

THE THEORY OF INDEX-BASED FUTURES AND OPTIONS MARKETS*

Robert J. Shiller

Yale University and NBER

Resumen: Este trabajo presenta una panorámica del desarrollo de mercados de futuros y opciones basados en índices económicos. La tesis es que, aun cuando dichos mercados han crecido dramáticamente, su desarrollo podría ser más extendido si algunos problemas de medición se pudieran resolver.

Abstract: This paper reviews the development of index-based futures and options markets. The thesis is that, while the growth of these markets to date has been dramatic, their development could be extended much further, if some problems of measurement can be solved.

1. The Theory of Index-Based Futures and Options Markets

I am honored to be invited to give the Miguel Sidrauski Memorial Lecture. I knew and admired Sidrauski when I was his student at the Massachusetts Institute of Technology. I was a student in the last class that he ever taught: the macroeconomic theory course that he co-taught with Robert Solow in the Spring of 1968. Two dozen years have passed since then. Still I remember his clear lectures (and very helpful, sometimes elaborate, graphical presentations). I also remember him as a very decent person, sympathetic and humble.

My subject today is the development of index-based futures and options markets. The last ten years have seen the proliferation of these markets, and the growth of these markets around the world seems more dramatic every day.

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My thesis here will be that, while the growth of these markets to date has been dramatic, the development of these markets could be extended much further, if some problems of measurement can be solved. I will discuss the development of markets that we have observed, the ways of approaching measurement problems used in existing markets, and extensions and improvements on these methods. In particular, I will discuss some repeated measures indices of prices that may be used to cash settle futures and options contracts.

2. Technological Progress in the History and Theory of Futures Contracts

I wish to argue first that the major events that affected the historical development of futures markets were technological: the invention of new concepts of contract settlement, and that these developments were in effect adoptions of new methods of creating statistical measures, measures that were meaningful to people desiring to hedge or diversify. The history of technological progress in these markets sets an example for what more sophisticated efforts to develop new market structures may do in the future.

Futures markets are not a new idea, although they have proliferated only recently. There is considerable dispute about when and where trading in futures markets first began, the dispute can never be cleanly resolved since the elements of what we call futures and options markets today did not appear all at once. It is commonly agreed, however, that a watershed event is the beginning of trade in agricultural futures at the Chicago Board of Trade in the 1860's.

The invention of these futures markets served to solve a sort of estimation problem: in the industry the markets are said to provide "price discovery," meaning that the market price is an estimator of some underlying parameter of interest to market participants. Creating markets to provide price discovery is a bit like creating an estimator in statistics, except that creating a market intimately involves the question of incentives for people to provide information on which estimates will be based.

Cash markets for agricultural commodities do not provide the best price discovery since these markets are not always liquid and do not always trade a standardized commodity. Often price changes from day to day in these markets may be meaningless, involving only very few, special, trades or reflecting only a change in the kind of commodity traded. Futures markets produce liquidity: liquidity is ease of trade: in a liquid market one can buy or sell large quantities at little cost, and with little effect on price. This liquidity encourages participation in a market, providing an incentive for people to reveal their information by participating in the market, in turn providing price discovery. An active liquid market provides meaningful prices on a

minute-by-minute basis. Futures markets also produce standardization, by defining precisely the qualities of the commodity that may be used for delivery. The prices generated in futures markets therefore carry information, information that can affect many economic decisions.

The standard futures contract then and now had the futures exchange functioning as a market maker and guarantor of the quality of the commodity delivered and of the fulfillment of the terms of the contract, in such a way as to open the participation in the contract to everyone. One who bought a futures contract in wheat, for example, would be guaranteed, in effect, the ability to buy wheat at the futures price (and disregarding daily resettlement) at the specified time in the future, one who sold would be in effect guaranteed the ability to sell wheat at this price at that time. And yet the market is able to function like an impersonal auction market, in which buyers and sellers need never meet each other, and have no need to try to assess each others' ability to meet the terms of the contract. The futures market is able to assure that both sides of the contract will be met by providing the service of assessing the quality of the commodity delivered and by requiring that buyers and sellers put up margin at the time that they enter the contract, and by establishing daily resettlement to these margin accounts. This arrangement guarantees the adherence to the terms of the contract to the extent of the margin; moreover, to the extent that the futures markets turn out to be liquid, the futures exchange is able to close out, via a reversing trade, anyone whose margin has been depleted and who does not come up with additional margin.

