A RELATIONSHIP BETWEEN EXTERNAL PUBLIC DEBT AND ECONOMIC GROWTH*

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Resumen: Se presenta un modelo de crecimiento endógeno con dos bienes, comerciable (manufacturero) y no-comerciable (no-manufacturero). El conocimiento tecnológico doméstico es producido únicamente en el sector comerciable. Este conocimiento se desborda hacia el sector no-comerciable. El gobierno emite deuda externa para financiar parte de su gasto en bienes comerciables. La tasa de interés domestica es igual a la tasa de interés mundial más la prima de riesgo país. El riesgo país depende positivamente del nivel de la deuda pública externa. Los hogares piden prestado al exterior y tienen una restricción de crédito externo. Se obtiene, en el estado estacionario, una relación no lineal entre la proporción deuda pública externa a PIB y la tasa de crecimiento, en forma de U invertida. Hay evidencia empírica que muestra la existencia de esta no linealidad entre deuda pública y crecimiento, tanto para países en desarrollo como desarrollados.

Abstract: An endogenous growth model with two goods, tradable (manufacturing) and non-tradable (non-manufacturing) is presented. Domestic technological knowledge is produced only in the tradable sector. This knowledge overflows into the non-tradable sector. The government issues external debt to finance part of its spending on tradable goods. The domestic interest rate equals the world interest rate plus the country risk premium. The country risk depends positively on the level of external public debt. Households can borrow abroad and have an external credit constraint. An inverted U-shaped nonlinear relationship between the external public debt to GDP ratio and the growth rate is obtained in the steady state. There is empirical evidence showing the existence of this non-linearity between public debt and growth, for both developing and developed countries.

Clasificación JEL/JEL Classification: F21, F36, F43, O41

Palabras clave/keywords: sector comerciable, aprendizaje por la práctica, deuda pública externa, crecimiento económico, tradable sector, learning by doing, external public debt, economic growth

Fecha de recepción: 29 V 2014 *Fecha de aceptación: 24 II 2015*

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1. Introduction

External public debt can have nonlinear impacts on economic growth. Thus, at low levels of indebtedness, an increase in the proportion of external public debt to GDP could promote economic growth; however, at high levels of indebtedness, an increase in this proportion could hurt economic growth. This article studies the non-monotonic relationship between external public debt and economic growth via a model of endogenous growth.

The theory of economic growth examines the relationship between external debt and growth using some contributions from international finance. Thus, Krugman (1989) shows the debt relief Laffer curve (with the shape of an inverted U), where the nominal value of debt of a country and its actual expected payment are related. On the upward segment of the curve, debt and expected payments increase because the risk of default is low; in the descending segment, the level of debt increases but expected payments begin to descend because the risk of default is very high. He concludes that when a country is on the descending segment of the curve, the country suffers from debt overhang.[1] In this situation of debt overhang, external debt obligations act as a tax on investment.

Using the above concepts, researchers have studied the effects of external over-indebtedness. Thus, Cohen (1993) extends the model of endogenous growth of Cohen and Sachs (1986) to formalize the relationship between external-indebtedness and investment.[2] Consequently, Cohen presents and compares three economic scenarios. In the first one, there is free access to the global financial market, and the rate of investment (and production) is greater than with financial autarky. In the second scenario, there is a credit restriction with soft repayment, and the investment rate is lower than the free access case but higher than the financial autarky case. In the third one, there is

[1] Krugman (1988) defines a debt overhang problem in a country as a situation where its debt is greater than the present value of future resource transfers that creditors expect.

[2] Cohen and Sachs (1986) develop a model of endogenous growth in which external debt can be repudiated. Starting without external debt, the economy will have two stages of indebtedness. The first, without restriction to external credit, is characterized by an increase in the ratio of external debt to product and an initial high growth rate of product, but progressively decreasing. The second stage, with restricted borrowing, is characterized by a constant ratio of external debt to product and by a lower economic growth rate than the growth rates of the first stage.

a credit restriction with forced repayment, and the investment rate is lower than with financial autarky. Therefore, the investment rate rises in the first scenario, before falling in the second and third scenarios. Thus, the relationship between external debt and investment (and growth) is non-linear. In addition, Cohen concludes that the third scenario would correspond to the concept of debt overhang, where external debt acts as a tax on investment, hurting economic growth.

Moreover, Saint-Paul (1992) shows an endogenous growth model with overlapping generations, where an increase in public debt reduces the growth rate of the economy. Adam and Bevan (2005) develop a model of endogenous growth with individuals who live two periods. They study various ways to finance public deficits. An increase in domestic public debt slows growth, while an increase in external public debt, financed in concessional terms, but rationed, helps growth. Aizenman, Kletzer and Pinto (2007) show a model of endogenous growth with restrictions in tax revenues and public debt. In general, they find that the higher the public debt the lower the growth. Finally, Checherita-Westphal, Hallett and Rother (2014) present a growth model with public capital and debt, where the public deficit is equal to public investment. In their model, the relationship between debt and growth is nonlinear. So, in the steady state, the optimal debt to GDP ratio can be determined where growth is maximized.

In order to study the relationship between external public debt and economic growth, this article presents an endogenous growth model for a small open economy.

The economy produces two goods, tradable (manufacturing) and non-tradable (non-manufacturing). The tradable sector produces domestic technological knowledge through learning by doing (Romer, 1989). This knowledge is used in the non-tradable good sector. Therefore, in this model there are two learning externalities.[3] The government taxes households with a lump-sum tax to finance spending on tradable goods and interest payments on its external debt, and the difference between these expenditures and tax revenue, if any, is covered by external public debt. The foreign lenders perceive a country risk that depends positively on the level of external public debt. More-

[3] This model is related to the productive structure of a dependent economy with externalities. For example, Brock and Turnovsky (1994) develop a model with a tradable capital good and another non-tradable capital good. Turnovsky (1996) presents a model of endogenous growth where physical capital is tradable and human capital is non-tradable. Korinek and Serven (2010) develop a model of endogenous growth in which the production of tradable goods generates higher learning externalities than the production of non-tradable goods.

over, the government collects taxes through another lump-sum tax on households to finance the purchase of non-tradable goods. Households consume a constant fraction of their disposable income, and own the two types of capital. They can borrow abroad, subject to a foreign credit constraint. In this article, country risk is fully transferred to the private sector.[4] Thus, interest rate parity adjusted by country-risk is assumed, and as a result, the interest rate on the two types of capital, external private debt and public debt is equal to the world interest rate plus the country risk premium.

