EFFECTS OF GROWTH ON RELATIVE PRICES IN A TWO-GOOD *N*-COUNTRY MODEL

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Resumen: Este trabajo analiza el efecto de un incremento en las dotaciones en un modelo con dos bienes y n países. Se muestra que si la dotación del bien uno se incrementa, entonces su precio relativo al bien 2 también puede incrementarse. Sin embargo, este efecto "perverso" no puede ocurrir si la economía está en un equilibrio de *tâtonnement*. También se muestra que hay "doble perversidad" en el caso de bienes normales cuando el equilibrio es inestable.

Abstract: This paper analyses the effect of an increase of the endowment in a two-good n-country model. It is shown that the endowment of good one can go up and the price of the same good relative to good two can also increase. However, this "perverse" effect cannot occur in both goods if the economy is at a *tâtonnement* stable equilibrium. This is a new restriction in the form of the equilibrium manifold of an exchange economy. It is also shown that this "double perversity" is the case with normal goods when the equilibrium is unstable.

1. Introduction

There are several problems in international trade which relate to how variations of endowments affects the equilibrium of the economy. The important questions are: *(i)* "The transfer problem": How does equilibrium vary when

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there is a gift from one country to another? *(ii)* "The growth problem": How does equilibrium change when the world grows?

Traditionally the transfer problem has been analysed in two ways:

(i) How do equilibrium prices change with a transfer?
(ii) How do welfare levels of countries change with a transfer?

These two questions have been thoroughly discussed in the literature. Basically Samuelson (1952 and 1954) posed them. The price changes were studied by Chipman (1974) and Jones (1970 and 1975). The welfare changes were studied by several authors. The state of the art is Dixit (1983) and Majumdar and Mitra (1985). The reader interested in bibliographical completeness should refer to these and other papers cited in here.

The growth problem literature has evolved in a different manner: Only questions related to welfare effects were raised. As it is, the fact that growth may cause "bad" welfare effects, normally understood as "immiserising growth", was point out by Bhagwati (1958). The state of the art is given by Mas-Colell (1976) and Mantel (1984). Again one should not take this as being a complete list of references for the problem. In particular one article, that of Jones (1985), gives a very coherent survey of the growth and the transfer problems, and how one can immerse an n-country growth analysis into an $n + 1$-country transfer analysis.

The question which this note addresses is the related price effect of growth. This is also a problem which is linked with the still open question of finding the structure of the graph of the Walras' correspondence, that is to say, the price-endowment equilibrium set of an exchange economy. On this see Balasko (1975 and 1988), Schecter (1979). In particular, one can easily see that the main proposition imposes some restriction on the Walras' correspondence graph that were to date ignored.

The point tackled here is: In a two-good, n-country economy, when is it possible that, given an increase of the initial endowment of a good of a given country, the relative price of this good will rise?

The example in the next section will show that such a perverse situation can occur. It will also be clear that this effect may be as high as one wishes.

Section three contains the main result of this note: This perverse effect cannot happen with both goods of a given country in economies with two goods if the equilibrium is stable, or downward sloping. One says an equilibrium is downward sloping, or stable, when an excess demand for a good results in an increase in its price to clear the market. This condition is generically verified at the equilibrium prices if equilibrium is unique. The economies which are considered here are such that equilibrium prices are strictly positive. This occurs, for example, if for each good there exists at leastone individual such that his demand for the given good goes to infinity when the price of the good goes to zero.

The last section argues that in the case of an equilibrium with upward sloping demand, or unstable equilibrium, "double perversity" is possible. Not only that, but also it is the normal case. This may be seen as another strong fact to consider such unstable equilibria as unreasonable.

Throughout the rest of the paper the word "consumer" will be used instead of "country". Also we are going to be dealing only with small changes of the endowments.

2. A Perverse Example

In this section the following perverse case is analysed: In a two-goods exchange economy it is possible that an increase in the endowment of a good causes a rise in its price, even in a downward-sloping demand equilibrium (or *tâtonnement* stable). Fix one of the consumers, say consumer 1, and increase her endowment of good 2. At first this results in a higher income for the consumer. If good 1 is inferior, then the consumer will demand less of it. Also good 1's supply remains unchaged, since we increased only the supply of good 2. Thus there is an excess supply of good 1. As the quilibrium is a stable one, this requires the price of good 1 to go down. This is the same as saying that the price of good 2 goes up. Hence one sees that it is perfectly reasonable that the perverse effect described above happens. One can also see that this can occur for good 2 only if good 1 is inferior.