What this futures market arrangement achieves is complete democracy and standardization in markets. By democracy, I mean that anyone can trade (who can come up with the margin) and by standardization that everyone knows that they are getting the same price on the same commodity as everyone else. In contrast, those who try to buy and sell commodities directly to another party by a forward contract must assess the creditworthiness of the other party, must deal with default by the other party at their own expense, and must themselves check out the characteristics of the commodity delivered.

What these organized futures markets then also offer, besides price discovery, is the opportunity for people who are subject to price risks to hedge these risks. Those who are long in a commodity (say, grain elevators storing wheat, or farmers planting corn) may take a short position in the futures market, so that they guarantee for themselves the ability to deliver at a established price. This means that the price risk can be borne by someone else, who is long in the futures market.

It was often remarked about the early futures markets that, while they were an excellent idea from a theoretical standpoint, there were remarkably few commodities actually represented by futures markets. As recently as two decades ago in the United States there were no more than a couple dozen

actively traded futures contracts, these only on agricultural commodities and primary materials. A great expansion of these markets occurred when the concept of cash settlement was introduced, and when sophisticated index numbers were used to settle.

The establishment of cash settled contracts was the next watershed event in the history of futures markets. This was another invention, another bit of technological progress in our financial markets. The first cash-settled futures contract was the Eurodollar contract offered at the Chicago Mercantile Exchange in 1981. One who sells in a futures market is, when the contract is specified as cash settled, obligated not to deliver the commodity but merely to deliver the difference between the cash-market price of that commodity and the contract price. One who buys in a cash settled market is obligated, not to take delivery of a physical commodity, but to receive the difference between the cash-market price of that commodity and the contract price. Cash settlement was an important development primarily since it paved the way for a variety of economic indices to be traded on financial markets.

The next watershed event in the history of futures markets came just a year later: the establishment of contracts settled on the basis of economic indices, rather than on individual prices. The first economic index futures market was the Value Line stock price index futures that began trading at the Kansas City Board of Trade in 1982. This very first index product was especially sophisticated: the index was based on a *geometric* average of prices, making the contract unlike any portfolio of stocks.¹

The newly discovered innovation of cash settlement based on economic indices raised the possibilities for futures and options contracts enormously, bringing it to the possibility of trading anything at all that econometricians can measure. This means that we can tailor futures and options contracts (and, for that matter, over-the-counter forward contracts as well) to the specific risk needs of individuals. Risk management is now something that can, and is, being pursued on a scientific basis.

The earlier forms of futures contracts, involving physical delivery, were in fact in effect creating some very primitive index numbers on which to base trade. In physical delivery of an agricultural commodity, the originators of futures markets had to contend with the fact that there are very many qualities and types of commodity. They could not demand delivery of any single quality or type, since there may not be enough of any one such type to guarantee that it can be delivered and that the price at which it can be delivered is representative of prices that hedgers are concerned with. So, futures exchanges allowed delivery of any of a number of grades and types.

¹ The Value Line contract has since been altered, so that the index is arithmetic rather than geometric.

Thus, the price at which a futures contract settles tends to be the minimum price of all these grades and types: those who deliver will do so in the cheapest way possible. An econometrician would hardly consider the minimum price over all these grades as a satisfactory index of their price. The minimum price is too heavily reliant on outliers, and may not correspond well to the price at which other users trade.

3. The Potential for Future Proliferation of Futures and Options Markets

Despite the obvious advantages that futures, and the associated options and over-the-counter forward markets, offer, these markets have been slower to grow than their potential usefulness would suggest. The question naturally arises whether any new technological innovations might serve to allow faster growth in the future.

It is certainly a fact that proliferation of futures and options markets is at this moment rapidly taking place around the world. Very much has happened in the past few years, or promises to happen in the next few years. Japan only started trading stock index options a few years ago. Germany began a futures market, the DTB, Deutsche Terminboerse, only few years ago. Brazil has shown dramatic recent interest in futures markets: the Bolsa de Mercadorias & Futuros in Sao Paulo maintains a large market in interbanking deposit interest rates, as well as a market in the U.S. Dollar - Cruzeiro exchange rate. There is also a growing futures exchange, the Bolsa Brasileira de Futuros of Rio de Janeiro. Exchanges may be launched shortly in Chile, Mexico, and other Latin American countries. The Bolsa Mexicana de Valores may commence trading stock index futures within a few months.