I study how the economy responds, in the steady state, to an increase in the proportion of external public debt to GDP and I obtain a nonlinear relationship between the ratio of external public debt to GDP and the growth rate. That is, my results show an inverted U-shaped curve connecting external public debt and economic growth. This nonlinearity is the result of two opposite effects on the growth rate of the economy when the proportion of external public debt to GDP increases. The positive effect is as follows: when the proportion of external public debt to GDP increases, the relative price of the non-tradable good decreases (the real exchange rate depreciates) and the tradable sector, leader in technological terms, attracts resources. Therefore, the proportion of labor employed in the manufacturing sector increases and the ratio of non-tradable to tradable capital diminishes, increasing the growth rate of the economy. The negative effect is as follows: when the proportion of external public debt to GDP increases, the country risk premium increases and interest payments on total external debt increases. Therefore, household disposable income falls, the proportion of savings to GDP declines and resources for capital accumulation decrease, thus the growth rate of the economy decreases.

At a high external public debt to GDP ratio, the economic growth that is stimulated by the depreciation of the real exchange rate and the attraction of resources towards the tradable sector, is offset by the exit of resources to the exterior (due to the burden of external debt), and the consequent decrease in the savings to GDP ratio.

The results of this paper are related to Cohen (1993) and Checherita-Westphal, Hallett and Rother (2014), where nonlinear relationships between debt and growth are also presented, although in their models, the external debt affects economic growth through different

[4] Pancrazi, Seoane and Vukotic (2014) show that, for five European economies, the relationship between the sovereign risk premium and the risk premium for private loans is negative for the period 2003-2008 and is positive during the crisis (2008-2011).

channels than those presented here. The result that public spending on tradable goods leads to a depreciation of the real exchange rate, stimulating the tradable sector, is related, inversely, with Korinek and Serven (2010). They affirm that the accumulation of international reserves (the extension of credit to foreigners for the purchase of domestic tradable goods) leads to a depreciation of the real exchange rate, stimulating the tradable sector and triggering the desired learning effects.

The results I obtained for an economy with endogenous growth, with two goods, two learning externalities, where the external public debt acts positively and negatively on economic growth, are not present in the literature and contribute to a better understanding of the relationship between external public debt and economic growth. Moreover, the existence of a maximum level of external public debt means that those responsible for public finances should be prudent in handling external public debt, to avoid high debt levels and prevent the kind of situations that occurred in Latin America in the 1980s and in the European periphery in recent years.

In the empirical literature, there is evidence showing the existence of this non-linearity between public debt and growth, for both developing and developed countries. Thus, Pattillo, Poirson and Ricci (2002, 2011) study the contribution of the proportion of external debt to GDP to the growth of per capita GDP for 93 developing countries between the years 1969-1998. They find that the contribution of external debt (current net value) on growth is nonlinear, in the form of an inverted U. The critical point, where the contribution of external debt to growth becomes negative, is between 35 and 40% of GDP. The negative impact of the high level of external debt on growth operates through adverse effects on the formation of physical capital and total factor productivity (see Pattillo, Poirson and Ricci 2004). Analyzing 55 low-income countries between the years 1970 to 1999, Clements, Bhattacharya and Nguyen (2003) argue that the servicing of external debt negatively affects public investment and, indirectly, growth.

Recently, Reinhart and Rogoff (2010) argue that the relationship between public debt to GDP ratio and growth for advanced and emerging countries is weak at levels of public debt to GDP ratio lower than 90%, but the relationship is negative for ratios greater than 90%. Similarly, Caner, Grennes and Koehler-Geib (2010) determine the critical level, where an increase in average public debt ratio to GDP decreases the average annual growth for developed and developing countries between the years 1980-2008. They conclude that for the total sample of countries, the threshold stands at 77.1% of GDP.

For developing economies, the critical level is at 64% of GDP. Also, Checherita-Westphal and Rother (2012) show that the relation between public debt to GDP ratio and growth in per capita income has an inverted U shape, for a sample of 12 countries in the euro area, with data since 1970. The threshold is between 90-100% of GDP, and could even begin at levels of 70-80% of GDP. The main channels through which public debt affects the rate of growth are private saving, public investment and total factor productivity. However, in a detailed review of the empirical literature, Panizza and Presbitero (2013) argue that the non-linear relationship between public debt and growth, with a threshold of 90% of GDP, is not robust across samples, specifications and estimation techniques.

It is important to mention that in the empirical literature there is also evidence of a relationship which is always negative between debt and growth; for example, Kumar and Woo (2010) show an inverse relationship between initial public debt and growth of per capita GDP for advanced and developing economies for the period 1970-2007.

The paper is organized as follows. In section 2, I develop an endogenous growth model of a small open economy. In section 3, I redefine the model in stationary variables. In section 4, I demonstrate the existence and stability of the steady state, as well as the nonlinear relationship between external public debt and growth. I present my conclusions in section 5.

2. The economy

In this model, the economy is small, so the world market determines the price of the tradable good and the world interest rate. Moreover, there is a country risk that depends positively on the external public debt. The tradable and non-tradable goods are produced using physical capital, labor and domestic technological knowledge. For simplicity, the tradable sector is the only one that generates domestic technological knowledge through learning by doing. Knowledge overflows to the non-tradable sector. Capital is specific in both sectors. The representative firm, in both the tradable and the non-tradable sector, maximizes profits taking the externality as given. The government imposes two lump sum taxes on households, one to finance spending on tradable goods and interest payments on the external public debt, and the other to finance the purchase of non-tradable goods. The government borrows from the rest of the world to purchase tradable goods. The representative household consumes a constant fraction of its disposable income. Households can borrow abroad, and have an external credit constraint. The total supply of labor is constant, and there is free mobility of labor between the two productive sectors.

2.1. *Production of the tradable good*

It is assumed that the production function of the tradable sector is Cobb-Douglas:

$$Y_T = A_T K_T^{\alpha} L_T^{1-\alpha} E_1 \tag{1}$$

where Y_T is the production of the tradable good, A_T is a positive parameter of efficiency, K_T is the stock of physical capital accumulated of the tradable good, L_T is the labor employed in the sector, α and $1-\alpha$ are the shares of K_T and L_T, respectively, with $0 < \alpha < 1$, and E_1 is a learning externality. It is assumed that K_T is used only in the tradable sector.

Domestic technological knowledge is created through learning by doing in the sector. Therefore, E_1 is the external effect of K_T in the production function of the tradable sector. In order to generate endogenous growth, it is assumed that $E_1 = K_T^{1-\alpha}$ so the production function of the tradable sector has constant returns in a broad measure of capital (see Romer, 1989).

