DECOMPOSING ELECTRICITY PRICES WITH JUMPS

Eduardo Martínez Chombo*

Banco de México

Resumen: Se propone un modelo para descomponer los precios de la electricidad en dos procesos estocásticos independientes: uno que representa el comportamiento "normal" de los precios y otro que capture los "saltos" temporales. Para cada componente se especifica un parámetro de reversión a la media. Para identificar tales componentes especificamos un modelo estado-espacio con cambio de régimen. Al utilizar los precios de la electricidad de Nueva Gales del Sur estimamos el modelo aplicando la metodología de Kim (1994). Finalmente, se realizaron simulaciones con el método *bootstrap* para estimar la contribución esperada de cada componente en el precios total de la electricidad.

Abstract: We propose a model that decomposes electricity prices into two independent stochastic processes: one that represents the "normal" pattern of electricity prices and other that captures temporary shocks, or "jumps", with non-lasting effects in the market. Each contains specific mean reverting parameters to estimate. In order to identify such components we specify a state-space model with regime switching and apply the Kim's (1994)filtering algorithm to estimate the model for the mean adjusted series of New South Wales' electricity prices. Finally, bootstrap simulations were performed to estimate the expected contribution of each of the components in the overall electricity prices.

Clasificación JEL: C51, C52, C53, Q41

Palabras clave: Electricity prices, mean-reversion, jump modelling, markov switching models, state-space representation, energy finance.

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1. Introduction

When considering deregulation of the electricity industry, it is first necessary to determine a mechanism to price electricity in a competitive framework given the non-storability of electricity and the permanent need for maintaining the balance between demand and supply. Now, after more than ten years of international experience in competitive electricity markets, a set of alternative mechanisms exist, based on the interaction between the demand and supply, that warrant the uninterruptible operation of the power market. However, the specific characteristics of the industry and the decentralized decisions about when, where and how much power to produce have resulted in greater price volatility, including huge spikes in prices. For example it is not uncommon to see power price levels that peak at 100 times the normal rate. These characteristics of electricity spot prices have encouraged the development of financial derivatives that help market participants to hedge price risks in the new and volatile environment. Pricing those financial instruments has become one of the main topics in the research agenda that traditional financial literature has yet to satisfactorily model. In particular, the high dependence of such derivatives on assumptions regarding the stochastic processes that follow the underlying assets has opened a discussion about what models best fit the particulars of electricity spot prices. This study tries to contribute to the still developing discussion on modeling electricity prices in a deregulated market.

The standard approach to modeling electricity prices has been taken from the theory of finance. Some of the first attempts to model electricity prices assumed standard diffusion processes such as Geometric Brownian motion or Ornstein-Uhlenbeck types of processes. However, although they aim to capture some characteristics of electricity prices, such as its strong mean reversion, they did not capture the presence of spikes in prices.[1] One natural method of modeling such spikes was to use the diffusion-jump model developed by Press (1967). Press considered that the daily (log) returns in security markets can be divided in two components: the continuous diffusion part, which can be described by a Wiener process, and a discontinuous jump that represents shocks in the market and that can be modeled as a compound Poisson process. Under this specification, the resulting distribution of the (log) prices becomes a Poisson mixture of normal

[1] For a review of the main characteristics of electricity prices in deregulated markets, see Clewlow and Strickland (2000).

distributions whose parameters have to be estimated simultaneously. This approach was later used to model all types of financial instruments and became one of the standard models for series that present continuous jumps in their paths. In modeling electricity prices this jump-diffusion part is, in general, added to mean reversion models to account for the relatively short life of the jumps.

Examples of applications of the diffusion-jump model include Johnson and Barz (1999), who fitted several diffusion models to electricity prices in different markets. They found that the best fit was obtained by mean-reverting models with jumps; Knittel and Roberts (2001), using US electricity prices, estimated jump diffusion models with fixed and time dependent jump intensity but they found that the performance of such models is poor; and Escribano, Peña and Villaplana (2002), who combined jump processes with stochastic volatility.

Although appealing, such jump-diffusion approaches have some limitations in practice. The main problems come from an identification problem, because the resulting distribution of the (log) prices is a mixture of normal distributions and the estimation methods imply the use of the same data to estimate the parameters of both processes simultaneously (see Clewlow and Strickland (2000), and Huisman and Mahieu (2003)). The outcome of estimating such models is well known. Bates (1995) has documented that the jump-diffusion specification tends to capture small and high-frequency jumps, which is exactly the opposite of what is relevant in the study of electricity prices.

