PRICE DYNAMICS IN A TWO-PERIOD REPUTATION MODEL

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Resumen: Se estudia la trayectoria de los precios en una versión de dos periodos del modelo reputacional de Milgrom y Roberts (1982a). Esta versión difiere del modelo mencionado en que suplanta el juego-etapa de entrada-diferida por un juego-etapa de señalización-vía-precios. Se demuestra que los precios pueden subir o bajar de un período al otro. También se demuestra que en este tipo de modelo no hay margen para introducir 'prendas'.

Abstract: This paper looks at the time-path of prices in a two-period modified version of the reputation model of Milgrom and Roberts (1982a) in which a non-standard price signalling stage game is substituted for the entry-deterrence game those authors work with. It shows that prices may rise or fall from one period to the next. Also, it shows that 'bonding' is not possible in this environment.

1. Introduction

Most consumer goods are traded in 'reputational' markets, that is, in markets where the quality of the item being traded, while known to the seller, cannot be costlessly ascertained prior to consumption by the buyer. In such markets buyers have an incentive to rely on the seller's fear of loosing repeat business to ensure delivery of the desired quality. In other words, they might rely on a seller's concern about his 'reputation' to ensure the quality of their purchase.

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This paper looks at the role of prices in this type of trade process. Prices represent a form of 'pre-trade communication', and might conceivably help in persuading a sceptical buyer to trade. In fact, one might think that a particular intertemporal pricing pattern can operate as a quality commitment device, say by incurring losses today which can only be recouped if high quality is provided today -a line of argument often referred to as 'bonding' (see Becker and Stigler (1974), Klein and Leffler (1981)). Even if prices play only a passive role (as will turn out to be the case in this model). I think it is important to characterize price paths associated with the formation of reputations. This because prices are directly and easily observable, unlike beliefs or information flows, and bringing them into the picture gives us a way of tracking the development of reputations empirically.

One major strand of literature dealing with the issue of reputation, which originated with the contributions of Kreps and Wilson (1982a), and Milgrom and Roberts (1982a),[1] has, as far as I am aware, ignored these issues, and concentrated instead on analyzing entry-deterrence stories in which there are no prices.

The present paper aims to start filling this gap by modifying the model presented in Milgrom and Roberts(1982a), substituting at each stage a 'price signalling' game for the entry deterrence one that those authors worked with.[2] This 'price signalling' game is not quite a signalling game in the usual sense as it lacks the 'single crossing' structure that those games normally have.

Consequently, all the equilibria of the game studied in this paper will be pooling equilibria, and thus the present work will have nothing to say on the interesting question of to what extent can prices serve to separate types (for contributions of that sort, see Milgrom and Roberts (1986) and Hertzendorf (1993)).[3]

The basic contribution of this paper is then to characterize the

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[1] In contrast to the literature that models reputations as a norm in an infinitely repeated game, see Klein and Leffler (1981).

[2] Note that the stage-game in this model has exactly the same structure of payoffs, given a price, as the Milgrom-Roberts entry-deterrence stage-game: Just identify 'selling high quality' with 'fighting entry', and 'selling low quality' with 'accommodating entry'. See figure 1.

[3] Actually, there is a sense in which reputation and separation (but not 'bonding') are opposites: If separation is possible to start with, then there is no role for reputation in the sense of Milgrom and Roberts (1982a) and Kreps and Wilson (1982a). Moreover, characterizing the price dynamic is immediate: Potential cheaters will no sell, while 'honest' sellers will sell at the highest possible price (under take-it-or-leave-it price offers).

time paths of prices in the two-period case when prices play no signalling role. It is shown that prices will not necessarily fall from the first to the second period. The possibility of rising prices would seem to run counter to the expectation that prices will track reputations, and hence, fall throughout ('reputation' being assumed strongest at the start of the game). In fact, this impression is wrong, and originates in two misunderstandings.

