# COMPLEMENTARITIES AND COMMITMENT IN A COURNOT SETTING

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*Resumen:* Cuando las empresas oligopólicas compiten invirtiendo tanto en I+D para reducir costos, como en publicidad creadora de demanda, su compromiso estratégico en dichas inversiones puede diferir notablemente del que tendrían si sólo utilizaran alguna de ellas. En particular, si el gasto en I+D (y en publicidad) se compromete con anterioridad a la elección del nivel de producción, pueden surgir casos de infrainversión en reducción de costos, y otros en los que no existe utilización estratégica diferenciada del I+D. Además, cuando la publicidad es una variable de inversión más de las empresas, el gasto en I+D que realizan puede igualar o superar el nivel socialmente óptimo de reducción de costos.

*Abstract:* When oligopolistic firms compete by investing simultaneously in cost-reducing R&D and in demand-creating advertising expenditures, their strategic commitment in such assets may differ qualitatively from the behavior pursued when only one of them is used. In particular, if R&D (and advertising) investment is decided on and made public before selecting the output, then cases of undercommitment in cost reduction can arise despite the non-existence of technological spillovers; and others in which there is no room for a differentiated strategic use of R&D. Furthermore, when advertising is included among the investment variables of firms, their R&D expenses may equal or even exceed the socially optimal level of cost reduction.

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## 1. Introduction

It is widely known that a great part of industrial organization literature on strategic precommitment games has focused on the analysis of oligopolistic competition modeled as a two-stage game with the usual timing. At the first stage or premarket phase, each player commits itself to some sunk investment (long-run variable), which is made observable before the quantity (or price) setting and then modifies Cournot-in-quantity (or Bertrand-in-price) behavior at the second phase or market stage (short-run variable). In this phase, a Nash equilibrium emerges given the inherited conditions coming from the first stage of the game. In fact, subgame perfection (Selten, 1975) requires the second stage actions to be a Nash equilibrium in this phase for all possible actions adopted in the first stage, i.e., for every subgame. Put in another way, it is supposed that firms correctly anticipate the effects of the rival first-stage actions on the second stage equilibrium no matter where they are in the game.

In the framework above described two issues complete the modelization. On the one hand, the role of sunk costs and public observability in some actions are critical for determining the rules of the game, as well as the economic performance. The second feature is the recognizition of how each firm can acquire a strategic advantage over its rivals through a sunk investment. For that, such strategic competition models adopt the one-stage game (myopic competition), in which the entire set of decisions is simultaneously chosen by each firm, as a benchmark for characterizing over or underinvestment patterns in such irreversible assets.

Since the seminal work of Spence (1977), Dixit (1980), Ware (1984) and others scholars in exploring the strategic entry deterrence problem, there have been a lot of contributions in this research field offering several stylized facts. The common idea in all of them is the recognition, more or less explicit, that the degree of flexibility of firms, in the sense of a higher or lower slope of their cost and/or demand functions, affects the nature and the performance of competition among such firms in the respective industry.

Briefly, in a symmetric duopoly in which firms produce a substitute good and play the two-stage game $\Gamma((u_i), q_i)$, $i = 1, 2$, –i.e., they compete by first committing themselves to an unalterable publicly observable cost-reducing R&D expenditure, $u_i$, and then picking output levels, $q_i$– firms spend more resources on cost reduction than in the one-stage game $\Gamma(u_i, q_i)$ –in which R&D and output choices are simultaneously set– for a low enough degree of technological spillovers (see

Brander and Spencer, 1983; De Bondt and Veugelers, 1991).[1] The intuition behind this result is simple. When firms act in a strategic competition framework, each one aims to make the output-setting of its competitor less aggressive. So, given that the cost-reducing effort of each firm increases the marginal profitability of its own output, and that outputs of both firms are strategic substitutes (see Bulow, Geanokoplos and Klemperer, 1985),[2] the so-called (positive) 'strategic effect'[3] entails the adoption of a 'top dog' behavior in cost reduction (see Fudenberg and Tirole, 1984). As a result, firms' output levels are higher than in the one-stage game, the price of goods is lower than in the one-stage game, and firms' profits are also lower because of the higher level of fixed (sunk) costs undertaken by them. On the other hand, a 'lean and hungry' look with respect to the investment in R&D is adopted by firms (to induce weak competition at the product market) if a high enough degree of technological spillovers prevails in cost reduction investments.

In the same line and from the social point of view, D'Aspremont and Jacquemin (1988) state that in the one-stage game $\Gamma(u_i, q_i)$ firms' investment in cost reduction is lower than that of a social planner, whose aim is the maximization of total surplus, given the existing market structure. This outcome is even true in the two-stage framework $\Gamma((u_i), q_i)$, despite the overinvestment in cost reduction with respect to the $\Gamma(u_i, q_i)$ game. The explanation for this conclusion lies in the fact that firms, when they choose their R&D expenditures, ignore any (positive) effect of cost reduction on the consumer surplus.

Things work similarly when it is advertising expenditure rather than cost-reducing effort that is the sunk cost of firms. In fact, if advertising expenses increase the marginal willingness to pay for the advertiser's product, then the optimal behavior of firms depends on

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[1] See result 1, further on, for the particular case of zero technological spillover.

[2] In fact, if firm $i$ increases its output, then not only does its marginal revenue decrease but so does firm $j$'s. This is because marginal revenue is an increasing function of price, which, in turn, is a decreasing function of total output. Nevertheless, this is not an entirely generalizable result, since when the demand function is convex (as in the case of an isoelastic demand function, for example), both output levels can be strategic complements for some values of the domain of the best reply functions. Particularly, if the output level of each firm is sufficiently low, the best each one can do when faced with an increase in the rival's output is to increase its own output.

[3] That is, an increase in firm $i$'s R&D reduces $j$'s output level, which entails an increase in firm $i$'s output level and thus increases firm $i$'s profit level (see the appendix).

whether advertising is of cooperative or competitive type in the sense of Friedman (1983).[4] Particularly, when each Cournot firm carries out a very cooperative advertising, i.e., when it acts upon total sales,[5] the rival becomes tougher at the market subgame and, as a result, under-commitment in advertising is strategically indicated. Output level is then lower than in the one-stage game, and prices of goods are also lower, but profits of firms are higher because of the underinvestment in sunk costs. On the other hand, when advertising is not very co-operative, independent or predatory, i.e., when it acts upon market share,[6] the negative marginal effect of the advertising of each firm on its rival's market share gives rise to aggressive competition at the product market. This allows each firm to increase its profits at the expense of its rival, by which the incentive to overadvertise is clear. In his turn, it leads to higher output and price levels, although firms' profits are lower because of the larger sunk costs in advertising.

Once more from a social welfare point of view, the level of advertising expenditure chosen in a one-stage game as $\Gamma(a_i, q_i)$ is too small when its effect on market size is sufficiently high, and too high when its dominant effect is upon market share. This is because of its component of public good in the former case, and its use to divide up the market in the latter. This pattern is still more clear in the two-stage setting $\Gamma((a_i), q_i)$ when advertising is very cooperative or very competitive, and, conversely, is attenuated in the intermediate cases, since the additional effect considered by firms under the two-stage framework of rivalry is the impact of advertising on market size.

Summing up, the analytical effort expended in the field of pre-commitment games to try to compare the predictions of two-stage vs. one-stage models of a single investment variable (in the spirit of Fudenberg and Tirole, 1984) has been considerable. Nevertheless, such models do not allow the use of several investment weapons, in order to determine whether, when their joint use is possible, the optimal behavior in each may differ from the optimal behavior when only one single type of investment is employed separately. As Kreps and Spence (1985) state:

> "Competition in reality is over many variables or in many dimensions at once. These analysis are extremely useful in focusing our attention on the

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[4] See result 2, further on.

[5] As happens in the cigarette industry (Roberts and Samuelson, 1988).

[6] As occurs in the saltine crakers industry (Slade, 1995) and in the soft-drink market (Gasmi *et al.*, 1992).

nature of competition in a single variable, but one wonders whether the nature of competition in one variable is not affected by what is happening with regard to a second. We despair of anyone's ability to model adequately the rich structure of realistic competition, and we therefore endorse the idea of studying one variable at a time. But there is a significant potential flaw in this method of analysis that should be kept in mind". (p. 353).

This type of criticism about the use of single investment models to stylize generic industrial behavior seems doubtless to be a good reason to develop models with several (more than one) investment variables at the disposal of firms. Clearly, in such a context, there are the same '(direct-) strategic effects' as in the single investment framework, but there are also complementarity (or substitutability) relationships between such variables, which cause 'indirect-strategic effects' that must also be taken into account. The reason is that these 'indirect-strategic effects' may place the firms' optimal investment in a level far different from the optimal conduct that holds when only one single long-run variable can be used. In fact, the introduction of several (more than one) investment variables is crucial, since it may give some additional degree of flexibility to firms and, as a consequence, lead to some important modifications in the conduct and the performance of firms. Put in other words, this brings together two strands of literature on industrial organization: one that looks at the (direct-) strategic effect of firms' investments, and the other that looks at the contemporaneous interaction between several investment variables. Clearly, one of the crucial goals in this context is to distinguish, in the 'total-strategic effect' of a given variable, what is due to the role of that variable ('direct-strategic effect') from what is due to the interaction of such a variable with others ('indirect-strategic effect').