Despite the rapid growth of futures markets, the real potential of these markets does not seem to be developing. The numbers of risks that are hedgeable in these markets are only a tiny fraction of the potential risks that could conceivably be hedged.

There is today no market in the world that allows people to hedge the risk of price change in real estate. There was an attempt in 1991 to start a real estate futures market at the London Futures and Options Exchange, but that effort was aborted by a scandal that shut down trading there after only five months. The value of real estate exceeds that of corporate equities in most countries; it is odd then that it is only on equities that organized futures contracts have been established. Real estate price risk is quite substantial, and has a dramatic effect on people's lives.²

² Further discussion of the potential for real estate futures and options markets is in Case, Shiller and Weiss (1993).

There is today no market in the world that allows people to hedge the risk of change in the price of labor. One might well argue that the risks of change in the price of labor are even more dramatic than those of real estate, since the bulk of people's incomes—and of firms expenses tend to be labor—related.

There are today no futures markets on prices of assets that are privately held, and infrequently traded. Such assets are of extreme importance, especially outside of the United States and the United Kingdom where liquid public stock markets have a long history of domination. Such assets may well have a price path that differs from the price paths of widely traded assets, and so the organized stock index futures markets may not serve the hedging needs of investors in these privately held assets. The establishment and proliferation of public stock markets around the world signals a trend towards greater public participation and liquidity in the market for corporate equity, but still this process is proceeding slowly. An organized futures market in indices of prices of privately-held assets could provide an alternative way for such public trading to get started.

There are today no markets on components of wholesale price indices or producer price indices, for example, such components as railroad equipment, or aircraft. There might plausibly be good reasons for such markets to be established—certainly the risks associated with these markets are substantial and there are plenty of firms that face risk associated with their price change.

An important reason why these are not available may be that it is just difficult to measure the prices in these markets.

For labor costs, there are no markets at all where we can observe the capital value of the asset. There are labor cost indices, but there are no prices that indicate the present value of expected future labor costs. What we observe with the available labor cost indices are in effect the dividend or rent on human capital. For real estate and privately held financial assets, there are observations of price, but the observations may be very infrequent for any given asset. A home or apartment building may not be sold for decades; privately held assets may be held for similarly long periods of time between sales.

One way to start a market in the asset value of labor costs, in human capital, would be to start a market for perpetual claims on the cash flow, a market that is cash settled in such a way that the market price represents a claim on the entire stream of index values, into the indefinite future. I have proposed elsewhere (1992) a perpetual futures contract that would achieve such an end. In such a market, there is a combination of both the normal daily resettlement in futures markets and a settlement representing dividends paid on the underlying asset. Such markets might also be used to create markets for commercial real estate or agricultural land, by using a rent index for these

properties to cash settle. The price discovery afforded by such markets may be very dramatic in its impact: we might for the first time learn the asset value of some of our major risks. Still, the establishment of such markets still requires wage or rent indices.

Futures markets may also be started based on indices that represent neither price nor wages or rents. Regression methods may be used to produce indices of even non-price indicators that represent risk of doing business for firms. Consider, for example, the automobile sales futures markets proposed by the Coffee-Sugar and Cocoa exchange in 1985 as the basis for cash settlement of futures contracts. The automobile sales variable was intended to be some measure of demand shifts in the automobile industry. But, of course, the automobile sales variable is reflective of more than just this. The sales variable is, of course, determined as the intersection of both supply and demand, and can be affected by shifts in either of these curves. The methods that econometricians have derived to identify supply and demand curves could then be used profitably to help create an index of automobile demand for cash settlement of contracts.

The problem that I will discuss here is the problem of composing index numbers of prices (or wages or rents or other variables) when the observations on price (or wage or rent) are either infrequent or of nonstandardized items on nonstandardized economic situations. The problem we have in real estate markets, for example, is that each property may be sold only at intervals of years, and the kinds of properties that are sold at different times varies through time.

4. Creating Index Numbers to Settle Contracts

It is the job of econometricians to create index numbers that will provide measures of risks that really affect individuals and firms.