First, there are two opposing forces at work here: While increasing incentives to cheat as the end of the game is approached will tend to lower the price, learning will tend to raise it. There is no obvious reason why one force should prevail over the other. On the other hand, strictly speaking, a seller does not carry a reputation at the start of the game. He or she adquires it only after the first (high quality) sale. So that, if anything, rising prices, not falling prices, represent the 'natural' outcome in this model.

There is still the question of whether one of these forces should systematically prevail over the other, and the main contribution of this paper is to show that this is not the case (a negative result, in a sense). The reason for this is that, as will be shown in the text, the decision to supply high quality today is totally independent of the price prevailing today, and, hence, there is no reason why the price today should be linked in any particular way to the price tomorrow.

Secondly, the analysis also throws light on the question of whether something analogous to the 'bonding' story of infinitely repeated games can emerge in this type of set-up. The answer is negative. The equilibrium displays the recursive structure of the equilibrium in Milgrom and Roberts (1982a), and, hence, will not allow for the intertemporal linkage of decisions implicit in the 'bonding' reasoning.

In the next section, the game is outlined and the solution concept used is discussed. The paper then characterizes the equilibria of the stage game. After this, the two period case is analyzed. Finally, existence and uniqueness are discussed, and conclusions are drawn.

2. The Game

In the stage game a seller, who lives for two periods, confronts a buyer with a life of one period (who shares information across generations). The seller can produce either high (H) or low (L) quality at a unit cost (disutility per unit) of c_H and c_L, respectively, with $c_H > c_L$. The buyer is endowed with v_H units of a non-produced good ('money'), which is assumed to enter linearly both buyers' and sellers' objectives.

Buyers have unit demands for the good produced by the seller, with reservation values v_H for a high quality unit and v_L for a low quality unit. The following inequality relates buyers' reservation values and sellers' unit costs of production, $v_H > c_H > c_L > v_L$. Note that buyers will only pay a price above costs if they expect to be supplied with a high quality unit. Buyers will be assumed not to be able to tell apart a high quality unit from a low quality one ex-ante, that is, before consuming the good. The seller might be of one of two basic types, Honest or Rational. The honest seller will always supply high quality (that is, as long as it is individually rational to do so),[4] the rational one might or might not, depending on the circumstances. Buyers assess prior probability δ that the seller is rational.

In order to avoid indeterminacy along the path of play (as opposed to the familiar multiplicity that results from the freedom to choose out of equilibrium beliefs, which I shall handle via a refinement -more on this, below), I will assume that a rational seller can be of one of a continuum of sub-types each with a different unit cost of producing the high quality good. These unit costs are assumed to range from c_L to c_H. More precisely, let each such rational type be indexed by $s \in [0,1]$, and let $c_H(s) : [0,1] \rightarrow [c_L, c_H]$ be a strictly decreasing, continuous function,[5,6]

In the stage game, the seller moves first offering to supply the buyer with one unit of the produced good in exchange for a certain amount of money. The buyer accepts or rejects the offer. If the buyer accepts, the buyer pays the price and the seller chooses (if he is rational) whether to produce high or low quality, after which he

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[4] Note that the seller will always be able to make sure the buyer rejects an offer by demanding a sufficiently high price. Hence, in order to keep things simple, I will not explicitly allow sellers to refuse to trade.

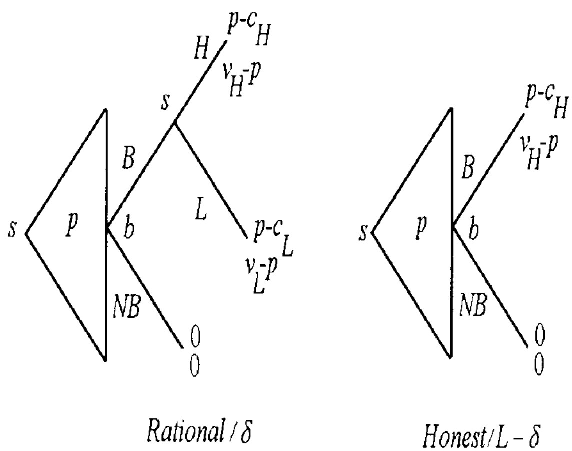
[5], The exact nature of the multiplicity I am referring to here is the following: In the two-period case, for initial beliefs above a certain critical value (specifically, $\bar{\delta}$, defined in section 3.1), sellers have to mix between providing high and low quality (for similar reasons as in Kreps and Wilson (1982a), but the mixture is not unique. This occurs because now it is possible to make buyers indifferent between buying or not (a necessary condition in order for sellers to be indifferent between supplying high or low quality) for a whole range of values of posterior beliefs by setting the price equal to the expected value of a purchase given those beliefs. This results, for any given initial beliefs, in a continuum of equilibria indexed by the probability that the seller provides high quality.