The aim of this paper then is to examine the circumstances under which the interaction effect between contemporaneous variables is strong enough to reverse the (direct-) strategic commitment effect. For that, I consider a duopoly in which firms compete by means of a cost-reducing investment with no technological spillovers, a demand-creating investment that intends to enhance consumer tastes for a certain product (the advertised good) and that may or may not affect the demand for the product of the rival,[7] and the quantities of

[7] With such variables I intend to model the combination of the two basic forms of competition used by firms: cost leadership and differentiation of its product from that of its rival.

output.[8] Next, results for the two-stage competition $\Gamma((u_i, a_i), q_i)$ – where R&D and advertising are the long-run variables (and each firm expects that the levels chosen by its rival will remain unchanged at the second stage), leaving output as the short-run variable– are compared with those for the one-stage regime $\Gamma(u_i, a_i, q_i)$, in which all three variables are simultaneously set. As we will see further on, interaction between both irreversible assets shows that strategic over- or under-commitment in each one does not depend only on the 'direct-strategic effect' as happens in the case of just a single long-run variable, but also on a new 'indirect-strategic effect', which may dramatically alter the commitment patterns mentioned above. Likewise, the socially eficient levels of investment in each asset can be reached under given circumstances in this several investments framework.

The rest of the paper runs as follows. In section 2 the model is built. In section 3 results for the two-stage game $\Gamma((u_i, a_i), q_i)$ of competition are compared with those for the one-stage regime $\Gamma(u_i, a_i, q_i)$. Section 4 is devoted to the analysis of firms' behavior in R&D and advertising as compared with those of a social planner. Finally, section 5 summarizes the main conclusions of the paper. Formal proofs of the results are collected in the appendix.

## 2. The model

Consider a differentiated symmetric duopoly in which firms $i$ and $j$, $i = 1, 2$; $j = 3 - i$, compete by choosing R&D and advertising levels, and by setting the quantity of their products. In order to lead to explicit solutions, we specify particular demand and cost functions. Regarding demand schedule, it is supossed that there is a continuum of consumers with quasilinear preferences defined by the utility function:

$$U(q_1, q_2; a_1, a_2; q_0) = u(q_1, q_2; a_1, a_2) + q_0, \tag{1}$$

where $q_1$ and $q_2$ denote quantities of the consumption goods 1 and 2, respectively, $a_1$ and $a_2$ are persuasive advertising expenses made by firms 1 and 2, and $q_0$ is the quantity of a numeraire good. Utility

[8] In industries such as seasonal clothing and automobiles, firms' conduct is probably correctly described by the Cournot rule, since they tend to commit themselves to production runs and then sell the output for whatever they can get for it.

on the quantities of the products and on the associated advertising levels is quadratic, as:

$$u(q_1, q_2; a_1, a_2) = (\alpha + \theta a_1 - \lambda a_2)q_1 + (\alpha - \lambda a_1 + \theta a_2)q_2 \qquad (2)$$
$$-\frac{1}{2}(\beta q_1^2 + 2\gamma q_1 q_2 + \beta q_2^2),$$

where parameters $\alpha, \beta, \gamma, \theta$ and $\lambda$ satisfy $\alpha > 0, \beta > \gamma > 0, \theta > 0$ and $\lambda \in (-\theta, \theta)$.[9]

The properties of the parameters of the utility function given in (2) ensure that consumers' demand for each product is well-defined. In fact, maximizing $\{U(\cdot) - p_1 q_1 - p_2 q_2\}$ subject to $(q_1, q_2) \neq (0,0)$ and the budget constraint $p_1 q_1 + p_2 q_2 + q_0 = I$, where $I$ stands for the consumer's income, yields the system of linear inverse demand functions:

$$p_i(a_i, a_j; q_i, q_j) = \max.\{\alpha + \theta a_i - \lambda a_j - \beta q_i - \gamma q_j, 0\}, \qquad (3)$$
$$i = 1, 2; \quad j = 3 - i,$$

where $p_i$ and $p_j$ denote unitary prices for products.

The expenditure on advertising made by each firm has two effects measured by parameters $\theta$ and $\lambda$. The parameter $\theta$ captures the effect of its advertising expenditure on its own product, i.e., the increment in the marginal willingness to pay for its product. On the other hand, the parameter $\lambda$ reflects the cross-effect of advertising on the rival's product and indicates the degree to which one firm's advertising, by comparing its product with that of its rival's, affects the marginal willingness to pay for the latter.

By inverting the system of equations given in (3), it follows that advertising made by each firm increases demand for its own product and may increase (in the case of cooperative advertising) or decrease (in the case of predatory or competitive advertising) demand for the rival's product (Friedman, 1983).[10] The threshold value

$$\lambda^{\bullet} = -\frac{\gamma}{\beta}\theta$$

[9] As can be seen, consumer preferences are not given, but they change with advertising. Indeed, $\alpha + \theta a_i - \lambda a_j$ can be understood as the consumer's reservation price for each good $i = 1, 2$; $j = 3 - i$. It is thus, in a sense, a measure of the market size for such a product.

[10] See also Chiplin and Sturgess (1981).

is important, since it separates cooperative advertising from predatory or competitive advertising. Specifically, it is said to be cooperative if $\lambda \in (-\theta, \lambda^\bullet)$, independent if $\lambda = \lambda^\bullet$, and predatory if $\lambda \in (\lambda^\bullet, \theta)$. Finally, the condition $\lambda \in (-\theta, \theta)$ rules out cases of perfectly cooperative and perfectly competitive advertising. Hence, a symmetric increase in both advertising expenditures shifts each residual demand curve outwards and, as a result, the price-elasticity of demand falls (in absolute value) at every price (see Dixit and Norman, 1978; Spence, 1980; Chiplin and Sturgess, 1981; Friedman, 1983; Fershtman, 1984).

On the other hand, the R&D investment undertaken by each firm shifts variable (and marginal) costs of output to fixed (sunk) costs (Brander and Spencer, 1983). In particular, I assume that all firms possess the production technology described by the cost function:

$$c_i(q_i; u_i, a_i) = (m - u_i)q_i + \phi u_i^2 + \omega a_i^2, \quad i = 1, 2, \tag{4}$$

where $m > 0$ is the marginal (and average) cost of output in the absence of any cost-reducing expenditure, and $\phi > 0$ and $\omega > 0$ are parameters denoting the productive efficiency (in the increase of the output) of investments in R&D and advertising, respectively. There are no technological spillovers in the cost-reducing effort.

As usual, I look for a Nash equilibrium of the one stage game $\Gamma(u_i, a_i, q_i)$, and for a subgame-perfect Nash equilibrium of the two-stage game $\Gamma((u_i, a_i), q_i)$. All through the paper, equilibrium levels in the former game will be labeled by a bar "-"and equilibrium values in the latter by a cap "$\wedge$".

To ensure interiority and stability of both equilibria, the following two technical assumptions are made:

**A1.** *Firm i's marginal (and average) cost of output when there is no* R&D *expenditure, m, is "moderate"in such a way that*

$$\frac{2\beta\alpha}{2\beta + \gamma} < m < \alpha$$

**A2.** R&D *and advertising cost parameters, $\phi$ and $\omega$, respectively, satisfy the condition $\phi\omega > \phi$*

$$\left[\max.\left\{\frac{\beta(2\beta+\gamma)(2\beta\theta+\gamma\lambda)(\theta+\lambda)}{(4\beta^2-\gamma^2)^2}, \frac{\beta(2\beta-\gamma)(2\beta\lambda+\gamma\theta)(\theta-\lambda)}{(4\beta^2-\gamma^2)^2},\right.\right.$$

$$\left.\left.\frac{\theta^2}{4\beta}\right\}\right] + \omega\left[\max.\left\{\frac{2\beta^2}{(2\beta-\gamma)(4\beta^2-\gamma^2)}, \frac{1}{4\beta}\right\}\right].$$

Assumption A1 is a regularity condition which guarantees that the cost-reducing level of equilibrium of one-stage and two-stage games, $\bar{u}_i$ and $\hat{u}_i$, respectively, are such that $0 < \bar{u}_i < m$ and $0 < \hat{u}_i < m$. In turn, assumption A2 ensures the strict concavity of the payoff function of each player in both games, i.e., it ensures that the solutions of games $\Gamma(u_i, a_i, q_i)$ and $\Gamma((u_i, a_i), q_i)$ are unique and stable (and, *a fortiori*, that the first order necessary conditions (FONCs, hereafter) are sufficient). In particular, the uniqueness of the equilibrium in the first stage of game $\Gamma((u_i, a_i), q_i)$ is ensured by conditions

$$\frac{\partial^2 g_i}{\partial a_i^2}\frac{\partial^2 g_j}{\partial a_j^2} - \frac{\partial^2 g_i}{\partial a_i \partial a_j}\frac{\partial^2 g_j}{\partial a_j \partial a_i} > 0$$

and

$$\frac{\partial^2 g_i}{\partial a_i^2}\frac{\partial^2 g_j}{\partial u_j^2} - \frac{\partial^2 g_i}{\partial u_i \partial u_j}\frac{\partial^2 g_j}{\partial u_j \partial u_i} > 0,$$

where $g_i$ ($g_j$, respectively) denotes the reduced or long-run payoff function for firm $i$ ($j$, respectively). These condition, together with the condition ensuring the uniqueness of the equilibrium in game $\Gamma(u_i, a_i, q_i)$, are contained in assumption A2. [11]

## 3. The game $\Gamma((u_i, a_i), q_i)$ compared with the game $\Gamma(u_i, a_i, q_i)$

Before examining the framework in which there are several investment variables in the strategy space of firms, it is worth taking as a baseline the two limit cases already mentioned in the Introduction. These cases replicate previous findings on optimal commitment when there is just a single investment variable, and they are therefore stated here without proof.[12] The firms' pattern in cost reduction is formally summarized in the following result.