P_T^w is defined as the world price of the tradable good, which is constant and given by the world market. The price of the tradable good is used as the *numéraire* ($P_T^w = 1$). Furthermore, r^w is defined as the world interest rate, which is constant. I introduce a country risk premium on r^w. Since this model is an endogenous growth model, I assume that an indicator that measures country risk is the ratio of external public debt to K_T, defined as d. The greater the proportion d, the higher the level of country risk. Thus, the model assumes that $d = D_G/K_T$ where D_G is the external public debt. Since this article assumes that the country risk is fully transferred to the private sector, a parity of returns adjusted for country risk is assumed. Therefore, the interest rate, r, on domestic assets and external private and public debt is:

$$r = r^w + \eta d \tag{2}$$

where η is a positive parameter that reflects country specific factors (see Eicher and Turnovsky, 1999, Eicher and Hull, 2004). Considering that the rate of depreciation of K_T is zero, we obtain $R_T = P_T^w \left(r - \dot{P}_T^w / P_T^w \right)$, where R_T is the rental price of K_T and $\dot{P}_T^w / P_T^w$ are the capital gains of K_T, with $\dot{P}_T^w = dP_T^w / dt$. Considering that P_T^w is the *numéraire*, the rental price of K_T is $R_T = r$. Firms in the tradable sector maximize profits taking the externality as given. The first order conditions are:

$$w_T = A_T K_T \left(1 - \alpha\right) L_T^{-\alpha} \tag{3}$$

$$R_T = r = A_T \alpha K_T^{\alpha - 1} L_T^{1-\alpha} \left[K_T^{1-\alpha} \right] = A_T \alpha L_T^{1-\alpha} \tag{4}$$

Equation (3) establishes that the wage is equal to the value of the marginal product of labor in the tradable good sector. Equation (4) states that the rental price of K_T is equal to the marginal product of K_T.

2.2. *Production of the non-tradable good*

With respect to the non-tradable sector, the production function is Cobb-Douglas:

$$Y_N = A_N K_N^{\beta} L_N^{1-\beta} E_2 \tag{5}$$

where Y_N is the production of the non-tradable good, A_N is a positive parameter of efficiency, K_N is the stock of physical capital accumulated from the non-tradable good, L_N is the labor employed in the sector, β and $1 - \beta$ are the shares of K_N and L_N, respectively, with $0 < \beta < 1$, and E_2 is a learning externality. The stock of K_N is used only in the non-tradable sector.

As the knowledge generated in the tradable sector is a public good, there is a spillover effect of knowledge between sectors. Thus, E_2 is the contribution of domestic technological knowledge in the production of the non-tradable good. Additionally, in order to have constant returns for a broad measure of capital in the sector, it is assumed that $E_2 = K_T^{1-\beta}$.

The variable p_N is defined as the relative price of the non-tradable to the tradable good. Considering that the rate of depreciation of K_N is zero, the rental price of K_N is $R_N = p_N(r - \dot{p}_N/p_N)$, where $\dot{p}_N/p_N$ is the growth rate of p_N, or the capital gains of K_N. Non-tradable firms maximize profits taking the externality as given. The first order conditions are:

$$w_N = p_N A_N K_N^{\beta} K_T^{1-\beta} (1-\beta) L_N^{-\beta} \tag{6}$$

$$\begin{aligned} R_N = p_N (r - \dot{p}_N/p_N) &= p_N A_N \beta K_N^{\beta-1} L_N^{1-\beta} \left[K_T^{1-\beta}\right] \\ &= p_N A_N \beta K_N^{\beta-1} K_T^{1-\beta} L_N^{1-\beta} \end{aligned} \tag{7}$$

Equation (6) states that the wage equals the value of the marginal product of labor in the non-tradable sector. Equation (7) is the dynamic equilibrium condition for K_N. Thus, the equation says that the rental price of K_N is equal to the value of the marginal product of K_N.

In models with tradable and non-tradable goods, the real exchange rate is defined as the level of relative prices of non-tradable goods in the foreign country in physical terms divided by the level of relative prices of non-tradable goods in the domestic country in physical terms. Considering that the level of relative prices of the foreign country is constant, the real exchange rate is inversely related to the level of relative prices of non-tradable goods in the domestic country in physical terms. Therefore, an increase in p_N corresponds to an appreciation of the real exchange rate.

2.3. *Government*

Regarding the tradable goods, the government spends a certain sum on consumption and interest payment on its external debt. Its spending is financed by a lump sum tax levied on households and by foreign loans. Consequently, the government budget constraint on the tradable good is:

$$\dot{D}_G = rD_G + G_T - T_T \tag{8}$$

where D_G is the external public debt, $\dot{D}_G$ is the increase in public debt over time, rD_G is interest payment on the public debt, G_T is spending on consumption on the tradable good, and T_T is a lump sum tax. The level of external public debt is measured as a constant fraction, θ_G, of Y_T, that is, $D_G = \theta_G Y_T$, where $\theta_G > 0$ and $\dot{D}_G = \theta_G \dot{Y}_T$. Furthermore, it is assumed that $G_T = \phi_T Y_T$; that is, public spending on tradable goods is a constant fraction, ϕ_T, of the product of the tradable sector, where $0 < \phi_T < 1$. Given that the inter-temporal budget constraint of the government, deducible from equation (8), is met by the appropriate adjustment of some residual fiscal variable (see Serven, 2007), I assume that the level of T_T is adjusted residually.[5] Considering the prior definitions given, I find that:

$$T_T = r\theta_G Y_T + \phi_T Y_T - \theta_G \dot{Y}_T \tag{9}$$

Regarding the non-tradable goods, the government has an expenditure on consumption and this consumption is financed by a lump sum tax charged to households. Therefore, the government budget constraint in the non-tradable good is $T_N = p_N G_N$, where T_N is a lump sum tax and $p_N G_N$ is consumption spending in the non-tradable good. I assume that $p_N G_N = \phi_N p_N Y_N$; that is, public spending on non-tradable goods is a constant fraction, ϕ_N, of the product of the non-tradable sector, where $0 < \phi_N < 1$. Considering the above definitions, we have:

$$T_N = \phi_N p_N Y_N \tag{10}$$

Therefore, the government only borrows from the rest of the world for the purchase of tradable goods.

[5] Alternatively, a rule of debt stabilization can be postulated, $T_T = \tau D_G$ where τ is the variable rate of adjustment. The end result is the same (see Brauninger 2005 and Heijdra and Ploeg 2002).