[6] This modification is in the same class as those introduced by Milgrom and Roberts (1982a) into the Kreps and Wilson framework in order to generate unique pure strategy equilibria.

delivers the good to the buyer. The buyer consumes, and by doing so, finds out if the good supplied to him was of high or low quality. This signalling game is depicted in the diagram below:

Figure 1

Stage Game



2.1. *Solution Concept*

The solution concept I shall use is the notion of sequential equilibrium (Kreps and Wilson 1982b). In order to deal with the multiplicity of equilibria resulting from the freedom to choose out of equilibrium beliefs, it is only natural to introduce a refinement. The idea here is the following: Since whenever an honest type gains from deviating, it is also profitable for a rational type to deviate, it seems reasonable to require that buyers stick to their prior beliefs when confronted with a deviation. In a similar vein, if the deviation could not possibly lead to a gain for either type, then beliefs remain unchanged.[7]

[7] This is an argument in the spirit of the 'divinity' refinement introduced by Banks and Sobel (1987). The sui-generis nature of this 'signalling' game makes

In a repeated signalling game this refinement amounts to the statement that, under whatever 'theory' a deviator of a given type entertains concerning the further development of the game after the deviation, if he expects to gain from deviating, so should the other type.

3. Equilibrium

In characterizing the equilibrium, I will proceed first by looking at the equilibrium of the stage game, then at the equilibrium in the two-period case. I will focus attention on equilibria involving sales, which will turn out to be unique and pooling such that a sale takes place every period until the game is over or low quality is supplied. Separating equilibria do exist, but no sales take place in them.

3.1 *Stage Game Equilibrium*

The equilibria of the stage game are illustrated in the next diagram. Clearly, in a one-shot situation, rational types will never supply high quality. The only equilibria involving a sale are pooling equilibria in the region $[0, \bar{\delta}]$, with prices in the shaded area marked (I). The reason for this is straightforward: In the complementary region, any price at or below the diagonal line representing the expected value of the good to a buyer lies below the cost of supplying high quality. Hence, a seller who demands such a price will be expected to supply low quality, and, consequently, no sales will take place.

It is easy to see that, with this refinement, the original stage game contains a unique selling price configuration for each value of δ in the region $[0, \bar{\delta}]$, given by $p_A^E = p_R^E = p(\delta)$, where $p(\delta)$ stands for the expected value of the good given initial beliefs δ; p_A^E for the price charged by the honest type (or automaton), and p_R^E for the price charged by the rational type.

The introduction of a continuum of rational types in the above fashion has no effect on the refined equilibrium of the one-stage game. In the two (or more) period case, though, it will be shown to induce a unique path of actual sales' prices.

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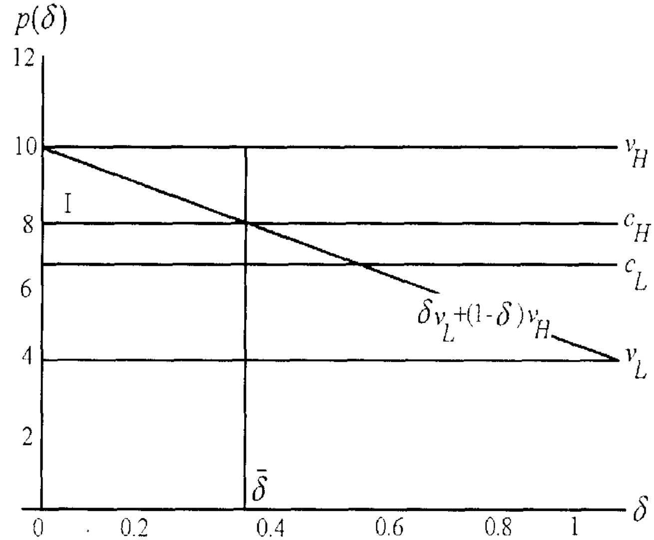
the conventional refinements of little use (i.e., Cho and Kreps (1987) intuitive criterium and extensions thereof).