Alternatively, there is a more natural approach to model such spikes in electricity prices which assumes a diffusion process augmented with regime-switching. Sudden jumps in electricity prices are always related to the state of the generation and transmission system. If there is a shortage of electricity (because some lines become congested or because of the sudden break-down of a generation plant), market prices adjust drastically to rebalance the supply and demand of electricity. This is the response in prices regardless of the policy with respect of the maximum level of prices that can be achieved in the market.

In the last few years there has been an increase in the use of regime-switching models in the literature. Examples of this trend include Deng (2000), who developed a general model in which the regime-switching is used to capture the seasonal components of electricity prices; Choudakis (2002), who generalized the idea of discrete regime-switching models to a continuous framework; and Huisman and Mahieu (2003) who observed the need for modeling jumps as regime-jumps as a way to separately estimate the parameters of the "normal" component of electricity prices.

However modeling electricity prices as a switching Markov process implies that the effect of a shock in price tends to die out rather quickly, even when new jumps are allowed for the following periods. A close inspection of electricity price raises some doubts about using only a switching Markov process, as the effects of price shocks in this market do not die out quickly. If that is the case, the scenario in which the effects of shocks in prices remain at least for a while would be an empirically testable feature of electricity as a stochastic process. For that event we propose a model in which electricity price is compounded by two parts: what we call the "normal" behavior of prices and its "jump" component. Specifying separately the "jump" component as a mean reverting process with regime-jumps will allow us to measure and test the significance of its reverting parameters. On the other hand, following most of the literature,[2] we employ an Ornstein-Uhlenbeck type process to represent the "normal" component. Such model specification is flexible enough to allow some extensions; for example, to allow the conditional probabilities to depend upon some exogenous or predetermined variables, or to specify a stochastic volatility or a functional form of the seasonal component of electricity prices.

In this study, in order to focus our attention on the jumps and spikes of electricity markets, we abstract from the seasonality and other components of prices and estimate the model for the Australian market. To estimate the probabilities of the regimes, the unobservable variables and the parameters of the diffusion processes, the model is treated as a state-space model with regime switching and the estimation is made using the algorithm developed by Kim (1994), who extended the Hamilton Markov-switching model to the state-space representation of dynamic linear models. Finally we propose a bootstrap simulation to estimate expected electricity prices.

The remainder of the article is organized as follows. In the next section we describe briefly how prices are determined in a deregulated electricity sector. In section three we present the model, and in section

[2] See for instance Lucia and Schwartz (2000), Knittel and Roberts (2001), Johnson and Barz (1999), Escribano, Peña and Villaplana (2002), Deng (2000), and Huisman and Mahieu (2003). In particular, Escribano, Peña and Villaplana (2002) reported unit root tests that reject its presence in several electricity prices in favor of the alternative of mean reverting. There are also some studies that allow for non-mean reverting behaviour, see for example De Vany and Walls (1999) and Leon and Rubia (2001).

four we describe the estimation method. Finally in section five we present the results for the demeaned Australian electricity prices, the estimation of the model and the bootstrap simulation.

2. How Power Prices are Determined

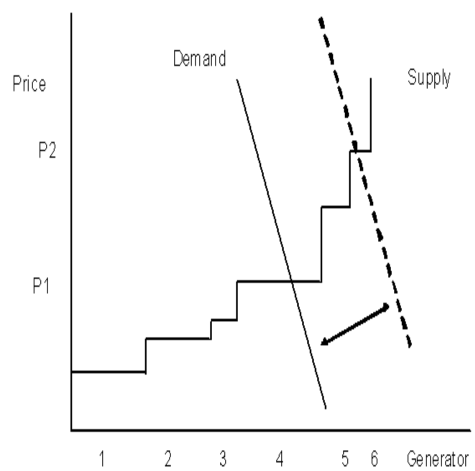
As in any other market, competitive electricity prices are determined by the interaction of demand and supply. Ideally the price clearing mechanism for this market will involve a two-side biding process, one for each side of the market. However, the atomization of the demand side has been one of the main obstacles to its complete implementation. Alternatively, many countries have adopted a one-side bid mechanism or have limited participation to customers with high electricity demand, including the distribution companies that buy electricity in the wholesale market and then distribute the power to small consumers. In fact, the implementation of a supply side bid mechanism, sometimes with the participation of large customers on the demand side, is considered the first step towards the liberalization of the sector.

In practice the one side bid mechanism may be described as follows. For each trading period (in general for each hour of the day) all the private power generators submit prices and the amount of power that they are willing to trade (in general in a day ahead market). Once the bids are submitted, the pool's regulators order the bids from the lowest to the highest prices and create an aggregate supply curve for the sector, which is matched with aggregate electricity demand to determine the spot prices and the dispatching order of generators. In this setting the marginal generator is the one which determines the clearing price in the market. One example of such a price clearing mechanism is illustrated in figure 1.