RESULT 1. (The game $\Gamma((u_i), q_i)$ *vs.* the game $\Gamma(u_i, q_i)$). *If* $a_i \equiv 0$, *then* $\hat{u}_i > \bar{u}_i$ *and* $\hat{q}_i > \bar{q}_i$ *(as well as* $\hat{p}_i < \bar{p}_i$ *and* $\hat{\pi}_i < \bar{\pi}_i$*),* $i = 1, 2$.

[11] Henriques (1990) shows that in the two-stage game, the second order conditions of the first stage are not sufficient to ensure stability in this stage. Such stability is ensured by assumption 2.

[12] These results are both in fact particular cases of proposition 3 (see corollary 1, further on). Obviously, in such situations, assumptions A1 and A2 have to be adapted to each particular case.

In general, the induction of a less aggressive rival at the market stage is profitable for each Cournot firm, which is internalized in the competition developed as a two-stage game. Hence, the re-allocation process from variable (and marginal) production costs to fixed costs benefits each firm since its output level increases as a result, whereas that of the rival falls. This is the so-called (positive) 'direct-strategic effect'. As was explained in the introduction, firms therefore spend more resources on cost reduction in the two-stage game than in the one-stage game, which parallels the overcommitment in cost-reducing shown in previous contributions in this field (see Brander and Spencer, 1983; Fudenberg and Tirole, 1984; Krouse, 1990).

Likewise, firms' behavior in demand-creating advertising is formally stylized as follows.

RESULT 2. (The game $\Gamma((a_i), q_i)$ *vs.* the game $\Gamma(a_i, q_i)$). *Suppose that* $u_i \equiv 0$, *and let*

$$\lambda^* = -\frac{\gamma}{2\beta}\theta.$$

*a) If* $\lambda < \lambda^*$, *then* $\hat{a}_i < \bar{a}_i$ *and* $\hat{q}_i < \bar{q}_i$ *(as well as* $\hat{p}_i < \bar{p}_i$ *and* $\hat{\pi}_i > \bar{\pi}_i$*),* $i = 1, 2$;

*b) If* $\lambda > \lambda^*$, *then all the above inequalities are reversed;*

*c) If* $\lambda = \lambda^*$, *there is no room for a differentiated strategic use of the advertising.*

A strategic undercommitment in demand-creating advertising by each firm, i.e., the adoption of a 'lean and hungry' advertising look, is desirable when advertising is very cooperative, in the sense that $\lambda < \lambda^*$, as part a) of result 2 shows. In such a case, the two effects of the advertising undertaken by a given firm are an increase in its own output, and an increase in the rival's output. Since the latter is the dominant effect of the two, each firm finds that it is profitable to underadvertise. As a result, firms' output is lower than in the one-stage game, as well as the price of goods. The threshold value of parameter $\lambda$ separating the two situations –advertising that increases the rival's output from advertising that decreases the rival's output– which is

$$\lambda^{\bullet} = -\frac{\gamma}{\beta}\theta$$

in the one-stage game (in which the output of equilibrium of each firm is not a function of the advertising expenditure previously made by the rival), changes to the value

$$\lambda^* = -\frac{\gamma}{2\beta}\theta$$

in the two-stage game. Accordingly, in the interval $\lambda \in (\lambda^*, \lambda^\bullet)$ each firm's advertising increases the rival's market share in the two-stage game and this (negative) effect outweighs the (positive) effect due to the increase in the market size.

The contrary holds when parameter $\lambda$ is such that $\lambda > \lambda^*$, in which case any investment in advertising reduces the rival's market share. This leads each firm to behave as a 'fat cat' in the advertising budget, as part b) of result 2 shows (see Fudenberg and Tirole, 1984).

Turning back to the two-investment variable setting, if neither R&D nor advertising expenditures are publicly observable, then the duopolists play the one-stage game $\Gamma(u_i, a_i, q_i)$. From (3) and (4), the short-run payoff function for each player comes straightforward. In fact, it is given by:

$$\pi_i(\cdot) = (\alpha - m + \theta a_i - \lambda a_j - \beta q_i - \gamma q_j + u_i)q_i \qquad (5)$$
$$-\phi u_i^2 - \omega a_i^2, \quad i = 1, 2; \; j = 3 - i,$$

and the FONCs for a Nash equilibrium in this game are

$$\frac{\partial \pi_i}{\partial u_i} = 0, \qquad (6)$$

$$\frac{\partial \pi_i}{\partial a_i} = 0, \qquad (7)$$

and

$$\frac{\partial \pi_i}{\partial q_i} = 0, \quad i = 1, 2 \qquad (8)$$

Resolution of the system of equations given by (6), (7) and (8) gives rise to the following proposition, which describes the firms' behavior in the absence of any strategic commitment.

PROPOSITION 1. *Under assumptions A1 and A2, a unique and interior Nash equilibrium of the one-stage game $\Gamma(u_i, a_i, q_i)$ exist and is defined as*

$$\left\{(\bar{u}_i, \bar{a}_i, \bar{q}_i) = \frac{\alpha - m}{\Theta}(\omega, \theta\phi, 2\omega\phi)\right\}, \quad i = 1, 2,$$

*where* $\Theta = 2(2\beta + \gamma)\omega\phi - \theta(\theta - \lambda)\phi - \omega.$

PROOF. See the appendix.

In this regime of competition, two facts need to be underlined. First, firms cannot acquire any strategic advantage from their $(u_i, a_i)$-investments because they cannot modify the (future) market behavior of the rival. Thus, they only consider the 'direct effects' of such investments on their own profits. Second, the advertising and R&D investments of each firm are strategic complements, in the sense that expenditure on one is encouraged by expenditure on the other (see Chiplin and Sturgess, 1981; Fershtman, 1984; Hula, 1988; Lunn, 1989).

On the other hand, when firms commit themselves to publicly observable investments before quoting output levels, then the two-stage game $\Gamma((u_i, a_i), q_i)$ is played. In the output game, the Nash-in-quantity equilibrium is defined by FONCs (8), i.e., the strategies that maximize profits of each firm at the market stage of the game, given the sunk investments. This results in:

$$\Big\{\hat{q}_i(\cdot) = \frac{1}{4\beta^2 - \gamma^2}[(\alpha - m)(2\beta - \gamma) + 2\beta u_i - \gamma u_j \tag{9}$$

$$+(2\beta\theta + \gamma\lambda)a_i - (2\beta\lambda + \gamma\theta)a_j]\Big\}, i = 1, 2; \quad j = 3 - i,$$

as such an equilibrium.

From the pair of Cournot-Nash strategies given in (9), firm $i$'s long-run (or reduced) profit function applicable at the first stage of the game can be written as:

$$g_i(\cdot) = \beta[\hat{q}_i(\cdot)]^2 - \phi u_i^2 - \omega a_i^2, \quad i = 1, 2, \tag{10}$$

and the FONCs defining the equilibrium in such a stage are given by:

$$0 = \frac{\partial g_i}{\partial u_i} \equiv \frac{\partial \pi_i}{\partial u_i} + \frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial u_i}, \tag{11}$$

and

$$0 = \frac{\partial g_i}{\partial a_i} \equiv \frac{\partial \pi_i}{\partial a_i} + \frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_i}, \quad i = 1, 2; \quad j = 3 - i, \tag{12}$$

where the envelope theorem is used. In short, firms' behavior in the entire game $\Gamma((u_i, a_i), q_i)$ is summarized in the following proposition.

PROPOSITION 2. *If assumptions A1 and A2 hold, then a unique and interior subgame-perfect Nash equilibrium of the two-stage game* $\Gamma((u_i, a_i), q_i)$ *exists and is given by*

$$\left\{((\hat{u}_i, \hat{a}_i), \hat{q}_i) = \frac{\alpha - m}{\Phi}((2\beta^2\omega, \beta(2\beta\theta + \gamma\lambda)\phi), (4\beta^2 - \gamma^2)\omega\phi)\right\},$$

$i = 1, 2$, *where* $\Phi = (2\beta+\gamma)(4\beta^2-\gamma^2)\omega\phi - \beta(2\beta\theta+\gamma\lambda)(\theta-\lambda)\phi - 2\beta^2\omega$

PROOF. See the appendix.