Figure 2
Equilibria of Stage Game



4. Characterizing the Equilibrium for the Two-Period Case

I start by introducing some additional notation: First of all, note that I will be counting time backwards: Period t will precede period $t-1$. Let T represent the horizon of the game (here $T = 2$), and let δ_2 represent the initial beliefs of the buyer (that is, the probability that the buyer alive at $t = 2$ assigns to the event that the seller is rational). Let $\widetilde{p}_t$ be the price charged at period t. $\delta_1(H)$ will designate the posterior probability assessment that the seller is rational, given that high quality was supplied at $t = 2$. Of course, $\delta_1(L) = 1$. Finally, $\rho_2(H)$ denotes the share of rational types aiming to supply high quality at $t = 2$, $\widetilde{p}_t(\delta)$ refers to the price charged at t as a function of initial beliefs, and $\delta_1(\delta)$ refers to the posterior value generated in the equilibrium corresponding to initial beliefs δ. The following proposition describes the unique (along the equilibrium path) sales equilibrium of the game as a function of initial beliefs:

Proposition 1. *The following beliefs and strategies form a sequential equilibrium for the two-period game. The equilibrium outcome is unique under the refinement introduced in the previous section.*

For all $\delta_2 \leq \min[1\widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$, *with* $\widehat{\delta}$ *given by the solution to* $\delta_1(\delta) = \overline{\delta}$ *(which is equivalent to* $\widetilde{p}_1(\delta) = c_H$*)* ; $\widetilde{\delta}$ *given by the solution to* $\widetilde{p}_2(\delta) = c_H - \beta(\widetilde{p}_1(\delta) - c_H)$, *and* $\overline{\overline{\delta}}$ *is given by the solution to* $\widetilde{p}_2(\delta) = c_L$*:*[8]

a) Beliefs:

Price deviations at any time leave beliefs unchanged; $\delta_1(L) = 1$ always; beliefs after purchasing a high quality unit are given by

$$\delta_1(H) = \frac{\rho_2(H)\delta_2}{\rho_2(H)\delta_2 + (1 - \delta_2)} \tag{1}$$

b) Strategies for sellers:

i) Pricing:

At $t = 2$, both rational and honest types charge

$$\widetilde{p}_2 = \delta_2[\rho_2(H)v_H + (1 - \rho_2(H))v_L] + (1 - \delta_2)v_H \tag{2}$$

At $t = 1$ after supplying high quality, a seller charges

$$\widetilde{p}_1 = \delta_1(H)v_L + (1 - \delta_1(H))v_H \tag{3}$$

At $t = 1$, a rational seller who supplied low quality charges any price above v_L.

ii) Qualities:

At $t = 2$, there exists a rational subtype indexed by s^* such that all subtypes with $s < s^*$ provide low quality, while the rest provide high quality, with s^* given by the solution to (1), (3) and (4), [9]

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[8] In words, $\widetilde{\delta}$ denotes a level of beliefs such that all beliefs exceeding it would yield a current price too low to guarantee a non-negative present value of sales; $\widehat{\delta}$ denotes the level of beliefs such that all beliefs above it will yield posteriors (after a high quality sale) too low to guarantee non-negative revenues in the last period; finally, $\overline{\overline{\delta}}$ denotes the level of beliefs such that all beliefs exceeding it would result in a current price below c_L.