2.4. *Households*

Households own K_T and K_N, and foreigners own the external debt of the households. The household budget constraint is:

$$w_T L_T + w_N L_N + R_T K_T + R_N K_N - T_T - T_N - rD_H \tag{11}$$
$$= C_T + p_N C_N + I_T + p_N I_N - \dot{D}_H$$

where $w_T L_T + w_N L_N$ is wage income, $R_T K_T + R_N K_N$ is the income from K_T and K_N, respectively, T_T and T_N are lump sum taxes, D_H is the external debt of the households, rD_H is the interest payment, C_T is consumption of the tradable good, C_N is consumption of the non-tradable good, $I_T = \dot{K}_T$ is the net investment in K_T, $I_N = \dot{K}_N$ is the net investment in K_N, and $\dot{D}_H$ is the increase in household debt through time.

I assume that only a constant and exogenous fraction, θ_H, of K_T can be used as collateral for loans in the world market, with $0 < \theta_H < 1$. Therefore, the borrowing constraint is $D_H = \theta_H K_T$. Thus, domestic residents own the entire stock of K_T, which is partially funded by the world market, and external residents own the debt of K_T (see Barro, Mankiw and Sala-i-Martin, 1995). Moreover, given that $D_H = \theta_H K_T$, one obtains $\dot{D}_H = \theta_H \dot{K}_T$.

Next, I deduce the consumption demands for the tradable and non-tradable goods. I assume that consumption demands result from the maximization of the utility function $u = C_T^{\gamma} C_N^{1-\gamma}$ subject to the constraint of the total consumer spending $C = C_T + p_N C_N$, where γ and $1-\gamma$ indicate the proportion of the expenditure in C_T and C_N with respect to aggregated consumption, C, respectively, with $0 < \gamma < 1$. Thus, the demands for C_T and C_N are: $C_T = \gamma C$ and $p_N C_N = (1-\gamma)\, C$.

For simplicity, I assume that households choose the level of aggregate consumption as a constant fraction of disposable income, $w_T L_T + w_N L_N + R_T K_T + R_N K_N - T_T - T_N - rD_H$ (there is no possibility for inter-temporal choice, which is a limitation of the model). Consequently, I obtain:

$$C = (1-s) \left[w_T L_T + w_N L_N + R_T K_T + R_N K_N - T_T - T_N - rD_H \right] \tag{12}$$

where s is the savings rate and $(1-s)$ is the consumption rate, s is constant and exogenous, with $0 < s < 1$. Note that since T_T, represented by equation (9), is a residual tax, the disposable income of the households is decreased by the interest payments on the external public debt and by public spending on the tradable good, but increased by $\dot{D}_G$, along with public spending on the non-tradable good, represented by equation (10).

2.5. *Equilibrium*

First, I deduce the aggregate condition of savings being equal to investment. Substituting w_T, w_N, R_T and R_N, equations (3), (4), (6) and (7), in equation (11), I obtain the resource constraint of the economy:

$$Y_T + p_N Y_N - T_T - T_N - rD_H = C_T + p_N C_N + I_T + p_N I_N - \dot{D}_H \quad (13)$$

where $Y_T + p_N Y_N - T_T - T_N - rD_H$ is equivalent to household disposable income. Thus, the aggregated consumption of households is:

$$C = (1-s)\left[Y_T + p_N Y_N - T_T - T_N - rD_H\right] \quad (14)$$

Substituting equation (14) in (13), with $C = C_T + p_N C_N$, one obtains:

$$s\left[Y_T + p_N Y_N - T_T - T_N - rD_H\right] + \dot{D}_H = I_T + p_N I_N \quad (15)$$

Equation (15) says that household savings plus the foreign credit extended to households serve to finance capital accumulation.

Next, I obtain the equilibrium conditions of the market of the tradable and non-tradable goods. Since the relative price of the non-tradable good is flexible, the supply of the non-tradable good is always equal to its demand. Therefore, the equilibrium condition for the market for the non-tradable good is:

$$p_N Y_N = p_N C_N + p_N G_N + p_N I_N \quad (16)$$

where $p_N G_N = T_N$. In order to obtain the equilibrium condition for the market of the tradable good, equation (16) is substituted into (13), yielding:

$$Y_T - T_T - rD_H = C_T + I_T - \dot{D}_H \tag{17}$$

Substituting the government budget constraint, equation (8), in the above equation results in:

$$Y_T - G_T - r(D_H + D_G) + \dot{D}_H + \dot{D}_G = C_T + I_T \tag{18}$$

Considering that $D = D_H + D_G$, where D is the total external debt, and that $\dot{D} = \dot{D}_H + \dot{D}_G$, the current account is defined as:

$$\dot{D} = rD - NX \tag{19}$$

where NX is the trade balance. Finally, substituting (19) into (18), one obtains:

$$Y_T = C_T + G_T + I_T + NX \tag{20}$$

The previous equation shows the equilibrium condition for the market of the tradable good. Regarding the labor market, I assume that the total labor supply, L, is constant. The equilibrium condition in the labor market is $L = L_T + L_N$.

3. The model in stationary variables

Given that the variables K_T and K_N show a constant, common rate of growth, it is necessary to define the model variables as stationary variables, that is, variables that are constant in the steady state. Thus, $z = K_N/K_T$ is defined as a stationary variable. Furthermore, given that L is constant, it is normalized to one ($L = 1$). Thus, the equilibrium condition in the labor market is: $n + (1 - n) = 1$, where n is the fraction of labor employed in the tradable sector and $(1 - n)$

is the fraction of the labor employed in the non-tradable sector. As n is constant in the steady state, the variable n is also stationary. Similarly, as the relative price of the non-tradable good must be constant in the steady state, p_N is another stationary variable.

Taking into consideration the externalities E_1 and E_2, the production functions in stationary variables are:

$$Y_T = A_T K_T n^{1-\alpha} \tag{21}$$

$$Y_N = A_N z^{\beta} K_T (1-n)^{1-\beta} \tag{22}$$

Also, since $D_G = \theta_G Y_T$, then $d = D_G / K_T = \theta_G A_T n^{1-\alpha}$. Using the equation (2), the rate of interest must be:

$$r = r^w + \eta\ \theta_G A_T n^{1-\alpha} \tag{23}$$

The marginal conditions for the tradable sector in stationary variables are:

$$w_T = A_T K_T\, (1-\alpha)\, n^{-\alpha} \tag{24}$$

$$r = A_T \alpha n^{1-\alpha} \tag{25}$$

The first order conditions for the non-tradable sector in stationary variables are:

$$w_N = p_N A_N z^{\beta} K_T\, (1-\beta)\, (1-n)^{-\beta} \tag{26}$$

$$r - \frac{\dot{p}_N}{p_N} = \frac{A_N \beta (1-n)^{1-\beta}}{z^{1-\beta}} \tag{27}$$

I assume that $\alpha > \beta$, so the tradable sector is more capital intensive than the non-tradable sector. Turnovsky (1997) shows that the dynamic in dependent economies changes when one sector is more capital intensive than the other.