Among the determinants of the supply curve are the number of generation plants, their technology and the transmission lines that connect generators with consumption centers. If there is a large number of generators with similar technology and unrestricted transmission capacity, we would expect to observe gradual changes in prices as long as demand changes gradually. In the real world, however, there is a big variation in technologies across power plants, some of them better suited to supply power under specific conditions. For instance, in order to recover their fixed costs, big plants with low variable costs are scheduled to operate most of the time. Meanwhile, plants with high marginal cost but low cost of capital are economically better suited to operate only during periods of peak demand.

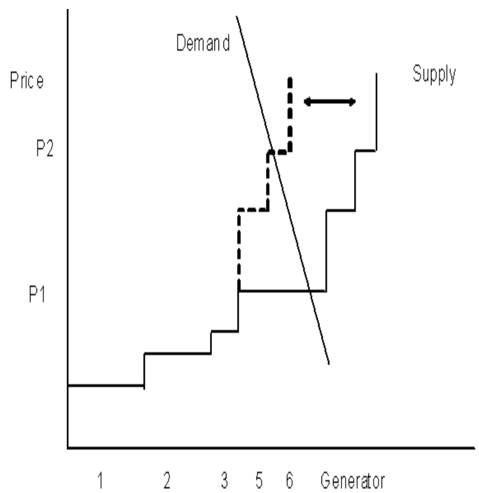
There are some peaking plants that work a relatively small number of hours during the year but charge a high price for their power as a way to cover fixed costs. If the maximum electricity demand is close to the total generation capacity of the system, such peaking plants will probably scheduled to operate. In such a case, electricity prices tend to rise drastically in the face of any increase in demand.

Figure 1
Balance between demand and supply in power markets: Shock in demand



Sudden and drastic changes in prices that quickly revert can be the result of a temporary surge in demand (for example, due to temporal changes in temperatures) or the result of temporary drops in supply (for example, due to temporal generators or transmission failures). These temporary movements are called the “jump” state in this article. Demand shocks may be identified with temporal movements of the demand curve to the right and the corresponding schedule of higher cost generators in the system. Figure 1 illustrates this movement: given the shock in demand, generator 6 at price P2 is dispatched. On the other hand, in the case of a shock on the supply side there would be a temporal movement of the supply curve to the left. Figure 2 illustrates this situation: if generator 4 temporarily goes off line, generators 5 and 6 will be activated at the higher price P2. It is even possible that because of such shocks, demand will not intersect the supply curve. In such a case electricity prices must be determined exogenously from the market mechanism, either by the regulators or by the price of exogenous power sources (i.e. the price charged by power plants external to the pool). Although there are several mechanisms for pricing electricity when demand is higher than the total capacity, generally prices are set at such a high level as to induce the entrance of new generator plants.

Figure 2
Shock in supply



A different kind of pattern in power prices is observed when electricity demand or supply is not exposed to extreme temporal shocks that require most or all the generation capacity. This condition is called in the study the "normal" state and it is characterized by the lack of extreme jumps in prices.

As in other competitive markets, electricity prices play the role of signals of the general conditions in the sector. Therefore it is important to distinguish between changes in prices that represent temporary shocks with non-lasting effects and changes in prices that correspond more closely to the intrinsic dynamics of the market.

It is possible that exogenous variables play an important role in the determination of prices, affecting simultaneously both components. That is the case, for example, when there is a shortage of fuel or when companies exercise some kind of market power. In such cases, neither component is independent and modeling explicitly such exogenous variables is necessary to fully capture the behavior of electricity prices. In this paper we assume that the behavior of each component is affected by different sources and therefore we treat them as independent. The goal of the next section is to explicitly differentiate these two components.

3. The Model

Taking into account that the extreme jumps which revert quickly correspond to different dynamics than the normal pattern in electricity prices, we consider a model that breaks apart such components. Specifically we express the electricity price P_t, as the sum of two independent stochastic processes, one that represents the normal behavior of prices (X_t) and other that represents the effect of temporary shocks (Y_t):

$$P_t = X_t + Y_t \tag{1}$$

We also assume that X_t and Y_t are governed by the following stochastic differential equations:

$$dX_t = k(a - X_t)dt + \sigma dB_t \tag{2}$$

$$dY_t = -\alpha Y_t dt + z_t dq_t \tag{3}$$

with dB_t representing an increment to a standard Brownian motion B_t and dq_t representing a Poisson process. Both the Poisson and the diffusion process are assumed to be independent of each other.