[9] The term s^* identifies the rational subtype for whom the cost savings achieved by supplying low instead of high quality today equals the present discounted value of tommorrow's net revenues when planning to supply low quality.

$$\rho_2(H) = 1 - c_H^{-1}[\beta(\widetilde{p}_1 - c_L) + c_L] = 1 - s^*. \tag{4}$$

At $t = 1$, rational types always supply low quality.

c) Strategies for Buyers:

At $t = 2$, for $\delta_2 \leq \min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$, buy only if

$$\widetilde{p}_2 \leq \delta_2[\rho_2(H)v_H + (1 - \rho_2(H))v_L] + (1 - \delta_2)v_H$$

At $t = 1$, for $\delta_1 \leq \overline{\delta}$, buy only if high quality was supplied the previous period and

$$\widetilde{p}_1 \leq \delta_1(H)v_L + (1 - \delta_1(H))v_H$$

Otherwise, don't buy unless $p_1 \leq v_L$.

In the region $[\min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}], 1]$, there are no equilibria involving a sale, but there are a multiplicity of no-sale equilibria, which I will not describe in detail.

Proof. See appendix.

Proposition 2. *A necessary and sufficient condition for the existence of selling equilibria (potential sales in both periods), for initial beliefs* $\delta_2 \in [0, \min(1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta})$ *is*

$$c_H(1) - c_L \leq \beta(v_H - c_L)$$

Proof. Rewrite equations (1, 3, 4) above as follows

$$[(c_H(s) - c_L) - \beta(v_L - c_L)]\delta_2(1 - s) + (1 - \delta_2)(c_H(s) - c_L)$$

$$= \beta(v_H - c_L)(1 - \delta_2)$$

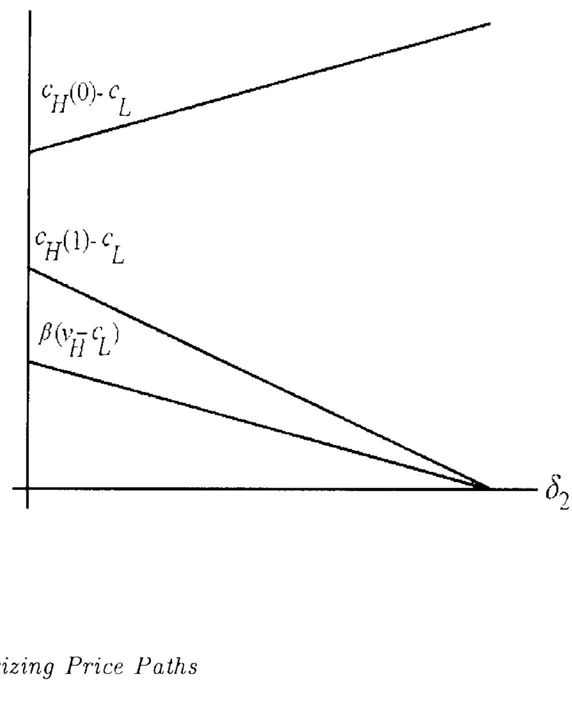
If one sets $s = 1$, the left-hand side expression will be everywhere falling in $\delta_2 \in [0, 1]$, starting at $c_H(1) - c_L$ and ending at 0. If now one sets $s = 0$, this same expression will be rising everywhere in $\delta_2 \in [0, 1]$, starting at $c_H(0) - c_L (> c_H(1) - c_L)$. For all $s \in (0, 1)$, the expression will take values in the area enclosed by the two schedules just outlined. So, if

$$c_H(1) - c_L > \beta(v_H - c_L),$$

the only point of intersection between the schedule defined by the right- hand side expression and the family of schedules defined by the left-hand side expression will be at $\delta_2 = 1$ and $s = 1$ (see diagram below), in which case no buyer will be prepared to pay a price above $v_L(< c_L)$. On the other hand, it is easy to see that $\widetilde{p}_2$ cannot be below c_L in equilibrium.