The static condition of efficient allocation of labor between the two sectors is obtained by equating (24) and (26):

$$A_T\,(1-\alpha)\,n^{-\alpha} = p_N A_N z^{\beta}(1-\beta)(1-n)^{-\beta} \tag{28}$$

This condition says that the value of the marginal product of labor in both sectors must be equal at all times. With equation (28), the level of p_N is:

$$p_N = \frac{A_T\,(1-\alpha)\,(1-n)^{\beta}}{A_N z^{\beta}\,(1-\beta)\,n^{\alpha}} \tag{29}$$

Then, the growth rates of K_T and K_N are obtained in stationary variables. Substituting $I_T = \dot{K}_T,\ I_N = \dot{K}_N$ and $\dot{D}_H = \theta_H \dot{K}_T$ into equation (15), and dividing by K_T, one obtains:

$$\frac{s\,[Y_T + p_N Y_N - T_T - T_N - rD_H]}{K_T} + \theta_H \frac{\dot{K}_T}{K_T} = \frac{\dot{K}_T}{K_T} + p_N z \frac{\dot{K}_N}{K_N} \tag{30}$$

Now, I determine $\dot{K}_N/K_N$ in function of $\dot{K}_T/K_T$. Taking logarithms and time derivatives of $z = K_N/K_T$, one obtains:

$$\frac{\dot{z}}{z} = \frac{\dot{K}_N}{K_N} - \frac{\dot{K}_T}{K_T} \tag{31}$$

As will be apparent later, n is always in a steady state and is constant. Taking logarithms and time derivatives of equation (29), I obtain:

$$\frac{\dot{z}}{z} = -\frac{1}{\beta}\frac{\dot{p}_N}{p_N} \tag{32}$$

Equating (31) and (32), I have:

$$\frac{\dot{K}_N}{K_N} = \frac{\dot{K}_T}{K_T} - \frac{1}{\beta}\frac{\dot{p}_N}{p_N} \tag{33}$$

Substituting equation (33) into equation (30), I have:

$$\begin{aligned} &\frac{s\left[Y_T + p_N Y_N - T_T - T_N - rD_H\right]}{K_T} + \theta_H \frac{\dot{K}_T}{K_T} \\ &= \frac{\dot{K}_T}{K_T} + p_N z\left[\frac{\dot{K}_T}{K_T} - \frac{1}{\beta}\frac{\dot{p}_N}{p_N}\right] \end{aligned} \tag{34}$$

Substituting T_T and T_N, equations (9) and (10), with $\dot{Y}_T = A_T n^{1-\alpha}\dot{K}_T$ in the above equation, I obtain:

$$\begin{aligned} &\frac{s\left[(1-\phi_T)\,Y_T + (1-\phi_N)\,p_N Y_N - r\,(D_H + \theta_G Y_T)\right]}{K_T} \\ &+ s\;\theta_G A_T n^{1-\alpha}\frac{\dot{K}_T}{K_T} + \theta_H \frac{\dot{K}_T}{K_T} = \frac{\dot{K}_T}{K_T} + p_N z\left[\frac{\dot{K}_T}{K_T} - \frac{1}{\beta}\frac{\dot{p}_N}{p_N}\right] \end{aligned} \tag{35}$$

Finally, substituting $D_H = \theta_H K_T$ and the production functions, equations (21) and (22), in equation (35) and solving for $\dot{K}_T/K_T$, the growth rate of K_T is obtained:

$$\begin{aligned} \frac{\dot{K}_T}{K_T} = &\left[\frac{1}{1 + p_N z - s\;\theta_G A_T n^{1-\alpha} - \theta_H}\right]\Big\{ s\Big[(1-\phi_T)A_T n^{1-\alpha} \\ &+(1-\phi_N)\;p_N A_N z^{\beta}\;(1-n)^{1-\beta} - r(\theta_H + \theta_G A_T n^{1-\alpha})\Big] + \frac{p_N z}{\beta}\frac{\dot{p}_N}{p_N}\Big\} \end{aligned} \tag{36}$$

where r is defined by equation (23). Similarly, I obtain the growth rate of K_N in stationary variables. Substituting $I_T = \dot{K}_T$, $I_N = \dot{K}_N$, and $\dot{D}_H = \theta_H \dot{K}_T$ into equation (15), dividing by K_N, using $\dot{K}_T/K_T$, equation (33), substituting T_T, equation (9) and T_N, equation (10), and using the production functions, equations (21) and (22), I obtain:

$$\frac{\dot{K}_N}{K_N} = \left[\frac{1}{1 + p_N z - s\ \theta_G A_T n^{1-\alpha} - \theta_H}\right] \Bigg\{ s \Big[(1-\phi_T) A_T n^{1-\alpha} + (1-\phi_N)\ p_N A_N z^{\beta}\ (1-n)^{1-\beta} - r(\theta_H + \theta_G A_T n^{1-\alpha}) \Big] - \frac{(1 - s\ \theta_G A_T n^{1-\alpha} - \theta_H)}{\beta} \frac{\dot{p}_N}{p_N} \Bigg\} \tag{37}$$

As n is always in a steady state and is constant, it is possible to show that the rate of growth of national income, $Y = Y_T + p_N Y_N - r\left(\theta_H K_T + \theta_G A_T K_T n^{1-\alpha}\right)$, is:

$$\frac{\dot{Y}}{Y} = \frac{\dot{K}_T}{K_T} + \frac{p_N Y_N}{Y} \left(\beta \frac{\dot{z}}{z} + \frac{\dot{p}_N}{p_N} \right) \tag{38}$$

where $p_N Y_N / Y$ is the participation of $p_N Y_N$ in the national income. In the next section, the steady state solution is shown.

4. The steady state and the relationship between external public debt and growth

The steady state solution implies the existence of the equilibrium. Therefore, the growth rates of z, n and p_N must be zero in the steady state, so their levels remain constant. Furthermore, the growth rates of K_T, K_N, Y_T, Y_N and Y must be equal to a constant rate in the steady state.