Much of the theory of price indices that has occupied economic theorists since the last century has been aimed at measuring the cost of living, that is, the cost of attaining a specified level of utility. This literature may have a payoff soon if futures markets are established around the world on consumer price indices. One such market was created by the Coffee, Sugar and Cocoa Exchange in 1985, but that market failed, apparently due to the decline in the variability of inflation that occurred about that time. Another such market, created at the Bolsa de Mercadorias & Futuros in Sao Paulo, was successful, probably because of the much higher variability in the consumer price index there. Index-linked bonds have, moreover, been traded in a number of countries, and their trade continues today to provide benefits. The existing theory of price indices may be well suited to constructing such indices,

although there are perhaps issues about the matching of horizons of futures market participants with the time the market basket was measured.

But we have seen that many more kinds of indices could be traded in organized futures markets. Attention on the theory of index numbers on measuring the cost of living has been partly due to the enormous variability of price levels, connected as it is to the stabilization policies of governments. There are other kinds of risks that may be outside of the control of governments, and which have received comparatively little attention from economists.

Futures markets could be started in terms of the costs that firms face in any of a number of dimensions. These futures markets would help firms to hedge a number of specific risks, and thereby concentrate their attention on the aspects of their business that they do control without their having to be vulnerable to sudden, and possibly devastating, surprises.

However, it is likely that existing index number construction methods could be improved upon if they are to be the basis for trade in futures markets. First of all, there is the issue of regional indices; firms in a given region are interested in hedging regional costs of production. The construction of such regional indices has not been the primary concern of constructors of producer price indices. Moreover, indices might be better attuned to the needs of firms than is the case with subindices of the producer price indices. For example, a good measure of labor costs for a firm is something that could provide great hedging opportunities to firms, if the index truly represents the total costs to the firm of hiring labor. Such costs are really the cost of hiring an amount of effective labor service, of input to production, and not the cost of an hour of work. Variations in the skill level of the labor force, of the quality of labor-management relations, should properly have an impact on these indices. A great deal of work is necessary to develop such indices for public trading. There are considerations of objectivity and manipulability to contend with; we do not want, for example, a single labor-management dispute to have a substantial effect on the index, since the labor or management parties may have taken positions in the futures market.

The problem we have alluded to above with producer price indices as the basis for cash settlement of futures contracts is that these indices are prices of new, rather than existing, commodities. Those people who hold inventories of commodities and wish to hedge their price risk may find that the price of new commodities does not represent well the market price of the stock of commodities that they own. The reason for the difference is that the new commodities may not be representative—in type or quality—of the commodities that are already in inventory. New commodities are produced only in forms that are currently profitable to produce. In times when the price of the inventory of commodities falls very low, there may be virtually no production of new items, except on a made-to-order basis for very special

needs. The producer price index for these new commodities may not fall, in such a circumstance, when the actual price of the hedgeable inventories has fallen, and thus the producer price index may not well serve hedging needs. The problem here is that the quality of the goods sold today may differ fundamentally from those of the goods on inventory.

5. Measured and Unmeasured Quality Variation

One solution to this problem is to attempt to correct for quality, by representing price as a function of quality variables. The hedonic regression formulation considers an asset as a bundle or portfolio of qualities, each of which has a price. To the extent that the quality variables are measured, we can price each one of them from sales of commodities that have qualities that span the space of quality variables. We can run a regression with price as the dependent variable and various quality variables as independent variables. The price index then can be taken as the price of a standard bundle of quality variables, that is, as the fitted value of the estimated regression for these quality variables. This method was first proposed by Court (1939). Rather than taking a fixed standard bundle of commodities, we could also compute a chain index, that represents through time the changing character of the average quality of outstanding assets. This would not entail changing any of the regression estimation methodology, merely adopting an index that involves changing weights to the different qualities.

The simple hedonic methods consist of running regressions, with one observation per sale of the asset, of price or transformed price on a set of independent variables representing, as well as the time of sale, measures of the quality of that asset. The former independent variables, representing time, may consist in the simplest formulation just of time dummy variables. The time dummy variable for time t is one if the property is sold at time t , zero otherwise. The latter independent variables are referred to variously as quality variables, characteristic variables, or hedonic variables. For example, with housing prices, these independent variables might be such things as number of square feet, number of bathrooms, and dummy variables for fireplaces, brick construction, and air conditioning. Interactions between the time-dummy variables and quality variables may also be included; these are products of the time-dummy variables with quality variables. The price index at time t is then taken as the fitted value for the regression where the values of the independent variables are taken as the values corresponding to a standard house at time t .