When firms' investments are precommitted, they have the same non-strategic or direct effects –reinforced solely by the complementarity between both assets– as in the one-stage game. In addition, each one of these investments has a 'total-strategic effect' that is composed by a 'direct-strategic effect' and an 'indirect-strategic effect' through the behavior in the other investment. Specifically, the 'total-strategic effect' of cost-reducing effort is formally given by:

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial u_j}\left(\frac{\partial u_j^*}{\partial u_i} + \frac{\partial u_j^*}{\partial a_i}\right), \quad i = 1, 2; \quad j = 3 - i, \tag{13}$$

where $u_j^*$ denotes the R&D best response function of player $j$, which is defined by FONC

$$\frac{\partial g_j}{\partial u_j} = 0.$$

In the same way, the 'total-strategic effect' of demand-creating advertising budget is given by:

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_j}\left(\frac{\partial a_j^*}{\partial a_i} + \frac{\partial a_j^*}{\partial u_i}\right), \quad i = 1, 2; \quad j = 3 - i, \tag{14}$$

where $a_j^*$ denotes the profit-maximizing level of advertising expenditures of firm $j$, which is determined by FONC

$$\frac{\partial g_j}{\partial a_j} = 0.$$

The difference in the optimum levels of investments between the game $\Gamma((u_i, a_i), q_i)$ and the game $\Gamma(u_i, a_i, q_i)$ is reflected in the following proposition.

PROPOSITION 3. *Suppose that assumptions A1 and A2 hold, and let*

$$\lambda^* = -\frac{\gamma}{2\beta}\theta,\ \omega^* = -\frac{\beta(\theta-\lambda)\lambda}{(2\beta+\gamma)\gamma}\ \textit{and}\ \phi^* = \frac{\beta\lambda}{(2\beta+\gamma)(2\beta\lambda+\gamma\theta)}.$$

*Then behavior of firms in terms of cost reduction and demand creation is as follows.*

*1) If* $\lambda < \lambda^*$, *as well as:*

*a)* $\omega < \omega^*$ *and* $\phi < \phi^*$, *then* $\hat{u}_i < \bar{u}_i$ *and* $\hat{a}_i < \bar{a}_i$;

*b)* $\omega = \omega^*$ *and* $\phi < \phi^*$, *then* $\hat{u}_i = \bar{u}_i$ *and* $\hat{a}_i < \bar{a}_i$;

*c)* $\omega > \omega^*$, *then* $\hat{u}_i > \bar{u}_i$ *and* $\hat{a}_i \gtreqless \bar{a}_i$ *(depending on whether* $\phi \lesseqgtr \phi^*$.

*2) If* $\lambda = \lambda^*$, *then* $\hat{u}_i \lesseqgtr \bar{u}_i$ *(depending on* $\omega \lesseqgtr \omega^*$, *respectively) and* $\hat{a}_i < \bar{a}_i$.

*3) If* $\lambda > \lambda^*$, *then* $\hat{u}_i > \bar{u}_i$ *and* $\hat{a}_i > \bar{a}_i$, $i = 1, 2$.

PROOF. See the appendix.

Proposition 3 constitutes one of the two main results of the paper. It shows that the 'indirect-strategic effects', derived from interaction (complementarity) of an investment variable with the other, may be strong enough to reverse the sign of 'direct-strategic effects' of each investment. In fact, undercommitment in cost reduction –as part 1a) of proposition 3 indicates– is rational when demand-creating advertising is cooperative enough, and advertising and R&D are both highly efficient in the increase of the output. The intuition behind this conclusion is as follows. When firms play strategically, each firm $i$ wants to affect $j$'s conduct at the market subgame by reducing the marginal profitability of both types of investments, $u_j$ and $a_j$. Clearly, levels of $u_i$ and $u_j$ are strategic substitutes, i.e.,

$$\frac{\partial\pi_i}{\partial q_j}\frac{\partial\hat{q}_j}{\partial u_j}\frac{\partial u_j^*}{\partial u_i} \equiv \frac{\partial\pi_i}{\partial q_j}\frac{\partial\hat{q}_j}{\partial u_j}\frac{\partial^2 g_j}{\partial u_j\partial u_i} > 0,$$

which results in overinvestment in cost reduction with respect to the one-stage game, $\hat{u}_i > \bar{u}_i$. Furthermore, in this case, $a_i$ and $u_j$ are strategic complements, i.e.,

$$\frac{\partial\pi_i}{\partial q_j}\frac{\partial\hat{q}_j}{\partial u_j}\frac{\partial u_j^*}{\partial a_i} \equiv \frac{\partial\pi_i}{\partial q_j}\frac{\partial\hat{q}_j}{\partial u_j}\frac{\partial^2 g_j}{\partial u_j\partial a_i} < 0,\ \text{when}\ \lambda \in \left(-\theta, -\frac{\gamma}{2\beta}\theta\right).$$

So, each firm has an incentive to underadvertise, $\hat{a}_i < \bar{a}_i$, which, in turn, decreases the marginal profitability of its R&D expenditure and therefore reduces the incentive to overcommit in such an asset. Indeed, if the productive efficiency of advertising expenditures is high enough, then the sign of equation (13) may become positive, i.e., $\hat{u}_i < \bar{u}_i$.

On the other hand, levels of advertising expenditures, $a_i$ and $a_j$, are strategic complements, i.e.,

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_j}\frac{\partial a_j^*}{\partial a_i} \equiv \frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_j}\frac{\partial^2 g_j}{\partial a_j \partial a_i} < 0, \text{ when } \lambda \in \left( -\theta, -\frac{\gamma}{2\beta}\theta \right),$$

and this constitutes an incentive to underadvertise, $\hat{a}_i < \bar{a}_i$. Furthermore, in this case, $u_i$ and $a_j$ are strategic substitutes, i.e.,

$$\frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_j}\frac{\partial a_j^*}{\partial u_i} \equiv \frac{\partial \pi_i}{\partial q_j}\frac{\partial \hat{q}_j}{\partial a_j}\frac{\partial^2 g_j}{\partial a_j \partial u_i} > 0,$$

which gives each firm an incentive to overcommit in cost reduction, $\hat{u}_i > \bar{u}_i$. In his turn, it is this overcommitment in cost reduction that provides an incentive to overadvertise. In fact, if the efficiency of R&D is high enough and that of advertising is sufficiently low, then the incentive to underadvertise arising from the 'direct-strategic effect' is outweighed by the incentive to overadvertise emerging from its combined use with a cost reduction investment. As a result, we have $\hat{a}_i > \bar{a}_i$, even for a very cooperative type of advertising, as part 1c) of the proposition shows. Parts 2 and 3 have a similar interpretation.

Finally, the following corollary can be shown fairly simply from proposition 3.

COROLLARY 1. *Result 1 (respectively, result 2) follows from proposition 3 by taking the limit when $\omega$ (respectively, $\phi$) tends to infinite.*

This corollary parallels the conclusions of Brander and Spencer (1983), Fudenberg and Tirole (1984), and Krouse (1990) for the unidimensional cases of strategic investment.

## 4. Welfare results

It is well known that the cost-reduction processes of firms benefit consumers, since the final price they pay for the goods falls, whereas investments of firms in demand-creation harm them due to the increase in the price of the output that they lead to. In addition, oligopolistic competition developed as in games $\Gamma(u_i, q_i)$ and $\Gamma((u_i), q_i)$ leads to a lower level of cost reduction than the social welfare maximizing level

$(fb)$. Thus, $\bar{u}_i < u_i^{fb}$ and $\hat{u}_i < u_i^{fb}$, for $i = 1, 2$ (see D'Aspremont and Jacquemin, 1988). Likewise, games as $\Gamma(a_i, q_i)$ and $\Gamma((a_i), q_i)$ give rise to a greater amount of demand-creating advertising than the social welfare maximizing level when such an advertising is predatory. Namely, $\bar{a}_i > a_i^{fb}$ and $\hat{a}_i > a_i^{fb}$.

These findings nevertheless raise the important question of whether they depend critically on the simultaneous choice of both expenditures. In this section, the consequences on market performance of considering the complementarity between demand-creating advertising and cost-reducing R&D expenditures are investigated. In this setting of cost-demand interaction, I also consider the behavior of a duopoly socially managed, in order to compare such a behavior with investment levels in a non-cooperative regime.

With respect to the first question, the comparative statics of the equilibrium price of goods in both games shows us that there are conditions under which demand-creating effort may be procompetitive, as well as conditions under which cost-reducing effort may be anticompetitive. This is the spirit of next proposition.

PROPOSITION 4. *Under assumptions A1 and A2 values of parameters that measure productive efficiency of advertising and R&D expenditures exist, respectively*

$$\omega^\bullet = \min.\left\{\frac{\theta(\theta-\lambda)}{2(\beta+\gamma)}, \frac{\beta(2\beta\theta+\gamma\lambda)(\theta-\lambda)}{(4\beta^2-\gamma^2)(\beta+\gamma)}\right\} \quad \textit{and} \quad \phi^\bullet = \frac{1}{2\beta},$$

*such that the following holds:*

*a) If $\phi < \phi^\bullet$, then a reduction in $\omega$ decreases the price of goods,*

*b) If $\omega < \omega^\bullet$, then a reduction in $\phi$ increases the price of goods.*

This conclusion holds by differentiating the price strategy in each equilibrium with respect to parameters $\omega$ (case a) and $\phi$ (case b). The result states that regardless of whether competition is developed in a one-stage or in a two-stage game, a reduction in the cost of advertising investment may reduce the final price of goods, as part a) of the proposition states. This is true when the productive efficiency of the cost-reducing investment is high enough. The explanation of this conclusion lies on the fact that the advertising effect on the price of goods through the (higher) R&D investment outweighs the advertising effect on the price through the (higher) output level. In the same way, if advertising is highly efficient, then a reduction in the cost of R&D

investment causes such an increase in the advertising expenditure that its (positive) effect on price outweighs the (negative) effect on price due to the higher level of output. Thus, the price of the output increases, as part b) of proposition 4 shows.