It may be of interest to ask whether this can happen to good 1 itself. There are two ways of checking that. The first one is to observe that we can interchange the names of the two goods. In this way good 2 is inferior and good 1 increases its endowment, which means a higher price of good 1. The other more cumbersome, but nevertheless very enlightening way is direct verification. Suppose the endowment of good 1 goes up. Then, as a first impact, income rises. Assume good 1 is normal. As a result demand for good 1 increases. However, its supply also went up. Suppose the increase in demand is larger than the increase in supply, what amounts to assuming that the second good is inferior, in a two-good world. Then there is an excess demand for good 1, and its price goes up as a result of being a stable equilibrium. This description is exactly the opposite of the one before, because in a world with two goods one is inferior if and only if the other is "supernormal", that is to say, an increase in its supply causes such an increase in its demand due to the higher income level, that demand surpasses supply.

It is interesting to note that such an effect can happen to any extent one wishes (as long as endowments and utilities are chocen adequately). Consider the following consumers. Let ξ_{ij} mean the demand for the j-th good by the i-th consumer, p_1 and p_2 are the prices of goods 1 and 2, I is income, I^1 and I^2 are the incomes of the first and second consumers, respectively.

(i) Consumer 1 is such that good 1 is inferior:

$$\xi_{11}(p_1, p_2, I^1) = \xi\left(\frac{p_1}{p_2}, \frac{I^1}{p_2}\right)$$

$$\xi_{12}(p_1, p_2, I^1) = \frac{I^1}{p_2} - \frac{p_1}{p_2}\xi\left(\frac{p_1}{p_2}, \frac{I^1}{p_2}\right)$$

where

$$\xi(p, I) = \frac{I}{p}e^{-I^n}$$

It is immediate to check that this is a neoclassical consumer.

(ii) Consumer 2 is Cobb-Douglas with income shares α of good 1 and $1-\alpha$ of good 2, $\alpha \in [0, 1]$:

$$\xi_{21}(p_1, p_2, I^2) = \frac{\alpha I^2}{p_1},$$

$$\xi_{22}(p_1, p_2, I^2) = \frac{(1-\alpha)I^2}{p_2}.$$

(iii) Endowments are (w_{11}, w_{12}) and (w_{21}, w_{22}) for consumers 1 and 2, respectively.

Then, setting $p_2 = 1$, $p_1 = p$, $I^1 = pw_{11} + w_{12}$, $I^2 = pw_{21} + w_{22}$, and equating excess demand for good 1 to zero, we obtain:

$$\frac{pw_{11} + w_{12}}{p}\exp[-(pw_{11} + w_{12})^n] + \alpha\frac{pw_{21} + w_{22}}{p} - (w_{11} + w_{21}) = 0.$$

Taking implicit derivatives:

$$\frac{\partial p}{\partial w_{12}} = \frac{\exp[-(pw_{11} + w_{12})^n]\,[1 - n(pw_{11} + w_{12})^n]}{-w_{11} - (1-\alpha)w_{21} + w_{11}\exp[-(pw_{11} + w_{12})^n]\,[1 - n(pw_{11} + w_{12})^n]}.$$

It is easy to see that by varying the parameters this value can be as negative as one wants, independently of p. Just make w_{12} large enough and w_{11} and w_{21} small enough.

3. Double Perversity Impossible at a Stable Equilibrium

Given one consumer in a two-goods economy, we will show that, if the equilibrium is stable, it is not possible that an increase in the initial endowment of each of the goods causes an increase in each of their relative prices. One can intuitively see why. Increase, as before, his endowment of good 2. In order for the described perversity to occur, it was shown that good 1 had to be inferior. Then good 2 has to be normal, and, therefore, by the same argument of the previous section, good 1 cannot exhibit such perverse effect. Obviously the argument also goes the other way around. Formally:

PROPOSITION. *Suppose there are n consumers and two goods. Consumption spaces are R^2_+ and demand functions are continuously differentiable in prices $(p_1, p_2)R^2_{++}$ and income IR_{++}. Also assume equilibrium prices are strictly positive. Suppose initial endowments are also strictly positive. Finally assume the equilibrium is stable, or downward sloping, in the sense that the derivative of the excess demand for good 1 with respect to its relative price is strictly negative. Then:*

(i) $\frac{\partial p}{\partial w_{11}}$ and $\frac{\partial p}{\partial w_{12}}$ both exist, the notation being the same as in the previous section;

(ii) it is not possible that $\frac{\partial p}{\partial w_{11}} \geq 0$ and $\frac{\partial p}{\partial w_{12}} \leq 0$.