The simplest hedonic regression method consists just of a regression of price (or log price) on the time dummies alone, with no quality variables at

all. This method, however, is hardly satisfactory; it produces an index of prices which is, for each time t , just the average price (or average log price if the dependent variable was log price) of all properties sold at time t . The method can be improved by adding quality variables; for houses such things as the square feet of living area, or, following Clapp and Giaccotto (1990), the assessed value of the house as of a certain fixed date. The estimated index for time t is taken to be the price of a standard house at time t , or, more simply, as the coefficient of time dummy for time t (normalized so that the index has a specified value in the base period). This method of index number construction can be called the time-dummy hedonic regression method. The extent of improvement above the simple regression on time dummies depends on the extent to which these additional variables capture quality of individual properties. If the additional quality variables are highly inaccurate or incomplete measures of quality, then the estimated index for time t may be nothing more than the average price of a property sold in that period.

The simple time-dummy regression method can be generalized by allowing interaction effects between the time dummies and the quality variables, that is, including the products of each time dummy with each quality variable. If all interaction effects are allowed, this reduces to what may be called a regression-per-period method. If we include as our independent variables the products of all time dummies with all quality variables, then the matrix of independent variables becomes block diagonal. This method then can be described as running a regression for each time period, where the dependent variable consists of observations on the price (or transformation of price) of the assets sold in that period. The estimated price index for time t is just the price of a standard property from the regression for time t . If the dependent variable is simple price (rather than log price), then the regression-per-period method may be regarded as motivated by the assumption that the asset is actually a portfolio of characteristics, each characteristic having a price, and the total price for the asset as the sum of the values of the characteristics it embodies. The regression-per-period method is more general than the time-dummy hedonic regression method in that it allows the price of all characteristics to change through time.

This method could be used with sales of newly-produced assets, as are priced for the purpose of computing producer price indices, so long as there are various qualities of the commodity represented, so that there is not multicollinearity in the regression. There are, however, some risks in doing this. First of all, the quality characteristics of the most newly made commodities may be systematically different from those of existing commodities in such a way that, while there may not be strict multicollinearity among the quality variables, we may get a poor estimate of the price of some of them. It is well known that the standard error of the fitted value of a regression model

is low when the fitted value is computed near the sample mean, at rises as the values of the independent variables are made to move away from this mean. A futures market has to rely on well defined, objective settlement criteria, lest there be complaints and even litigation when the settlement procedure does not appear to perform well. A contract that was based on such fitted values of hedonic regressions would have to confront the risk that at some future date multicollinearity could become a serious problem.

Another problem with the hedonic regression method as applied to prices of newly-produced commodities is that these newly produced commodities are likely also to differ systematically in *unmeasured ways* from those of commodities that are already in inventory. It is very hard for a statistical authority to produce a complete measure of the quality of any product, doubly hard when the product is evolving through time to new technology and changing public tastes.

It may, therefore, be advantageous to adopt a different tack, and to use prices of existing, rather than newly produced, commodities in the hedonic regression formula. The whole inventory of existing commodities may be more inherently representative of the hedgeable stock than are newly-produced. This is certainly likely to be true for real estate.

In dealing with sales of already-produced commodities, we also have the option of observing repeat sales of the same asset through time. In a hedonic regression framework, appending property dummy variables (a variable for each property sold, whose value is one if the sale is for that property, and zero otherwise) can protect us from any unmeasured quality variables that are constant through time for a given property. In doing this, if we had before using the property dummy variables used a separate regression for each time period, then we must stack these separate regression models into one model that is block diagonal in the matrix of independent variables. Adding property dummies to such a regression has the effect of breaking the block diagonality.

6. Repeat Sales or Repeated Measures Indices

Index number construction methods in use today to take account of repeated measures data on unchanging items have been called "repeat sales" methods, since it has been presumed that the index is a price index. Since the same index number construction methods may be used to generate wage, rent, or even other values, it is unduly restrictive to call them repeat sales indices. We will call them repeated measures indices, using the term from the literature on the analysis of variance. In this section, however, we will suppose that the quantity to be measured is the price of housing in a given city, and that each observation we have is the sale price of a house in that city.