On the other hand, investment patterns of firms $\bar{a}_i > a_i^{fb}$ and $\hat{a}_i > a_i^{fb}$, for an advertising encouraging consumers to switch their purchases from one brand to another (when there is no cost reduction effort), as well as $\bar{u}_i < u_i^{fb}$ and $\hat{u}_i < u_i^{fb}$ (when there is no advertising), may also be reversed when both cost-reducing and demand-creating expenditures are used simultaneously by firms. This is the message of the following proposition, which compares the investment behavior of a duopoly (in one-stage and two-stage games) with the investment behavior of a socially managed duopoly.

PROPOSITION 5. *When assumptions A1 and A2 hold, the following occurs:*

*1) Values of parameters $\lambda, \omega$ and $\phi$ exist, namely*

$$\bar{\lambda} = \frac{\beta}{2\beta+\gamma}\theta, \quad \bar{\omega} = \frac{(\theta-\lambda)\lambda}{2\beta} \text{ and } \bar{\phi} = -\frac{\lambda}{2[\beta\theta-(2\beta+\gamma)\lambda]},$$

*such that, in the one-stage game $\Gamma(u_i, a_i, q_i)$,*

*a) If $\lambda \in (\bar{\lambda}, \theta), \omega = \bar{\omega}$ and $\phi < \bar{\phi}$, then $\bar{u}_i = u_i^{fb}$ and $\bar{a}_i > a_i^{fb}$;*
*b) If $\lambda \in (\bar{\lambda}, \theta), \omega > \bar{\omega}$ and $\phi = \bar{\phi}$, then $\bar{u}_i < u_i^{fb}$ and $\bar{a}_i = a_i^{fb}$.*

*2) Values of parameters $\lambda, \omega$, and $\phi$ exist, namely*

$$\hat{\lambda} = \frac{4\beta^3-2\beta\gamma^2-\gamma^3}{8\beta^3+6\beta^2\gamma-\gamma^3}\theta, \quad \hat{\omega} = \frac{\beta(2\beta+\gamma)(\theta-\lambda)\lambda}{4\beta^3-2\beta\gamma^2-\gamma^3},$$

*and*

$$\hat{\phi} = -\frac{\beta(2\beta+\gamma)\lambda}{(4\beta^3-2\beta\gamma^2-\gamma^3)\theta-(8\beta^3+6\beta^2\gamma-\gamma^3)\lambda},$$

*such that, in the two-stage game $\Gamma((u_i, a_i), q_i)$,*

*a) If $\lambda \in (\hat{\lambda}, \theta), \omega = \hat{\omega}$ and $\phi < \hat{\phi}$, then $\hat{u}_i = u_i^{fb}$ and $\hat{a}_i > a_i^{fb}$,*
*b) If $\lambda \in (\hat{\lambda}, \theta), \omega > \hat{\omega}$ and $\phi = \hat{\phi}$, then $\hat{u}_i < u_i^{fb}$ and $\hat{a}_i = a_i^{fb}$, $i = 1, 2$.*

PROOF. See the appendix.

This is the other main result of the paper. Conditions under which advertising encourages the level of cost-reduction undertaken in a non-cooperative setting until that one maximizing the social welfare are: a very competitive advertising, a highly efficient advertising investment (in the increasing of the output), and a highly efficient R&D investment (in the increasing of the output). The intuition is that when advertising is predatory firms tend to advertise more deeply than the social welfare maximizing level, and this reinforces the complementarity relationship between advertising and R&D expenses when both are efficient enough. Thus, the existence of advertising in the investment space of firms and its complementarity with R&D expenses leads to a welfare-improving increase in the effort on cost reduction.

Similarly, the combination of cost-reducing R&D expenditures with demand-creating advertising may lead duopolistic firms to reduce their levels of predatory advertising in such a way that they pick the same level as that of a socially managed duopoly.

## 5. Concluding remarks

In a Cournot setting with no technological spillovers in cost reduction investments, an undercommitment to cost-reducing R&D may be rational with the fact that firms are engaging in more investment variables than R&D (the expenditures on demand-creating advertising, for instance). This paper has examined this issue in a two-stage model and compared the results obtained with those of a one-stage game. In the former, firms choose a level of cost-reducing effort simultaneously with a level of demand-creating advertising at the first stage and then they set the quantities of output at the second phase. In the latter, all variables are chosen simultaneously. Despite the lack of technological filtrations in the cost-reducing expenditure, overinvestment in R&D is not the only possible behavior emerging from the two-stage setting (with respect to the one-stage game) as occurs when cost-reducing effort is the firms' sole investment. Indeed, there are values of the parameters measuring the spillover effect of advertising and the efficiency of advertising and R&D expenditures that eliminate the possibility of using strategically the cost-reducing investment (i. e. in a different amount than in the one-stage game). In the same way, undercommitment in cost-reducing R&D is possible.

Regarding the behavior on advertising that boosts the demand for the advertised product, its combination with an R&D investment leads firms to overadvertise when investment in cost reduction is highly efficient and expenditures in advertising are low efficient, for even a cooperative or a low predatory type of advertising.

Finally, the interaction between cost-reducing and demand-creating efforts also allows us to clarify the role played by each variable on market performance in a setting of several investments. In particular, and depending on parameters of the model, a decrease in the cost of advertising may produce lower prices of the output, whereas a decrease in the cost of R&D may be followed by higher prices of goods. In addition, advertising may be socially beneficial, since it restores the first best level of cost-reducing investment. In the same way, the existence of a cost-reducing expenditure in the investment space of firms may cause the advertising effort of each one (when it is very predatory) to coincide with the level chosen by a social planner.

As a general conclusion, these findings show that there are grounds for arguing that the consideration of possible complementarities in the investment space of firms has particular significance to evaluate their strategic commitments. Otherwise, we may incur in an excessive risk of offering patterns of behavior far different from the firms' real conduct.

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## Appendix

PROOF OF PROPOSITION 1. Each firm $i$ chooses the level of $u_i, a_i$, and $q_i$ to maximize profits. From its payoff function, given by equation (5) in the text, the FONCs defining a Nash equilibrium, given in equations (6), (7), and (8) in the text, are:

$$q_i - 2\phi u_i = 0 \tag{6'}$$

$$\theta q_i - 2\omega a_i = 0 \tag{7'}$$

and

$$\alpha - m + \theta a_i - \lambda a_j - 2\beta q_i - \gamma q_j + u_i = 0 \quad i = 1, 2; \quad j = 3 - i. \tag{8'}$$

The existence and uniqueness of the solution $\{(\bar{u}_i, \bar{a}_i, \bar{q}_i)\}$ are proved by solving the system of equations given in (6'), (7') and (8'). The description of the equilibrium is completed with the price of each good, which is given by

$$\bar{p}_i = \frac{2(\alpha\beta + m(\beta + \gamma))\omega\phi - m\theta(\theta - \lambda)\phi - \omega}{\Theta},$$

and the profit level

$$\bar{\pi}_i = \left(\frac{\alpha - m}{\Theta}\right)^2 (4\beta\omega\phi - \theta^2\phi - \omega)$$

for each firm $i$.

The stability of such an equilibrium is proved by inspection of the Hessian of function $\pi_i$ with respect to R&D, advertising, and output strategies.

For interiority of $(\bar{u}_i, \bar{a}_i)$-strategies, $0 < \bar{u}_i(\omega, \phi) < m$ and $0 < \bar{a}_i(\omega, \phi)$ must hold. That $0 < \bar{u}_i(\omega, \phi)$ and $0 < \bar{a}_i(\omega, \phi)$ follows from assumption A1, by which $m < \alpha$. So, it remains to prove that $\bar{u}_i(\omega, \phi) < m$, which holds since the gradient of $\bar{u}_i(\omega, \phi)$ is nonzero by assumption A2. Then, sup. $\bar{u}_i(\omega, \phi)$ lies at the boundary of the domain. Two cases must be considered:

a) If the type of advertising is such that

$$\lambda \in \left( -\theta, -\frac{\gamma}{2\beta}\theta \right),$$

then $4\beta\omega\phi - \theta^2\phi - \omega > 0$ by assumption A2, and the boundary may be explicitly rewritten using the strictly increasing and convex function

$$\phi(\omega) = \frac{\omega}{4\beta\omega - \theta^2}.$$

Plugging this function into the expression of $\bar{u}_i(\omega, \phi)$ we obtain that $\bar{u}_i(\omega)$ obeys the equation:

$$\bar{u}_i(\omega) = \frac{(\alpha - m)(4\beta\omega - \theta^2)}{2\gamma\omega + \theta\lambda}, \tag{15}$$

and since the function given in (15) is strictly increasing and concave, we have that

$$\text{sup. } \bar{u}_i(\omega) = \lim{}_{\omega\to\infty}\bar{u}_i(\omega) = \frac{(\alpha - m)2\beta}{\gamma}.$$

Thus, sup. $\bar{u}_i(\omega) < m \iff$ assumption A1 holds, which it does.