Figure 3

Existence Conditions for a Fixed Point



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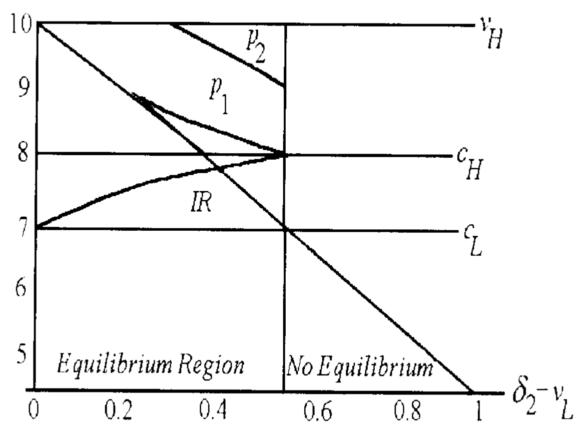
4.1 *Characterizing Price Paths*

4.1.1. An Example of Falling Prices

The diagram below illustrates a parametric example, with $c_H(s) = sc_L + (1 - s)c_H$ and $v_H = 10; c_H = 8; c_L = 7; v_L = 4$, and $\beta = \frac{1}{2}$:

Figure 4a
Falling Prices



In the figure the schedule IR plots the individual rationality constraint for the honest seller,

$$c_H - \beta(\widetilde{p}_1(\delta) - c_H)$$

(if the individual rationality constraint of honest sellers is satisfied, it must also be satisfied for dishonest sellers). The x-axis represents initial beliefs, i.e., δ_2. The vertical line corresponds to the $\min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$. Note that in this example, $\min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}] = \widehat{\delta}$, and prices are falling over time.

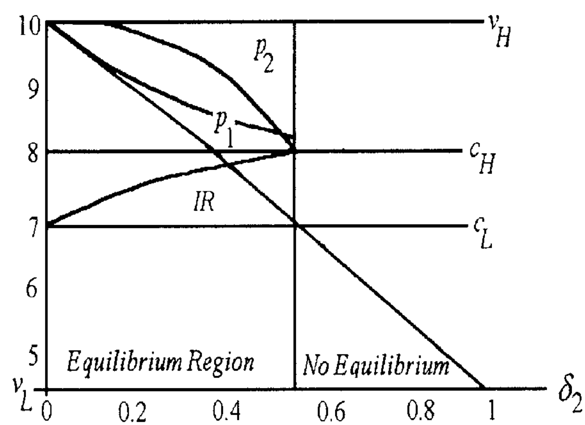
4.1.2. An Example of Rising Prices

With all parameter values as above except for the discount factor (here $\frac{1}{3}$), the situation depicted in the diagram below arises.

Here again, IR stands for individual rationality (of the honest seller). Note that just to the left of the vertical line (denoting the $\min[1, \tilde{\delta}, \hat{\delta}, \bar{\delta}] = \tilde{\delta}$), the $\tilde{p}_2$ -schedule is below the $\tilde{p}_1$ -schedule, i.e., prices rise. (At the vertical line itself, $\tilde{p}_2$ is even below c_H, meaning that losses are incurred in that period by honest sellers. Also, the IR schedule intersects the $\tilde{p}_2$-schedule at exactly the vertical line -by construction).

Figure 4b

Rising Prices



4.1.3. Interpretation

One way of looking at this is just to concentrate on the formal structure of this two-period equilibrium, and note that the decision to supply high quality today is totally independent of the price prevailing today, and that, hence, there is no reason why the price today should be linked in any particular way to the price tomorrow (of course, individual rationality will link both prices but as long as today's price covers the cost of producing high quality it will not require that prices rise or fall over time).

More intuitively, one can note that there two 'forces' operating in opposite directions on prices over time: The first stems from the fact that as the end of the game is approached, incentives for rational types to supply high quality fall, i.e., the share of dishonest types who sell high quality falls. This tends to lower the price. On the other hand, every time high quality is supplied, buyers become more 'bullish' on the likelihood of the seller being honest, which tends to raise the price. There is no reason in this two period case why one of these forces should systematically prevail, and so it should not be mysterious that the price tomorrow might end up exceeding the price today.