The Bailey-Muth-Nourse (BMN) index number construction method produces an index of log prices by regressing log price changes for individual houses on dummy variables denoting nothing other than the identity of the houses.³ In the levels form of the estimator, the matrix of independent variables is the $n \times T$ matrix Z_L whose ij th element is -1 if the first sale of house i occurred in period j , is 1 if the second sale of house i occurred in period j , and is zero otherwise. The dependent variable vector y has as its i th element the change in log price for the i th house. Here, lower-case letters will denote logs: $p_{ij} = \ln(P_{ij})$ where P_{ij} is the price of the i th house at time j . The index will be based on the model $y = z_\gamma + e$, and for the purpose of computing standard errors it will be assumed that the elements of the vector of error terms e are independent of each other. It is reasonable to suppose that individual house price variations unrelated to the city-wide variations are due to idiosyncratic value changes, although of course we might also properly account for heteroskedasticity due to the time interval between sales (Case and Shiller 1987), or to some variance components representing neighborhood or property types.

The estimated log price index for time t is the t th element of the ordinary least squares regression coefficient vector $\hat{\gamma} = (Z_L'Z_L)^{-1}Z_L'y$ are making the log price index for time zero equal zero since time 0 is the base year for the index. It is helpful to illustrate this setup with an example. If houses 1 and 2 were each sold in periods 1 and 2, houses 3 and 5 were each sold in periods 0 and 1, and house 4 was sold in period 0 and 2, then the matrices would be:

$$Z_L = \begin{bmatrix} -1 & 1 \\ -1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \quad y = \begin{bmatrix} p_{12} - p_{11} \\ p_{22} - p_{21} \\ p_{31} - p_{30} \\ p_{42} - p_{40} \\ p_{51} - p_{50} \end{bmatrix} \quad (1)$$

The difference form for the BMN estimator produces a vector $\hat{\delta}$ of estimated changes in the log price index; it is produced by regressing the same vector y on a matrix Z_D whose ij th element is 1 if house i was between sales at time j , i.e., time j was after the first sale but not after the second sale:

³ Extensions of the original BMN method are in Case and Shiller (1987, 1989), Goetzmen (1989), and Webb (1988). See also case and Quigley (1991) and case, Pollarowski and Wachter (1992).

$$Z_D = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \quad (2)$$

The estimate of the vector of changes in the log price is given by $\hat{\delta} \equiv (Z_D'Z_D)^{-1}Z_D'y = S\hat{\gamma}$ where S is a $T \times T$ lower triangular matrix with ones along the main diagonal and -1 along the first off diagonal. The normal equations for the BMN estimator has an easily interpreted form if written in the form $Z_D'Z_L\gamma = Z_D'y$: the i th equation gives the i th value of the log price index based on prices of homes that were between sales at time i as an average of log price changes from time zero, where missing values on homes not sold at time i are inferred from using the estimated index. In the example here, these normal equations are:

$$\hat{\gamma}_1 = 1(p_{31} - p_{30}) + (p_{42} - p_{40} - \hat{\gamma}_2) + (p_{51} - p_{50}) / 2 \quad (3)$$

$$\hat{\gamma}_2 = 1(p_{12} - p_{11} + \hat{\gamma}_1) + (p_{22} - p_{21} + \hat{\gamma}_1) + (p_{42} - p_{40}) / 3 \quad (4)$$

Note that the estimated log price index is a sort of average of log price changes of individual houses, so that taking $\exp(\hat{\gamma})$ as an index of the level of housing prices, this index will be a sort of *geometric* average of individual house price relatives.

One potential problem with BMN indices for the purpose of cash settling futures or options contracts is that the resulting indices are logarithmic. This means that small properties, even insignificant properties, are given the same weight as large properties in constructing the index. Properties whose value drops nearly to zero have a devastating impact on the index. Moreover, the antilog of the log index, based as it is on geometric averages rather than arithmetic averages, has a bias relative to portfolio values, which are themselves always arithmetic averages. Because of dissatisfaction with geometric averages for the purpose of settling futures contracts, the original Value Line stock price index contract traded at the Kansas City Board of Trade was replaced later by an arithmetic index. We seek, therefore, a way of modifying the BMN method to produce arithmetic indices.

