{g}} \left[1 + \frac{1}{\eta} \right] + m^i \frac{dP}{dm} \frac{dm^{-i}}{dW^i} \frac{dW^i}{d\bar{g}}\end{aligned}$$

where the first term is negative (as $d\bar{m}/d\bar{g} < 0$ and $|\eta| \geq 1$ for usual demand curves or appropriate bounds on the reservation wages) and the second term is negative (from $D > 0$ and $dW^i/d\bar{g} > 0$). Thus, $d\pi(\bar{g})/d\bar{g} < 0$. Restricting our attention (for tractability) to (approximately) linear surplus functions so that $\pi(\bar{g}) = a + b\bar{g}$, $a > 0$ and $b < 0$, for W^{-i} given, enable us to reexpress (B.1) and (B.2) as

$$[1 - g](a + bg) = \frac{[1 - g']}{1 + \tau_f Z} (a + bg') \quad (\text{B.5})$$

$$g'(a + bg') = \frac{g}{1 + \tau_u Z} (a + bg) \quad (\text{B.6})$$

Moreover, it is useful to renormalize our choice of units of the good (in effect inducing upward or downward translations of the surplus function π) until the linear surplus function crosses the $\bar{g}$ axis at 1 so that $\pi(1) = a + b = 0$ where (B.5) and (B.6) are given therefore by (B.7) and (B.8) below

$$[1 - g][1 - g'] = \frac{[1 - g']}{1 + \tau_f Z} [1 - g'] \quad (\text{B.7})$$

$$g'[1 - g'] = \frac{g[1 - g']}{1 + \tau_u Z} \quad (\text{B.8})$$

(B.7) implies

$$(1 - g') = (1 - g) (1 + \tau_f Z)^{1/2} \quad (\text{B.9})$$

$$g' = 1 - (1 - g) (1 + \tau_f Z)^{1/2} \quad (\text{B.10})$$

Substituting (B.9) and (B.10) into (B.8) we obtain

$$[1 - (1 - g) (1 + \tau_f Z)^{1/2}][1 - g] (1 + \tau_f Z)^{1/2} = \frac{g[1 - g]}{1 + \tau_u Z} \quad (\text{B.11})$$

(B.11) becomes

$$g[(1 + \tau_u Z)(1 + \tau_f Z) - 1] = (1 + \tau_u Z) (1 + \tau_f Z) - (1 + \tau_u Z) (1 + \tau_f Z)^{1/2} \quad (\text{B.12})$$

$$g[Z(\tau_f + \tau_u + \tau_f \tau_u Z)] = (1 + \tau_u Z)[(1 + \tau_f Z) - (1 + \tau_f Z)^{1/2}] \quad (\text{B.13})$$

(B.13) can be further simplified by using the power series expansion of $(1 + \tau_f Z)^{1/2}$ which corresponds to (B.14) below

$$(1 + \tau_f Z)^{1/2} = 1 + \frac{\tau_f Z}{2} + \frac{(\frac{1}{2})(\frac{1}{2} - 1)}{2} \tau_f^2 Z^2 + \frac{(\frac{1}{2})(\frac{1}{2} - 1)(\frac{1}{2} - 2)}{3!} \tau_f^3 Z^3 + \dots \quad (\text{B.14})$$

Using (B.14) into (B.13) yields (B.15) below (for scalars k_j , $j \in \mathbb{N}$)

$$gZ(\tau_f + \tau_u + \tau_f \tau_u Z) = (1 + \tau_u Z)[1 + \tau_f Z - 1 - \frac{\tau_f Z}{2} - K_1 Z^2 - K_2 Z^3 - \dots] \quad (\text{B.15})$$

$$gZ(\tau_f + \tau_u + \tau_f \tau_u Z) = (1 + \tau_u Z)Z [\frac{\tau_f}{2} - K_1 Z - K_2 Z^2 - K_3 Z^3 \dots] \quad (\text{B.16})$$

As $Z \rightarrow 0^+$ (continuous bargaining), (B.16) becomes

$$\begin{aligned} g(\tau_f + \tau_u) &= \frac{\tau_f}{2}, \\ g &= \frac{\tau_f}{2(\tau_f + \tau_u)} \end{aligned} \quad (\text{B.17})$$

Appendix C

Proof of Lemma 1

The proof of Lemma 1 is simplified with the following notation:

$\sigma^i \equiv (\Sigma^i, v^i)$.

$\sigma \equiv (\sigma^1, \sigma^2)$.

σ^c = Reward (carrot) strategy profile.