b) Similarly, if

$$\lambda \in \left( -\frac{\gamma}{2\beta}\theta, \theta \right),$$

then sup. $\bar{u}_i(\omega, \phi)$ lies on the graph of the function

$$\phi(\omega) = \frac{(4\beta^2 - \gamma^2)^2\omega}{4(2\beta - \gamma)[(2\beta + \gamma)(4\beta^2 - \gamma^2)\omega - \beta(2\beta\lambda + \gamma\lambda)(\theta - \lambda)]}$$

and following the reasoning above, it is straightforward to show that:

$$\bar{u}_i(\omega) = \frac{(\alpha - m)4\beta[(4\beta^2 - \gamma^2)^2\omega - \beta(2\beta - \gamma)(2\beta\lambda + \gamma\theta)(\theta - \lambda)]}{2(4\beta^2 - \gamma^2)^2\gamma\omega - (2\beta - \gamma)(\theta - \lambda)[(2\beta + \gamma)(4\beta^2 - \gamma^2)\theta - \beta^2(2\beta\lambda + \gamma\theta)]} \tag{16}$$

From (16) it follows that

$$\text{sup. } \bar{u}_i(\omega) = \lim{}_{\omega\to\infty}\bar{u}_i(\omega) = \frac{(\alpha - m)2\beta}{\gamma} < m$$

by assumption A1. This completes the proof of the proposition. ∎

PROOF OF PROPOSITION 2. Given the investments $(u_i, a_i)$ precommitted by each firm $i$ at the first stage of the game, the equilibrium at the second stage is the one defined by equation (9) in the text. Thus, the long-run payoff function for each firm $i$ is defined by:

$$g_i(\cdot) = \beta[\hat{q}_i(\cdot)]^2 - \phi u_i^2 - \omega a_i^2, \quad i = 1, 2, \tag{17}$$

and FONCs for equilibrium at the first stage of the game are:

$$0 = \frac{\partial g_i}{\partial u_i} \equiv \frac{2\beta^2}{4\beta^2 - \gamma^2}[\hat{q}_i(\cdot)] - \phi u_i \tag{18}$$

and

$$0 = \frac{\partial g_i}{\partial a_i} \equiv \frac{\beta(2\beta\theta + \gamma\lambda)}{4\beta^2 - \gamma^2}[\hat{q}_i(\cdot)] - \omega a_i, \quad i = 1, 2; \;\; j = 3 - i. \tag{19}$$

By solving the 4x4 system of equations defined by (18)-(19) and substituting the solution into equation (9), the subgame-perfect Nash equilibrium of the entire two-stage game is obtained. The description of such an equilibrium is completed with the price of each good, given by

$$\hat{p}_i = \frac{4(\beta^2 - \gamma^2)[\alpha\beta + m(\beta + \gamma)]\omega\phi - \beta(\theta - \lambda)(2\beta\theta + \gamma\lambda)m\phi - 2\alpha\beta^2\omega}{\Phi},$$

and the level of profits

$$\hat{\pi}_i = \left(\frac{\alpha - m}{\Phi}\right)^2 \omega\phi\beta[(4\beta^2 - \gamma^2)^2\omega\phi - \beta(2\beta\theta + \gamma\lambda)^2\phi - 4\beta^3\omega]$$

for each firm $i$.

The stability of the first-stage equilibrium $(\hat{u}_i, \hat{a}_i)$ is ensured by assumption A2. Finally, the interiority of such an equilibrium is proved in the same way as for the $(\bar{u}_i, \bar{a}_i)$-strategies in proposition 1. This completes the proof of the proposition. ∎

PROOF OF PROPOSITION 3. Comparison of the equilibrium values of R&D and advertising in the two-stage game with respect to the one-stage game yields:

$$\hat{u}_i \gtreqless \bar{u}_i \iff \omega(2\beta + \gamma)\gamma + \beta\lambda(\theta - \lambda) \gtreqless 0, \tag{20}$$

and

$$\hat{a}_i \gtreqless \bar{a}_i \iff \phi(2\beta + \gamma)(2\beta\lambda + \gamma\theta) - \beta\lambda \gtreqless 0, \quad i = 1, 2. \tag{21}$$

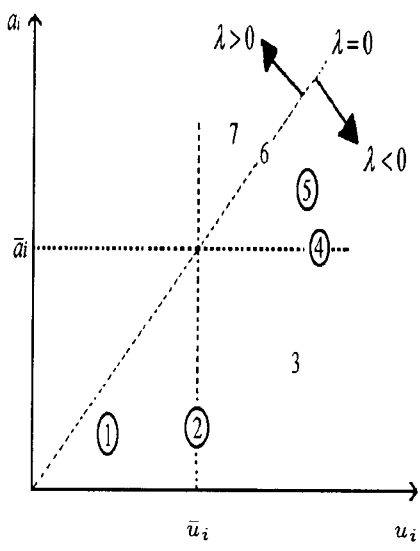
Finally, the proposition follows from (20) and (21), the assumption A2, and the relationship between ratios

$$\frac{\hat{a}_i}{\hat{u}_i} \quad \text{and} \quad \frac{\bar{a}_i}{\bar{u}_i}.$$

This comparison between $(\hat{u}_i, \hat{a}_i)$ and $(\bar{u}_i, \bar{a}_i)$ levels is depicted in the following figure.

**Figure 1**

*Levels of investment in $(\hat{u}_i, \hat{a}_i)$ with respect to $(\bar{u}_i, \bar{a}_i)$, $i = 1, 2$*



Taking as the benchmark the investments undertaken by each duopolist $i$ in the game $\Gamma(u_i, a_i, q_i)$, figure 1 may be divided into several regions:

- Case

$$\lambda \in \left( -\theta, -\frac{\gamma}{2\beta}\theta \right).$$

Under this type of advertising the way to have a soft competitor at the market subgame is to overinvest both in $u_i$ and $a_i$ (region 5 of figure 1); or to overinvest in $u_i$ and to invest in $a_i$ the same quantity as in the one-stage game (region 4 of figure 1); or to overinvest in $u_i$ and underinvest in $a_i$ (region 3 of figure 1); or to invest in $u_i$ the same level than in the one-stage game and to underinvest in $a_i$ (region 2 of figure 1); or even to underinvest in both $u_i$ and $a_i$ (region 1 of figure 1). A commitment located in regions 4 or 5 (both marked by circles), in which $\hat{a}_i \geq \bar{a}_i$ despite the fact that such an advertising is highly cooperative, reflects that interaction between R&D and advertising offsets or outweighs the 'direct-strategic effect' of advertising. For that, parameter $\omega$ must be high, whereas parameter $\phi$ must be low. On the other hand, a commitment located in regions 1 or 2 (each one also depicted with a circle), in which $\hat{u}_i \leq \bar{u}_i$, arises when effects of underadvertising (in the decreasing of the marginal profitability of rival's R&D and advertising) outweighs or offsets effects of underinvestment in R&D (in the increasing of the marginal profitability of the rival's advertising and R&D). For that, parameters $\omega$ and $\phi$ must be low enough.

- Case

$$\lambda = -\frac{\gamma}{2\beta}\theta.$$

In this case, each firm is located, in terms of R&D and advertising investments, in regions 1, 2 or 3 of figure 1.

- Case

$$\lambda \in \left( -\frac{\gamma}{2\beta}\theta, \theta \right).$$

When advertising investments are of this type the only way for firms to get a strategic advantage is to overinvest in both $u_i$ and $a_i$ (regions 5, 6 or 7 of figure 1), given the substitutability that exists in all the relationships between R&D and advertising expenditures.

Separation of 'direct-strategic effects' from 'indirect-strategic effects' of each investment in the differential effect between both regimes of competition follows from the FONCs of the first stage of the game $\Gamma((u_i, a_i), q_i)$, given in equations (11) and (12) in the text. By applying the implicit function theorem to this system, we obtain:

$$\frac{\mathrm{d}\hat{q}_j}{\mathrm{d}u_i} = \frac{1}{H}\frac{\partial^2\pi_i}{\partial q_i\partial u_i}\frac{\partial^2\pi_j}{\partial q_j\partial q_i} + \frac{1}{H}\left(\frac{\partial^2\pi_i}{\partial q_i\partial a_i}\frac{\partial^2\pi_j}{\partial q_j\partial q_i} - \frac{\partial^2\pi_i}{\partial q_i^2}\frac{\partial^2\pi_j}{\partial q_j\partial a_i}\right)\frac{\mathrm{d}a_i}{\mathrm{d}u_i}, \tag{22}$$

and

$$\frac{\mathrm{d}\hat{q}_j}{\mathrm{d}a_i} = \frac{1}{H}\left(\frac{\partial^2\pi_i}{\partial q_i\partial a_i}\frac{\partial^2\pi_j}{\partial q_j\partial q_i} - \frac{\partial^2\pi_i}{\partial q_i^2}\frac{\partial^2\pi_j}{\partial q_j\partial a_i}\right) + \frac{1}{H}\frac{\partial^2\pi_i}{\partial q_i\partial u_i}\frac{\partial^2\pi_j}{\partial q_j\partial q_i}\frac{\mathrm{d}u_i}{\mathrm{d}a_i}, \quad i = 1,2; \quad j = 3-i, \tag{23}$$

where $H = \beta^2 - \gamma^2$ is the determinant of the Hessian matrix of function $\pi_i$ with respect to output levels, and

$$\frac{\mathrm{d}a_i}{\mathrm{d}u_i} \quad (\text{respectively, } \frac{\mathrm{d}u_i}{\mathrm{d}a_i})$$

is the increasing or complementarity effect that the use of $u_i$ (respectively, $a_i$) causes on $a_i$ (respectively, on $u_i$).