Note that in the example presented, prices rise over time only for sufficiently pessimistic beliefs. This seems intuitive as well. The stronger the 'reputation' effect, i.e., the higher the share of rational sellers aiming to supply high quality in equilibrium, the higher the expected value of a purchase at $t = 2$. On the other hand, the higher the share of rational sellers intending to supply high quality, the less informative a high quality purchase (for given priors), and, hence, the lower the price obtainable in the last period. As priors worsen, in order to induce rational types to sell high quality, it becomes necessary to lower the measure of rational sellers so as to make a high quality purchase highly informative about the seller's type. This lowers the expected value of the good today, while simultaneously making the price tomorrow correspondingly higher.[10]

4.1.4. The 'Bonding' Issue

This two period example suffices to illustrate why the 'bonding' does not apply in this type of set-up, as, evidently, the price charged at $t = 2$ is totally unrelated to the quality provided by rational sellers at that date.[11] Mathematically, this expresses itself in the fact that one can solve for all the other endogenous variables in the system $(\widetilde{p}_1 \rho_2(H), \delta_1(H))$, without using the equation generating $\widetilde{p}_2$. In other words, it is the recursivity of the equilibrium, i.e., the fact that actions at any time depend only on the value of the beliefs held at that time,

[10] One should make an important caveat regarding the interpretation of the results in this two-period case: It can be shown that it is wrong to presume that these results apply in an end-phase of a longer game, even if the equilibrium of the longer game can be shown to be recursive in beliefs.

[11] This property is robust to longer horizons.

plus the fact that these beliefs depend only on the actual quality provided the previous period, and not in any way on the price charged at that time, that accounts for the absence of 'bonding'.

The nearest this model seems to get to the 'bonding' intuition is when $\widetilde{p}_2$ dips below c_H, as in one of the diagrams above. The reason for that temporary loss being, as already pointed out, that in order to make a high quality sale sufficiently informative as to the type of seller, it might be necessary for relatively few rational types to supply high quality. This then feeds back into $\widetilde{p}_2$. Note that it is not the losses per se that matter, but the 'informativeness' of the signal. The losses incurred at $t = 2$ are merely incidental. What remains true is that in order to satisfy individual rationality, if losses are incurred today, the seller of high quality must be hoping to make a sale tomorrow (though I want to emphasize again that the reverse does not apply). Finally, note in particular that the observation of a price below c_H cannot in any way be taken as a 'signal' that high quality will be provided, for sellers of low quality will not incur losses in equilibrium.

5. Conclusions

This paper has established two results: 1) Price can fall or rise in this two period model; 2) There cannot be 'bonding' in this environment. Of course, this work should just be considered a first approximation to the subject. Not just because it analyzes only the two-period case, but also because the cost structure of dishonests is assumed to differ systematically from that of honest types, a rather ad-hoc assumption from an economic point of view, but one that eases the analysis of the game substantially.

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Appendix

Proof of Proposition 1

Consistency of Beliefs

If low quality is supplied, buyers know that the seller must be rational, for honest sellers never provide low quality. Consistency of beliefs is evident as a rational seller of type t will only supply high quality in the region $[0, \min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]]$, if

$$c_H(t) - c_L \leq \beta(\widetilde{p}_1 - c_L)$$

Then the formula

$$\delta_1(H) = \frac{\rho_2(H)\delta_2}{\rho_2(H)\delta_2 + (1 - \delta_2)}$$

is just Bayes' Rule given initial beliefs δ_2.

Sequential Rationality

Buyers

At any time, buyers should never pay more than the expected value of the good. Clearly, it always pays to buy at a price below v_L.

Sellers

By the assumption that price deviations leave beliefs unchanged, sellers must charge the expected value of the good to the buyers in an equilibrium at each period. If low quality was supplied, then buyers will not buy at any price above v_L. Since $v_L < c_L$, it just does not pay to sell. The condition$c_H(s) - c_L \leq \beta(\widetilde{p}_1 - c_L)$ just says that the gain from supplying low quality today is smaller than the loss associated with doing so.