Therefore, with equations (23) and (25), I have:

$$n^* = \left[\frac{r^w}{A_T\,(\alpha - \eta\ \theta_G)}\right]^{\frac{1}{1-\alpha}} \tag{39}$$

Given that r^w, η, θ_G, A_T and α are constant, the level of n^* is constant in the steady state (steady state levels are denoted with *). It can be shown that $\partial n^*/\partial\theta_G > 0$; that is, an increase in θ_G increases n^*. Also, given an increase in θ_G, the level of n^* will jump immediately to the new steady state level and, thus, n^* will always be in a steady state, as mentioned above. With equation (23), the interest rate in the steady state is $r^* = r^w + \eta\theta_G A_T n^{*(1-\alpha)}$.

Using equation (27), with $\dot{p}_N = 0$ and $r^* = A_T\alpha n^{*(1-\alpha)}$, I obtain:

$$z^* = \left[\frac{A_N\beta}{A_T\alpha}\right]^{\frac{1}{1-\beta}} \frac{(1-n^*)}{n^{*(1-\alpha)/(1-\beta)}} \tag{40}$$

As z^* depends on parameters, the level of z^* is constant in the steady state. It can be shown that $\partial z^*/\partial\theta_G = (\partial z^*/\partial n^*)(\partial n^*/\partial\theta_G) < 0$; that is, an increase in θ_G decreases z^*.

Using equation (29), and substituting the level of z^*, the relative price of the non-tradable good, p_N, in the steady state is:

$$p_N^* = \left[\frac{A_T\,(1-\alpha)}{A_N\,(1-\beta)}\right]\left[\frac{A_T\alpha}{A_N\beta}\right]^{\frac{\beta}{1-\beta}} \frac{n^{*\beta(1-\alpha)/(1-\beta)}}{n^{*\alpha}} \tag{41}$$

As p_N^* depends on parameters, the level of p_N^* is constant in the steady state. Given that $\alpha > \beta$, it is possible to demonstrate that $\partial p_N^*/\partial\theta_G = (\partial p_N^*/\partial n^*)\,(\partial n^*/\partial\theta_G) < 0$; that is to say that an increase in θ_G, causes p_N^* to decrease. Thus, for the stationary variables, the existence of the steady state has been analytically shown.

As in the steady state $\dot{p}_N = 0$, the growth rate of K_T in the steady state, equation (36), equals the growth rate of K_N in the steady state, equation (37). Similarly, as in the steady state $\dot{z} = 0$ and $\dot{p}_N = 0$, the rate of growth of national income, equation (38), is equal to $\dot{K_T}/K_T$. Also, the growth rate of Y_T and Y_N is equal to $\dot{K_T}/K_T$. Therefore, the steady state growth rate of the economy, g^*, is:

$$g^* = \left[\tfrac{1}{1+p_N^* z^* - s\ \theta_G A_T n^*(1-\alpha)-\theta_H}\right]\left\{ s\Big[(1-\phi_T)A_T n^{*(1-\alpha)} \right. \tag{42}$$

$$\left. +(1-\phi_N)\ p_N^* A_N z^{*\beta}\ (1-n^*)^{1-\beta} - r^*(\theta_H + \theta_G A_T n^{*(1-\alpha)})\Big]\right\}$$

Given that g^* depends only on parameters, the level of g^* is constant in the steady state.

In order to study the relationship between public debt and growth, I define the proportions of external public debt to GDP and savings to GDP in the steady state. Since the values of n^*, z^* and p_N^* are known, the ratio of external public debt to GDP in the steady state, $D_{G,GDP}^*$, is

$$D_{G,GDP}^* = \frac{\theta_G A_T n^{*(1-\alpha)}}{\left[A_T n^{*(1-\alpha)} + p_N^* A_N z^{*\beta}(1-n^*)^{1-\beta}\right]}$$

As $D_{G,GDP}^*$ depends on parameters, its level is constant in the steady state. Moreover, since g^* is known, the proportion of savings to GDP in the steady state, $S_{H,GDP}^*$, is:

$$S_{H,GDP}^* = \frac{s\left[(1-\phi_T)\, A_T n^{*(1-\alpha)} + (1-\phi_N)\, p_N^* A_N z^{*\beta}(1-n^*)^{1-\beta}\right]}{\left[A_T n^{*(1-\alpha)} + p_N^* A_N z^{*\beta}(1-n^*)^{1-\beta}\right]}$$

$$+\frac{s\left[-r^*\left(\theta_H + \theta_G A_T n^{*(1-\alpha)}\right) + \theta_G A_T n^{*(1-\alpha)} g^*\right]}{\left[A_T n^{*(1-\alpha)} + p_N^* A_N z^{*\beta}(1-n^*)^{1-\beta}\right]}$$

As $S_{H,GDP}^*$ depends on parameters, its level is constant in the steady state. Thus for g^*, $D_{G,GDP}^*$ and $S_{H,GDP}^*$, the existence of the steady state has also been analytically demonstrated.

Next, I numerically analyze, in the steady state, the relationship between the proportion of external public debt to GDP and the rate of growth of the economy. The values of the parameters α and β are taken from Valentinyi and Herrendorf (2008) where, for the US economy, they show that the tradable sector is more capital intensive, $\alpha = 0.37$, than the non-tradable sector, $\beta = 0.32$. Given that there is only net investment, the level of s is the rate of net national saving for OECD economies of 8% relative to national income, $s = 0.08$ (1980-2012 average, calculated using the world development indicators of the

World Bank). I calculated the levels of public expenditure, the final consumption expenditure of the general government sector of OECD economies, to be 18% of their GDP (1980-2013 average, calculated using the world development indicators of the World Bank). Given the lack of disaggregated data for expenditure on final consumption of the general government sector in tradable and non-tradable goods, I opted for $\phi_T = 0.10$ and $\phi_N = 0.18$. The level of $r^w = 0.031$ is the average level of the 10-year Treasury Bonds in the United States (2007-2013 average). As the levels of θ_H, η, A_T and A_N depend on the unique characteristics of each economy, they are set only for explanatory purposes.