An arithmetic estimator analogous to the BMN is produced by defining a matrix of independent variables X so that X_{ij} equals minus the price of the first sale of house i if the time of the first sale was j , equals the price of the second sale of house i if the time of the second sale was j , and otherwise equals zero. The vector Y of observations on the dependent variable is specified so that Y_i equals the price of the first sale of house i if the first sale was in period

0, and is otherwise zero. The coefficient vector β now has its i th element equal a *reciprocal* price index for time i , equal to the estimated price at time zero divided by the estimated price at time i . By estimating reciprocal price indexes, rather than price indexes themselves, we have that the elements of $X\beta$ are all based on prices expressed in base-year units. The base period is again period 0, although now the index at time 0 is 1 instead of zero. In the example here, one may write the X and Y matrices as:

$$X = \begin{bmatrix} -P_{11} & P_{12} \\ -P_{21} & P_{22} \\ P_{31} & 0 \\ 0 & P_{42} \\ P_{51} & 0 \end{bmatrix} \quad Y = \begin{bmatrix} 0 \\ 0 \\ P_{30} \\ P_{40} \\ P_{50} \end{bmatrix}$$

This X matrix has zeros wherever Z_L did, and replaces -1 with minus a price and replaces $+1$ with a price.

The instrumental variables estimator for beta, taking either Z_L or Z_D as instruments, is $\hat{\beta} = (Z'X)^{-1}Z'Y$, where the subscript D or L under Z is omitted since it has no effect on the estimate. As in the BMN case, the normal equations have a particularly simple form if Z_D is used; in our example:

$$\hat{\beta}_1^{-1} = \text{Index}_1 = \frac{P_{31} + (\hat{\beta}_2 / \hat{\beta}_1)P_{42} + P_{51}}{P_{30} + P_{40} + P_{50}} \quad (6)$$

$$\hat{\beta}_2^{-1} = \text{Index}_2 = \frac{P_{12} + P_{22} + P_{42}}{P_{40} + \hat{\beta}_1(P_{11} + P_{21})} \quad (7)$$

These normal equations are analogous to those above for the BMN case, but the index is based on ratios of average prices rather than averages of log price changes. As with the BMN index, "missing" prices of houses between sales in the relevant interval but not sold at the ends of the interval are inferred using the estimated index itself. Of course, the normal equations, one for each time period, could just as well have been presented above with $\hat{\beta}_2$ on the left hand side of both equations. In effect, the T elements of $\hat{\beta}$ are inferred by writing $\hat{\beta}$ in terms of prices and $\hat{\beta}$ itself T different ways, allowing for solution for all T values of $\hat{\beta}$. These normal equations look rather more like indices of portfolio values, the numerator representing the value at time t of a portfolio of houses, the denominator representing the value of the same portfolio in the base period.

Of course, the standard stock price indices are also taken as having a portfolio interpretation: the Standard and Poor Composite Stock Price Index, for example, is the value of a portfolio of 500 stocks. Still, the analogy of these repeated measures indices (as in (6) and (7) above) to the wellknown stock price indices is not complete in the sense that the prices in the ratios above are not weighted by numbers of "shares" outstanding; these indices allow the composition of houses sold to affect the final index. In Shiller (1992) some methods of controlling for characteristics of houses in the context of a repeated measures index, effectively methods of weighting the prices by the numbers of shares, are discussed.

Further work in constructing index number methods for the purpose of cash settlement of contracts could deal with such issues as the possible variance components for the error terms discussed above, of methods of handling revisions in the indices as new data come in, of methods of dealing with delayed reporting of basic data, and of reducing the scope for manipulability of the indices.

7. Conclusion — A Future for Futures Markets

The proliferation of futures markets that we have seen has tended to follow conservative lines - showing a gradual pushing at the boundaries of existing markets, but only infrequent major innovations. We have now many different stock price index futures markets, but still no futures markets in wage costs, for real estate or for cost indices for production. There is much that theoretical econometricians have to offer in facilitating further development of these markets. The measurement problems that econometricians will have to confront are many and varied — as there should be demand for many regional or industry-specific indices; the data and measurement problems that are confronted in these various circumstances will likely vary widely. Much remains to be done on developing good methods of index number construction that are adapted to the various data and measurement problems we will encounter as these markets proliferate.

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