If $\delta > \min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$, then either the constraint $\delta \leq \overline{\delta}$ ($\widehat{\delta} = \min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$) is violated, or the individual rationality constraint for the honest type is violated ($\widetilde{\delta} = \min[1, \widetilde{\delta}, \widehat{\delta}, \overline{\delta}]$), or $\widetilde{p}_2(\delta) < c_L$. The price at $t = 2$ cannot be below c_L, for, in that case, sellers for whom it does not pay to supply high quality[12] would rather not sell. This cannot represent equilibrium behavior, as the sellers who are not selling are earning zero profits, while they would earn positive profits if they mimicked the behavior of the high quality suppliers (that is charged their price and supplied high quality). This must be so since the individual rationality constraint of honest types must be satisfied at the selling price (the high quality price), but this implies that the individual rationality constraint of the rational type with the highest costs is also satisfied at that price.

Uniqueness

Note that the equation (derived from (1),(2),(3) and (4))

$$(c_H(s) - c_L)[(1-s)\delta_2 + (1-\delta_2)] - (1-s)\delta_2\beta(v_L - c_L) \qquad (5)$$

$$= \beta(1-\delta_2)(v_H - c_L)$$

is monotone falling in s for $s \in [0,1)$, i.e., the sales equilibria are unique (if they exist). Again, from equations (1)-(4), one can write

$$\delta_1 = \frac{[1 - c_H^{-1}(\beta(\widetilde{p}_1(\delta_1) - c_L) + c_L)]\delta_2}{[1 - c_H^{-1}(\beta(\widetilde{p}_1(\delta_1) - c_L) + c_L)]\delta_2 + (1-\delta_2)} \qquad (6)$$

[12] There must be some, for otherwise, if everyone is supplying high quality, p_2 cannot be below v_H, under the pricing rule being used.

Since δ_2 enters linearly, this equation takes the value 0 at $\delta_2 = 0$, the value 1 at $\delta_2 = 1$, and is continuous, it follows that the solution to the equation $\delta_1(\delta) = \bar{\delta}$ is unique, if it exists. Since the candidate selling equilibrium is unique, it follows that $\delta_1(\delta)$ takes only value the range [0,1) for $\delta \in [0, 1]$. Further note that $\delta_1(\delta)$ in $[0, 1] \times [0, 1)$ must be strictly increasing (since this equation takes the value 0 at $\delta_2 = 0$, and the value 1 at $\delta_2 = 1$, and is continuous), and, so, $\widetilde{p}_2(\delta)$ must be strictly decreasing,[13] and $c_H - \beta(\widetilde{p}_1(\delta) - c_H)$ strictly increasing. It follows that, if a solution to the equation $\widetilde{p}_2(\delta) = c_H - \beta(\widetilde{p}_1(\delta) - c_H)$ exists, it must be unique. Such a solution exists if sales equilibria exist, since $\widetilde{p}_2(0) = v_H$, while $c_H - \beta(\widetilde{p}_1(0) - c_H) < c_H$, if sales equilibria exist.

Since all sales equilibria must be pooling (for the same general reason as in the one-shot game: It will always be advantageous and always be feasible for the rational type to mimic the strategy of the honest type in any candidate separating equilibrium), they must take the above form. By definition then, there cannot be sales equilibria outside the region $[\min[1, \widetilde{\delta}, \widehat{\delta}, \bar{\delta}], 1]$ Note finally that there cannot be pooling no-sale equilibria in the region $[0, \min[1, \widetilde{\delta}, \widehat{\delta}, \bar{\delta}]]$ because of the refinement being used here: It pays to deviate to some price below the expected value of the good but above its cost. ∎

[13] To see this: Note that

$$p_2 = \delta_2[\rho_2(H)v_H + (1 - \rho_2(H))v_L] + (1 - \delta_2)v_H$$

with

$$\rho_2(H) = 1 - c_H^{-1}[\beta(p_1 - c_L) + c_L]$$

Note that ρ_2 (*H*) is falling in initial beliefs as p_1 is falling in that variable. Hence the expression in square brackets multiplying initial beliefs is falling. In the equation for p_2 the weight on the smaller expression is increasing while that on the bigger one (v_H) is falling, so, the overall expression must be falling.