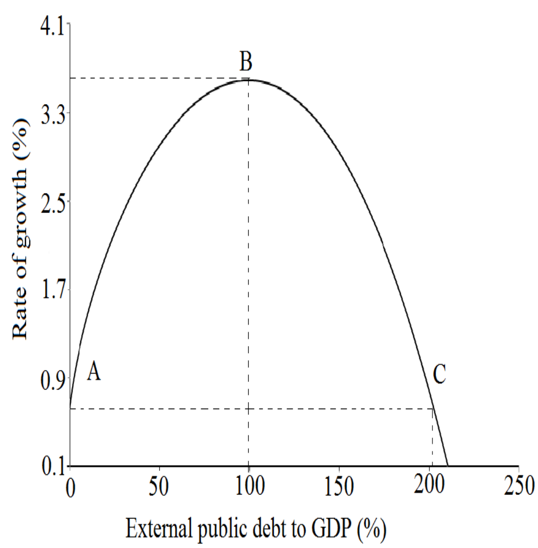
Therefore, I present a representative simulation, where the parameter values are: $\alpha = 0.37$, $\beta = 0.32$, $s = 0.08$, $\phi_T = 0.18$, $\phi_N = 0.18$, $r^w = 0.031$, $\theta_H = 0.1$, $\eta = 0.09$, $A_T = 1.5$ and $A_N = 0.3$. In the simulation, I used increasing levels of θ_G and $D^*_{G,GDP}$ to clearly illustrate the effect of the increasing external public debt on economic growth. Thus, in figure 1, the relationship between the proportion of external public debt to GDP and the growth rate in the steady state is shown. As shown in figure 1, there is a nonlinear relationship between the external public debt to GDP ratio and the growth rate. That is to say, there is an inverted U-shaped curve between the proportion of external public debt to GDP and growth. A large number of simulations were carried out and the result holds. This nonlinearity is explained by two opposite effects on the rate of growth of the economy when θ_G and $D^*_{G,GDP}$ increase. The positive effect is as follows: when θ_G and $D^*_{G,GDP}$ increase, p^*_Ndecreases; that is to say, the real exchange rate depreciates. Therefore, the tradable sector employs more labor and accumulates relatively more capital than the non-tradable sector; n^* increases and z^* decreases. Given that the tradable sector is the technological leader, the growth rate of the economy is benefited. The negative effect is as follows: when θ_G and $D^*_{G,GDP}$ increase, the country risk premium increases, and interest payments on private and public debt increase. Thus, national income and the savings to GDP ratio decrease, and the resources for capital accumulation decrease. Therefore, the growth rate of the economy is damaged.

At point A, in figure 1, $\theta_G = 0.0001$ and $D^*_{G,GDP} = 0\%$. The levels of the stationary variables are: $n^* = 0.0102$, $z^* = 5.217$ and $p^*_N = 14.812$. The growth rate in the steady state is 0.63% per annum. The country risk premium is zero and r^* is 3.1%. The level of savings to GDP ratio is 6.55%.

Point B, in figure 1, is where the maximum rate of growth is reached by increasing the external public debt to GDP ratio. The level of θ_G is 3.5 and the levels of the stationary variables are: $n^* = 0.2642$, $z^* = 0.191$ and $p_N^* = 11.665$. Now, the level of $D_{G,GDP}^*$ is 100% of the GDP and the growth rate is 3.59% per annum. If this case is compared with the previous one, it can be seen that the stationary variables move in the direction analytically predicted. In particular, p_N^* decreases from 14.812 to 11.665 (the real exchange rate depreciates), so the tradable sector attracts resources and that n^* increases from 0.0102 to 0.2642 and $z^* = K_N/K_T$ declines from 5.217 to 0.191. Given that the tradable sector is the sector that generates technical progress, this flow of resources to the tradable sector stimulates economic growth (positive effect). However, the negative effect is that the country risk premium increases 2089 basic points, and r^* is 23.99%. Thus, the payment of interest on external private and public debt causes the ratio of savings to GDP to decrease from 6.55 to 4.84%. It is important to note that at country risk levels of 2089 basic points, the probability of default is very high and the process of accumulation of external debt could stop abruptly, although this mechanism is not present in this model.

Figure 1

Relationship of external public debt to GDP and growth in the steady state



Point C, in figure 1, is where the increase in the external public debt to GDP ratio produces the same rate of growth as the case without external public debt, $g^* = 0.63\%$. The corresponding level of θ_G is 3.7638 and the levels of the stationary variables are: $n^* = 0.5185$, $z^* = 0.067$ and $p_N^* = 11.101$. In this case the level of $D_{G,GDP}^*$ is 202.3% of GDP and the growth rate is 0.63% per annum. If this case is compared with the case $\theta_G = 3.5$, it is observed that n^*, z^* and p_N^* move in the direction predicted analytically. In particular, p_N^* decreases from 11.665 to 11.101 (the real exchange rate depreciates even further). Therefore, the tradable sector attracts more resources, n^* increases from 0.2642 to 0.5185, and $z^* = K_N/K_T$ decreases from 0.191 to 0.067. This flow of resources to the tradable sector stimulates economic growth (positive effect). However, the country risk premium is 3359 basic points and r^* is 36.6%. Thus the interest payments on private and public external debt reduce disposable income and the savings to GDP ratio goes from 4.84 to 0.56% (negative effect). Finally, when the level of $D_{G,GDP}^*$ is 211% of GDP, the growth rate is zero.

In sum, an inverted U-shaped curve has been presented here, relating the external public debt with economic growth. In the upward segment of the curve, an increase in the external public debt to GDP ratio increases growth. However, in the downward segment of the curve, an increase in the external public debt to GDP ratio decreases growth. Depending on the levels of θ_H, η, A_T and A_N (whose values depend on specific factors in each country), the maximum level of $D_{G,GDP}^*$ may increase or decrease.

In order to study the stability and the transitional dynamics, equations (32) and (27) are used, yielding:

$$\dot{z} + (1/\beta)\left(r^w + \eta\theta_G A_T n^{1-\alpha}\right) z = A_N(1-n)^{1-\beta} z^\beta$$

The above equation is a Bernoulli equation, which can be solved by a change of variable $v = z^{1-\beta}$ and calculating $\dot{v} = (dv/dz)(dz/dt)$, we obtain:

$$\dot{v} + \left[(1-\beta)/\beta\right]\left(r^w + \theta_G A_T n^{1-\alpha}\right) v = (1-\beta)A_N(1-n)^{1-\beta}$$

where $\left[(1-\beta)/\beta\right]\left(r^w + \theta_G A_T n^{1-\alpha}\right)$ is a positive constant. Also, I find that $(1-\beta)A_N(1-n)^{1-\beta}$ is a positive constant. Thus, when θ_G

increases, z slowly decreases to its new lower steady state level. Using equation (29), one can see that at $t = 0$, p_N decreases instantaneously when n jumps, while z stays at the same level. Using equation (32), and since $\dot{z}/z < 0$, it can be seen that $\dot{p}_N/p_N > 0$. By all of the above, at $t = 0$, it follows that p_N decreases and overshoots its new steady state level. Thus, in the transition, p_N rises slowly to its new steady state level.