σ^p = Punishment (stick) strategy profile.

$\Lambda^i(\sigma)$ = Discounted sum of payoffs accruing to country i 's firm under the σ strategy profile.

$\sigma^{i*c}(t)$ = The period t strategy which solves the problem for firm i under deviation. Specifically, $\sigma^{i*c}(t)$ solves $\max \pi^i(\sigma^i(t), \sigma^{-i,c}(t))$ given all other agents play according to $\sigma^{-i,c}(t)$.

$\pi(\bar{m}^*)$ = One half of (world) monopolist's payoffs.

$\pi(\sigma^{N*})$ = (Unique) Nash equilibrium payoffs of the static game.

In what follows the condition $\pi(\bar{m}^*) > \pi(\sigma^{N*})$ is used. Previous comments and notation leads to the following definition.

DEFINITION 1. A strategy profile (σ^c, σ^p) is said to support subgame perfect equilibrium payoffs $\Lambda^i(\sigma^c, \sigma^p)$, $i = 1, 2$, if and only if the discounted net gain to firm i if it chooses to deviate from (σ^c, σ^p) is negative. Formally, it is required that $\pi^i(\sigma^{i*c}(t), \sigma^{-i, c}(t)) - \pi^i(\sigma^c(t)) - \delta^i[\Lambda^i(\sigma^c) - \Lambda^i(\sigma^p)] < 0$, for $i = 1, 2$.

PROOF. The Lemma is simple. We include the proof for completeness. By Definition 1, $(\sigma^{c*}, \sigma^{p*})$ must satisfy the supportability condition for $i = 1, 2$. The latter implies that for Lemma 1 to hold, the following inequality must be shown to hold for some $\delta = \delta^* \in (0, 1)$, namely,

$$\pi(\sigma^{i*c*}(t), \sigma^{-i, c*}(t)) - \pi^i(\sigma^{c*}(t)) \stackrel{(?)}{<} \delta^i[\Lambda^i(\sigma^{c*}) - \Lambda^i(\sigma^{p*})] \text{ for } t \in \mathbb{N}, i = 1, 2 \quad (\text{C.1})$$

But we know that the payoff accruing from any single period deviation is bounded from above by the (world) monopolist's payoffs. Namely,

$$\pi(\sigma^{i*c*}(t), \sigma^{-i, c*}(t)) - \pi^i(\sigma^{c*}(t)) < \pi^i(\sigma^{c*}(t)). \quad (\text{C.2})$$

It follows, therefore, that any δ that satisfies expression (C.3) below, must also satisfy (C.1).

$$\pi^i(\sigma^{c*}(t)) \stackrel{(?)}{<} \delta^i[\Lambda^i(\sigma^{c*}) - \Lambda^i(\sigma^{N*})]. \quad (\text{C.3})$$

Given previous notation, (C.3) is equivalent to (C.4) below

$$\pi(\bar{m}^*) \stackrel{(?)}{<} \frac{\delta^i}{1 - \delta^i} [\pi(\bar{m}^*) - \pi(\sigma^{N*})] \quad (\text{C.4})$$

in turn equivalent to

$$\pi(\bar{m}^*) \stackrel{(?)}{<} \delta^i [2\pi(\bar{m}^*) - \pi(\sigma^{N*})]. \quad (\text{C.5})$$

For (C.5) to hold, we need to show the existence of a scalar $\delta \in (0, 1)$ such that

$$\delta^i [2\pi(\bar{m}^*) - \pi(\sigma^{N*})] = \pi(\bar{m}^*) + \varepsilon \quad \varepsilon > 0 \text{ and close to zero.} \quad (\text{C.6})$$

Defining $\alpha > 0$ so that $\pi(\sigma^{N*}) = \pi(\bar{m}^*) - \alpha$ (C.6) becomes

$$\delta^i [2\pi(\bar{m}^*) - \pi(\bar{m}^*) + \alpha] = \pi(\bar{m}^*) + \varepsilon$$

so that

$$\delta^i(\alpha) = \frac{\pi(\bar{m}^*) + \varepsilon}{\pi(\bar{m}^*) + \alpha} \quad i = 1, 2.$$

Since ε is arbitrarily close to zero and $\pi(\bar{m}^*)$, $\alpha > 0$, $\delta'(\alpha) \in (0, 1)$. We take

$$\delta^* \equiv \delta^*(\alpha) = \max\{\delta^1(\alpha), \delta^2(\alpha)\}.$$

Since (C.5) holds for δ^a if it holds for δ^b where $\delta^a > \delta^b$, the result obtains. ■

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