The first term on the right-hand side of equation (22) captures the marginal effect of $u_i$ on $\hat{q}_j$, and the product of such a factor by

$$\frac{\partial\pi_i}{\partial q_j}$$

represents the 'direct-strategic effect' of cost-reducing investment. If a conventional negatively sloped demand exists and R&D investment reduces the marginal cost of the output, then

$$\frac{\partial^2\pi_i}{\partial q_i\partial u_i} > 0$$

when zero or low enough spillovers prevail;

$$\frac{\partial^2\pi_i}{\partial q_i\partial q_j} < 0,$$

for substitute goods; $H > 0$, by stability of the Nash equilibrium of the second stage; and

$$\frac{\partial \pi_i}{\partial q_j} < 0. \text{ Thus, it follows that } \frac{\partial \pi_i}{\partial q_j}\frac{\mathrm{d}\hat{q}_j}{\mathrm{d}u_i} > 0,$$

which encourages overinvestment in cost reduction in the absence of any advertising investment. This parallels the conclusions of Brander and Spencer (1983) and De Bondt and Veugelers (1991), among others (see result 1 in this paper).

The second term on the right-hand side of (22) represents the marginal effect of $u_i$ on $\hat{q}_j$ via the other long-run variable, $a_i$, and the product of such a term by

$$\frac{\partial \pi_i}{\partial q_j}$$

gives us the 'indirect-strategic effect'. When it is negative and sufficiently large (in absolute value), the 'direct-strategic effect' may be outweighed and therefore undercommitment in cost reduction may occur. Indeed, this happens when advertising increases the rival's residual demand and its productive efficiency is high enough.

Similar considerations with regard to advertising investment follow from the analysis of equation (23).

Explicit results for the model we have can be obtained by applying the mean value theorem to the FONCs given in equations (18) and (19), by which we obtain:

$$\Delta\left(\frac{\partial g_i}{\partial u_i}\right) = \frac{\partial}{\partial u_1}\left(\frac{\partial g_i}{\partial u_i}\right)\Delta u_1 + \frac{\partial}{\partial a_1}\left(\frac{\partial g_i}{\partial u_i}\right)\Delta a_1 + \frac{\partial}{\partial u_2}\left(\frac{\partial g_i}{\partial u_i}\right)\Delta u_2 + \frac{\partial}{\partial a_2}\left(\frac{\partial g_i}{\partial u_i}\right)\Delta a_2, \tag{24}$$

and

$$\Delta\left(\frac{\partial g_i}{\partial a_i}\right) = \frac{\partial}{\partial u_1}\left(\frac{\partial g_i}{\partial a_i}\right)\Delta u_1 + \frac{\partial}{\partial a_1}\left(\frac{\partial g_i}{\partial a_i}\right)\Delta a_1 + \frac{\partial}{\partial u_2}\left(\frac{\partial g_i}{\partial a_i}\right)\Delta u_2 + \frac{\partial}{\partial a_2}\left(\frac{\partial g_i}{\partial a_i}\right)\Delta a_2, \quad i = 1, 2, \tag{25}$$

where

$$\Delta\left(\frac{\partial g_i}{\partial u_i}\right) \equiv \frac{\gamma^2}{4\beta^2-\gamma^2}q_i$$

is the 'total-strategic effect' of R&D expenditures,

$$\Delta\left(\frac{\partial g_i}{\partial a_i}\right) \equiv \frac{\gamma(2\beta\lambda+\gamma\theta)}{4\beta^2-\gamma^2}q_i$$

is the 'total-strategic effect' of the advertising investment, $\Delta u_i \equiv \hat{u}_i - \bar{u}_i, \Delta a_i \equiv \hat{a}_i - \bar{a}_i$, and

$$\frac{\partial}{\partial a_i}(\cdot) \text{ and } \frac{\partial}{\partial u_i}(\cdot)$$

are evaluated at some point in the region defined by $(\hat{u}_i, \hat{a}_i)$ and $(\bar{u}_i, \bar{a}_i)$. The resolution of the 4x4 system defined by equations (24) and (25) yields

$$\Delta u_i = \frac{1}{Z}\gamma^2\frac{\omega(4\beta^2-\gamma^2)^2-\beta(2\beta\theta+\gamma\lambda)(2\beta-\gamma)(\theta-\lambda)}{4\beta^2-\gamma^2}q_i \qquad (24')$$

$$+\frac{1}{Z}\frac{2\beta^2\gamma(2\beta\lambda+\gamma\theta)(2\beta-\gamma)(\theta-\lambda)}{4\beta^2-\gamma^2}q_i$$

and

$$\Delta a_i = \frac{1}{Z}\gamma(2\beta\lambda+\gamma\theta)\frac{\phi(4\beta^2-\gamma^2)^2-2\beta^2(2\beta-\gamma)}{4\beta^2-\gamma^2}q_i \qquad (25')$$

$$+\frac{1}{Z}\frac{\gamma^2(2\beta\theta+\gamma\lambda)(2\beta-\gamma)}{4\beta^2-\gamma^2}q_i, \quad i=1,2,$$

where

$$Z = 2[(4\beta^2-\gamma^2)^2\omega\phi-\beta(2\beta\theta+\gamma\lambda)(2\beta-\gamma)(\theta-\lambda)\phi-2\beta^2(2\beta-\gamma)\omega].$$

The first term on the right-hand side of equation (24') is the contribution of cost-reducing expenditure to the direct output-mediated effect ('direct-strategic effect'), which is related to the efficiency of advertising. This 'direct-strategic effect' is positive, which means that $\Delta u_i > 0$.

The second term on the right-hand side of (24') measures the contribution of cost reduction to the output-mediated effect via advertising ('indirect-strategic effect'). It is positive when

$$\lambda > -\frac{\gamma}{2\beta}\theta,$$

which reinforces the 'direct-strategic effect' ensuring that $\Delta u_i > 0$, and is higher than in the absence of advertising. On the other hand, if

$$\lambda < -\frac{\gamma}{2\beta}\theta,$$

then the 'indirect-strategic effect' is negative and outweighs the 'direct-strategic effect' when $\omega$ is sufficiently low, in which case $\Delta u_i \leq 0$. In turn, if

$$\lambda < -\frac{\gamma}{2\beta}\theta,$$

and $\omega$ is large enough, then $\Delta u_i > 0$ but smaller than in the absence of advertising.

Similar considerations with regard to equation (25') help us to explain the firms' advertising behavior.

The conduct of firms in output terms in the two-stage game with respect to the one-stage game is such that:

$$\hat{q}_i \gtreqless \bar{q}_i \iff \phi(2\beta\lambda + \gamma\theta)(\theta - \lambda) + \omega\gamma \gtreqless 0, \quad i = 1, 2, \tag{26}$$

by which it follows that when

$$\lambda < -\frac{\gamma}{2\beta}\theta$$

and $\omega$ is low enough compared with $\phi$ in such a way that

$$\frac{\omega}{\phi} < \frac{(\theta - \lambda)(\gamma\theta - 2\beta\lambda)}{\gamma},$$

then $\hat{q}_i < \bar{q}_i$; if

$$\frac{\omega}{\phi} > \frac{(\theta - \lambda)(\gamma\theta - 2\beta\lambda)}{\gamma},$$

then $\hat{q}_i > \bar{q}_i$; if

$$\frac{\omega}{\phi} = \frac{(\theta - \lambda)(\gamma\theta - 2\beta\lambda)}{\gamma},$$

then $\hat{q}_i = \bar{q}_i$, and, on the other hand, if

$$\lambda > -\frac{\gamma}{2\beta}\theta,$$

then $\hat{q}_i > \bar{q}_i$. Finally, the pattern of firms' profits in each regime of competition moves in the opposite direction compared to the output. ■

PROOF OF PROPOSITION 5. When the duopolistic industry is socially managed, the social planner has, as the objective function, the total welfare function, $W$, defined as the sum of the consumer surplus and the firms' profits, i.e.:

$$W(\cdot) = (\alpha + \theta a_i - \lambda a_j - m + u_i)q_i \tag{27}$$

$$+(\alpha - \lambda a_i - \theta a_j - m + u_j)q_j - \frac{1}{2}(\beta q_i^2 + 2\gamma q_i q_j + \beta q_j^2)$$

$$-\phi(u_i^2 + u_j^2) - \omega(a_i^2 + a_j^2), \quad i = 1,2; \quad j = 3 - i$$

His goal is to choose $u_i, a_i$ and $q_i, i = 1,2$, so as to maximize this function, given the existing duopolistic market structure. Hence, his (non-strategic) behavior is characterized by the resolution of FONCs system:

$$0 = \frac{\partial W}{\partial u_i} \equiv q_i - 2\phi u_i, \tag{28}$$

$$0 = \frac{\partial W}{\partial a_i} \equiv \theta q_i - \lambda q_j - 2\omega a_i, \tag{29}$$

and

$$0 = \frac{\partial W}{\partial q_i} \equiv \alpha + \theta a_i - \lambda a_j - m \tag{30}$$

$$+u_i - \beta q_i - \gamma q_j, \quad i = 1,2; \quad j = 3 - i.$$

The resolution of this system of equations yields the socially efficient levels of cost-reducing effort, demand-creating advertising, and output for each firm. These are given by:

$$\left\{(u_i^{fb}, a_i^{fb}, q_i^{fb}) = \frac{\alpha - m}{F}(\omega, (\theta - \lambda)\phi, 2\omega\phi)\right\}, i = 1, 2, \qquad (31)$$

where $F = 2(\beta + \gamma)\omega\phi - (\theta - \lambda)^2\phi - \omega$.