5. Conclusions

A model of endogenous growth was developed with two productive sectors, where the tradable sector is the only one that generates domestic technical progress. The knowledge generated in the tradable sector is used in the non-tradable sector. The tradable sector and the non-tradable sector can accumulate physical capital. The government spends on tradable goods and on interest payments on its external debt. This expenditure is financed by a lump sum tax levied on households, and by external public debt. The government also spends on non-tradable goods, which are financed by a lump sum tax. I assume that the country risk premium increases with the level of external public debt, and that households borrow from abroad, and face a restriction on foreign credit.

I demonstrate analytically that an increase in the external public debt to GDP ratio has a positive impact on the tradable sector by reducing the relative price of the non-tradable good. Thus, with the depreciation of the real exchange rate, the fraction of labor employed in the tradable sector increases and the proportion of non-tradable capital to tradable capital decreases The relationship between external public debt and economic growth is shown to have an inverted U-shape. Two opposite effects on the growth rate of the economy explain this nonlinearity between the external public debt to GDP ratio and growth. The positive effect is that, when the external public debt increases, the relative price of the non-tradable good decreases, so the tradable sector attracts resources, and since the tradable sector is the technological leader, the growth rate benefits. The negative effect is that, when the external public debt increases, the country risk premium increases, and interest payments on the private and public debt increase. Thus, the household disposable income and the savings to GDP ratio decrease, and the resources for capital accumulation are reduced; consequently, the growth rate is damaged.

Thus, it has been shown that at low levels of indebtedness, an increase in the external debt to GDP ratio could promote growth;

however, with high levels of indebtedness, an increase in the external debt to GDP ratio could hurt economic growth. This theoretical result resembles certain empirical results that have demonstrated a nonlinear relationship between debt and growth.

Furthermore, the inverted U-shaped relationship between external debt and economic growth indicates the existence of a maximum level of external debt, which the policy makers should avoid reaching, in order to prevent situations such as Latin America in the eighties and the European periphery in recent years.

References

Adam, C.S. and D.L. Bevan. 2005. Fiscal Deficits and Growth in Developing Countries, *Journal of Public Economics*, 89: 571-597.

Aizenman, J., K. Kletzer and B. Pinto. 2007. Economic Growth with Constraints on Tax Revenues and Public Debt: Implications for Fiscal Policy and Cross-Country Differences, NBER Working Paper Series, no. 12750.

Barro, R.J., N.G. Mankiw and X. Sala-i-Martin. 1995. Capital Mobility in Neoclassical Models of Growth, *American Economic Review*, 85: 103-115.

Brauninger, M. 2005. The Budget Deficit, Public Debt, and Endogenous Growth, *Journal of Public Economic Theory*, 7(5): 827-840.

Brock, P.L. and S.J. Turnovsky. 1994. The Dependent-Economy Model with Both Traded and Nontraded Capital Goods, *Review of International Economics*, 2: 306-325.

Caner, M., T. Grennes and F. Koehler-Geib. 2010. Finding the Tipping Point-When Sovereign Debt Turns Bad, The World Bank, Policy Research Working Papers, no. 5391.

Checherita-Westphal, C. and P. Rother. 2012. The Impact of High Government Debt on Economic Growth and its Channels: An Empirical Investigation for the Euro Area, *European Economic Review*, 56(7): 1392-1405.

Checherita-Westphal, C., A.H. Hallett and P. Rother. 2014. Fiscal Sustainability Using Growth-Maximizing Debt Targets, *Applied Economics*, 46: 638-647.

Clements, B., R. Bhattacharya and T.Q. Nguyen. 2003. External Debt, Public Investment, and Growth in Low-Income Countries, International Monetary Fund, IMF Working Papers, no. 249.

Cohen, D. 1993. Low Investment and Large LDC Debt in the 1980's, *American Economic Review*, 83(3): 437-449.

Cohen, D. and J. Sachs. 1986. Growth and External Debt Under Risk of Debt Repudiation, *European Economic Review*, 30: 529-560.

Eicher, T. and L. Hull. 2004. Financial Liberalization, Openness and Convergence, *Journal of International Trade & Economic Development*, 13: 443-459.

Eicher, T. and S.J. Turnovsky. 1999. International Capital Markets and Non-Scale Growth, *Review of International Economics*, 7: 171-188.

Heijdra, B. and F. van der Ploeg. 2002. *The Foundations of Modern Macroeconomics*, Oxford University Press.

Korinek, A. and L. Serven. 2010. Undervaluation Through Foreign Reserve Accumulation. Static Losses, Dynamic Gains, World Bank, Policy Research Working Papers, no. 5250.

Krugman, P. 1988. Financing vs. Forgiving a Debt Overhang, *Journal of Development Economics*, 29: 253-268.

——. 1989. Market-Based Debt-Reduction Schemes, in J.A. Frenkel, M.P. Dooley and P. Wickham (eds.) *Analytical Issues in Debt*, pp. 258-278.

Kumar, M.S. and J. Woo. 2010. Public Debt and Growth, International Monetary Fund, IMF Working Papers, no. 174.

Pancrazi, R., H.D. Seoane and M. Vukotic. 2014. Sovereign Risk, Private Credit, and Stabilization Policies , University of Warwick.

Panizza, U. and A.F. Presbitero. 2013. Public Debt and Economic Growth in Advanced Economies: A Survey, *Swiss Journal of Economics and Statistics*, 149(2): 175-204.

Pattillo, C., H. Poirson and L. Ricci. 2002. External Debt and Growth, International Monetary Fund, IMF Working Papers, no. 69.

——. 2004. What Are the Channels Through Which External Debt Affects Growth?, International Monetary Fund, IMF Working Papers, no. 15.

——. 2011. External Debt and Growth, *Review of Economics and Institutions*, 2: 1-30.

Reinhart, C.M. and K.S. Rogoff. 2010. Growth in a Time of Debt, *American Economic Review*, 100(2): 573-578.

Romer, P.M. 1989. Capital Accumulation in the Theory of Long Run Growth, in R. Barro (ed.), *Modern Business Cycle Theory*, Basil Blackwell.

Saint-Paul, G. 1992. Fiscal Policy in an Endogenous Growth Model, *Quarterly Journal of Economics*, 107: 1243-1259.

Serven, L. 2007. Fiscal Rules, Public Investment, and Growth, The World Bank, Policy Research Working Papers, no. 4382.

Turnovsky, S.J. 1996. Endogenous Growth in a Dependent Economy with Traded and Nontraded Capital, *Review of International Economics*, 4: 300-321.

——. 1997. *International Macroeconomic Dynamic*, The MIT Press.

Valentinyi, A. and B. Herrendorf. 2008. Measuring Factor Income Shares at the Sectorial Level, *Review of Economic Dynamics*, 11: 820-835.