PROOF. Let $\xi^1(p_1, p_2, I^1)$ and $\xi(p_1, p_2, I^2, \ldots, I^n)$ be the demands for good 1 of the first consumer and the rest of the economy, respectively. Set $p_1 = p$ and $p_2 = 1$. Then, as before, $I^1 = pw_{11} + w_{12}$, and $I^2, \ldots, I^n$ are independent of w_{11} and w_{12}. Define the excess demand for good 1:

$$Z(p, w_{11}, w_{12}) = \xi^1(p, 1, pw_{11} + w_{12}) + \xi(p, 1, I^2, \ldots, I^n) - \sum_{i=1}^{n} w_{j1}$$

As by hypothesis $\frac{\partial Z}{\partial p} < 0$ at equilibrium (stable equilibrium), the implicit function theorem can be applied, and one has:

$$\frac{\partial p}{\partial w_{11}} = -\left(\frac{\partial Z}{\partial p}\right)^{-1} \frac{\partial Z}{\partial w_{11}}$$

and

$$\frac{\partial p}{\partial w_{12}} = -\left(\frac{\partial Z}{\partial p}\right)^{-1} \frac{\partial Z}{\partial w_{12}}.$$

But

$$\frac{\partial Z}{\partial w_{11}} = p\frac{\partial \xi^1}{\partial I} - 1$$

and

$$\frac{\partial Z}{\partial w_{12}} = \frac{\partial \xi^1}{\partial I} .$$

Thus

$$\frac{\partial p}{\partial w_{11}} = p\,\frac{\partial p}{\partial w_{12}} + \left(\frac{\partial Z}{\partial p}\right)^{-1} .$$

Again, as $\frac{\partial Z}{\partial p} < 0$, it follows that

$$\frac{\partial p}{\partial w_{11}} < p\,\frac{\partial p}{\partial w_{12}} .$$

Hence $\frac{\partial p}{\partial w_{11}} \geq 0 \Rightarrow \frac{\partial p}{\partial w_{12}} > 0$ and $\frac{\partial p}{\partial w_{12}} \leq 0 \Rightarrow \frac{\partial p}{\partial w_{11}} < 0.$

By the verbal argument at the beginning of this section, it should be clear that the differentiability hypothesis is merely technical, and is not necessary to prove the result. The proposition abone shows a restriction on the graph of the Walras' correspondence that was not known: When the Walras' correspondence is a function, generically it occurs that excess demand is dowsword sloping at the equilibrium price (a consequence of the index theorem). Thus, for such economics it cannot happen that both $\frac{\partial p}{\partial w_{11}} \geq 0$ and $\frac{\partial p}{\partial w_{12}} \leq 0.$

4. Double Perversity Possible with Unstable Equilibrium

As one can see from the preceding section, the crucial hypothesis is that of stability. In this section it is shown that with an unstable equilibrium it is possible to have "double perversity". If the endowment of good 2 is increased, this increases the income, and hence demand for good 1. Then, as its supply is fixed, there is an excess demand for it. But the equilibrium is unstable (or upward sloping), which implies that the relative price of good 1 falls, which in turn means that the relative price of good 2 goes up.

To see that the same happens with an increase in good 1's endowment (of the same consumer), one can do as before: Either rename the goods, or do it directly. In order to do so, observe that good 2 being normal, if good 1 is also normal, then good 1 cannot be "supernormal" in the sense described in section 2. Thus an increase in good 1's endowment causes its demand to increase less than its supply. Hence there is an excess supply of good 1. The fact the equilibrium is unstable assures that the relative price of good 1 goes up, and the double perversity is verified.

It remains to be shown that it is possible to have an unstable equilibrium where both goods are normal. For this one invokes a result by Mantel (1976) which essentially says that any excess demand function in a l-good economy can be obtained by means of l strictly convex and monotonic consumers, all of which are homothetic. Thus one can think of an economy wit two goods and three equilibria, two of which are stable, and one unstable. By Mantel's result one can get it with homothetic consumers. For a homothetic consumer all goods are normal. By the verbal argument earlier it follows that at the unstable equilibrium of this economy double perversity occurs.

From the reasoning above it is easy to attest that "double perversity" is actually a very common case at an unstable equilibrium. This should be an additional fact to consider such equilibria as unreasonable.

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