The stability of such an equilibrium, as well as the interiority of $(u_i^{fb}, a_i^{fb})$-strategies are proved as just in proposition 1.

From here, we can compare investments of a duopoly developed in one- and two-stages games with those of a socially managed duopoly.

• Comparison between $(\bar{u}_i, \bar{a}_i)$ and $(u_i^{fb}, a_i^{fb})$. By comparing $(\bar{u}_i, \bar{a}_i)$-levels of proposition 1 with $(u_i^{fb}, a_i^{fb})$-levels given in (31) we have that:

$$\bar{u}_i \gtreqless u_i^{fb} \iff 2\beta\omega - \lambda(\theta - \lambda) \lesseqgtr 0, \quad i = 1, 2, \qquad (32)$$

and

$$\bar{a}_i \gtreqless a_i^{fb} \iff 2[\beta\theta - (2\beta + \gamma)\lambda]\phi + \lambda \lesseqgtr 0, \quad i = 1, 2. \qquad (33)$$

From (32) and (33) and considering the relationship between

$$\frac{\bar{a}_i}{\bar{u}_i} \text{ and } \frac{a_i^{fb}}{u_i^{fb}} \text{ ratios, the following occurs.}$$

a) $\bar{u}_i < u_i^{fb}$ and $\bar{a}_i < a_i^{fb}$ when $\lambda < 0$ (region 1 of figure 2), $\lambda = 0$ (region 2 of figure 2) or

$$\lambda \in \left(0, \frac{\beta}{2\beta + \gamma}\theta\right) \text{ (region 3 of figure 2).}$$

b)If

$$\lambda \in \left(\frac{\beta}{2\beta + \gamma}\theta, \theta\right) \quad \text{and} \quad \phi > -\frac{\lambda}{2[\beta\theta - (2\beta + \gamma)\lambda]},$$

then $\bar{u}_i < u_i^{fb}$ and $\bar{a}_i < a_i^{fb}$ (region 3 again).

c)If

$$\lambda \in \left(\frac{\beta}{2\beta+\gamma}\theta, \theta\right) \quad \text{and} \quad \phi = -\frac{\lambda}{2[\beta\theta - (2\beta+\gamma)\lambda]},$$

then $\bar{u}_i < u_i^{fb}$ and $\bar{a}_i = a_i^{fb}$ (region 4 of figure 2);

d) When

$$\lambda \in \left(\frac{\beta}{2\beta+\gamma}\theta, \theta\right) \quad \text{and} \quad \phi < -\frac{\lambda}{2[\beta\theta - (2\beta+\gamma)\lambda]},$$

three cases must be considered:

d.1) If

$$\omega > \frac{(\theta-\lambda)\lambda}{2\beta}, \text{ then } \bar{u}_i < u_i^{fb} \text{ and } \bar{a}_i > a_i^{fb} \text{(region 5 of figure 2);}$$

d.2) If

$$\omega = \frac{(\theta-\lambda)\lambda}{2\beta}, \text{ then } \bar{u}_i = u_i^{fb} \text{ and } \bar{a}_i > a_i^{fb} \text{ (region 6 of figure 2);}$$

d.3) If

$$\omega < \frac{(\theta-\lambda)\lambda}{2\beta}, \text{ then } \bar{u}_i > u_i^{fb} \text{ and } \bar{a}_i > a_i^{fb} \text{ (region 7 of figure 2).}$$

This comparison between $(\bar{u}_i, \bar{a}_i)$ and $(u_i^{fb}, a_i^{fb})$ levels is illustrated in figure 2, where regions 4 and 6 are both indicated by a circle.

• Comparison between $(\hat{u}_i, \hat{a}_i)$ and $(u_i^{fb}, a_i^{fb})$. Comparison between $(\hat{u}_i, \hat{a}_i)$-levels of proposition 2 and $(u_i^{fb}, a_i^{fb})$-levels given in condition (31) is similar to the previous one and yields that:

$$\hat{u}_i \gtreqless u_i^{fb} \iff (4\beta^3 - 2\beta\gamma^2 - \gamma^3)\omega \tag{34}$$

$$-\beta(2\beta+\gamma)\lambda(\theta-\lambda) \lesseqgtr 0, \quad i = 1, 2,$$

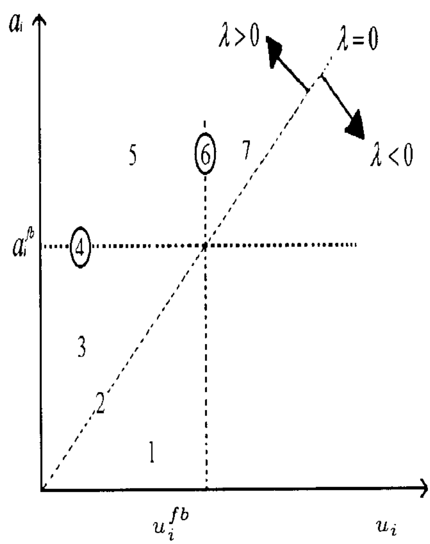
and

$$\hat{a}_i \gtreqless a_i^{fb} \iff [(4\beta^3 - 2\beta\gamma^2 - \gamma^3)\theta - (8\beta^3 + 6\beta^2\gamma - \gamma^3\lambda)]\phi \quad (35)$$

$$+\beta(2\beta+\gamma)\lambda \lesseqgtr 0, \quad i = 1, 2.$$

**Figure 2**

*Levels of investment in $(\bar{u}_i, \bar{a}_i)$ with respect to $(u_i^{fb}, a_i^{fb})$ levels, i = 1, 2*




The result is as follows.

a) When $\lambda < 0$, $\lambda = 0$ or $\lambda \in$

$$\left(0, \frac{4\beta^3 - 2\beta\gamma^2 - \gamma^3)}{8\beta^3 + 6\beta^2\gamma - \gamma^3}\theta\right), \quad \hat{u}_i < u_i^{fb} \text{ and } \hat{a}_i < a_i^{fb}$$

b)When

$$\lambda \in \left( \frac{4\beta^3 - 2\beta\gamma^2 - \gamma^3\theta}{8\beta^3 + 6\beta^2\gamma - \gamma^3}\theta, \theta \right)$$

and

$$\phi > \frac{\beta(2\beta + \gamma)\lambda}{(4\beta^3 - 2\beta\gamma^2 - \gamma^3)\theta - (8\beta^3 + 6\beta^2\gamma - \gamma^3)\lambda},$$

then $\hat{u}_i < u_i^{fb}$ and $\hat{a}_i < a_i^{fb}$.

c) When

$$\lambda \in \left( \frac{4\beta^3 - 2\beta\gamma^2 - \gamma^3}{8\beta^3 + 6\beta^2\gamma - \gamma^3}\theta, \theta \right)$$

and

$$\phi = \frac{\beta(2\beta + \gamma)\lambda}{(4\beta^3 - 2\beta\gamma^2 - \gamma^3)\theta - (8\beta^3 + 6\beta^2\gamma - \gamma^3)\lambda},$$

then $\hat{u}_i < u_i^{fb}$ and $\hat{a}_i = a_i^{fb}$.

d) When

$$\lambda \in \left( \frac{4\beta^3 - 2\beta\gamma^2 - \gamma^3}{8\beta^3 + 6\beta^2\gamma - \gamma^3}\theta, \theta \right)$$

and

$$\phi < \frac{\beta(2\beta + \gamma)\lambda}{(4\beta^3 - 2\beta\gamma^2 - \gamma^3)\theta - (8\beta^3 + 6\beta^2\gamma - \gamma^3)\lambda},$$

three cases must be distinguished:

d.1) If

$$\omega > \frac{\beta(2\beta + \gamma)(\theta - \lambda)\lambda}{4\beta^3 - 2\beta\gamma^2 - \gamma^3}, \text{ then } \hat{u}_i < u_i^{fb} \text{ and } \hat{a}_i > a_i^{fb};$$

d.2) If

$$\omega = \frac{\beta(2\beta + \gamma)(\theta - \lambda)\lambda}{4\beta^3 - 2\beta\gamma^2 - \gamma^3}, \text{ then } \hat{u}_i = u_i^{fb} \text{ and } \hat{a}_i > a_i^{fb};$$

d.3) If

$$\omega < \frac{\beta(2\beta + \gamma)(\theta - \lambda)\lambda}{4\beta^3 - 2\beta\gamma^2 - \gamma^3}, \text{ then } \hat{u}_i > u_i^{fb} \text{ and } \hat{a}_i > a_i^{fb} \quad i = 1, 2.$$

This may be illustrated in a similar way as in figure 2 and completes the proof of the proposition. ∎