Notice that X_t follows an Ornstein-Uhlenbeck process, with instantaneous variance σ^2, long-run mean a, and a speed of adjustment $k > 0$. This specification of the normal pattern of prices attempts to capture the mean reverting property which is a characteristic of electricity prices. One straightforward extension of such specification is to allow a to change over time; either because it is a function of exogenous variables (such as the average price of the inputs used to generate electricity) or because of the seasonal pattern of electricity demand. In such a case the model would be specified with a varying parameter a_t instead of a fixed a. For the purposes of the present study, which focusses on modeling the “jump” component of prices, we assume that the long-run mean is constant.

On the other hand, Y_t is also specified to evolve as a mean-reverting process, with a long-run mean of zero and reverting rate $\alpha > 0$. However, its stochastic part is defined as a Poisson error component (dq_t) with an arrival frequency parameter λ and jump size z_t. Finally, z_t is assumed to be drawn from a normal distribution, with mean μ_z and variance δ^2, independent of the diffusion and Poisson processes.

In order to estimate the model we approximate the Poisson error component of Y_t with a Markov-switching model. Consider the following specification of (3) which involves the latent variable S_t:

$$dY_t = -\alpha Y_t dt + z_{t,S_t} \tag{4}$$

where

$$z_{t,0} = 0, \text{ with probability } 1 - \lambda\Delta t + o(\Delta t)^2,$$

$$z_{t,1} = z_t, \text{ with probability } \lambda\Delta t + o(\Delta t)^2,$$

$$z_{t,n} = n \cdot z_t, n \geq 2 \text{ with probability } o(\Delta t),$$

$$S_t = 0, 1, 2, ..., n \text{ is a latent variable, and}$$

$$z_t \tilde{} N(\mu_z, \delta^2) \text{ is the size of the jump.}$$

Notice that the expressions of the probabilities that govern z_{t,S_t} are obtained from the Taylor series expansion of the Poisson density; i.e. the probability of no jump in a "small" interval of time is approximately $1 - \lambda\Delta t$, and of one jump, $\lambda\Delta t$.

To translate this specification to a Markovian switching model we construct a manageable transition probability matrix to define the evolution of the state variable. In order to specify a Markov-switching model with two states, the "normal" state with $S_t = 0$ and the "jump" state with $S_t = 1$, we first assume that the probability of more than one jump in one unit of time is negligible.[3] We also assume a first order Markov-switching process for S_t, that is, the discrete variable S_t will depend only upon S_{t-1}. In order to capture the spikes

[3] In fact if data is available with high enough frequency, as is the case of electricity prices (for instance, hourly data), we can assume that in a short period of time only one jump may occur.

in prices due to short-lived shocks we assume that once the state variable indicates a jump at period t it will return to the "normal" state at period $t+1$ with probability m_{10} and that it will jump again with probability $m_{11} = 1 - m_{10}$. Therefore the relevant transition probabilities for the model are $\Pr[S_t = 0|S_{t-1} = 0] = m_{00}$ (that approximates the probabilities of the Poisson distribution $1 - \lambda\Delta t$) and $\Pr[S_t = 1|S_{t-1} = 1] = m_{11}$, with the complementary probabilities $\Pr[S_t = 1|S_{t-1} = 0] = m_{01} = 1 - m_{00}$ and $\Pr[S_t = 0|S_{t-1} = 1] = m_{10} = 1 - m_{11}$.

Notice that in (4) the mean reverting component of Y_t is not affected by the latent variable, S_t, hence there is a continuous adjustment even when the temporal shock has disappeared. This characteristic of the model gives some flexibility to capture possible lags in the effect of supply and demand shocks on the behavior of immediate future prices.

4. The Estimation Method

The proposed model contains inferences about the undistinguished variables X_t and Y_t, as well as inferences about the evolution of the state variable S_t. To solve the model we can take advantage of its state-space representation and use Kim's (1994) filtering algorithm, which merges switching states with dynamic models involving unobservable variables.

Assuming a Euler approximation to the stochastic differential equations (2) and (4), we can write the state-spacerepresentation of the system as follows:

Measurement Equation:

$$P_t = H\beta_t, \tag{5}$$

Transition Equations:

$$X_t = ka\Delta t + (1 - k\Delta t)X_{t-\Delta t} + \sigma\Delta t\varepsilon_t \tag{6}$$

$$Y_t = (1 - \alpha\Delta t)Y_{t-\Delta t} + e_{S_t} \tag{7}$$

with $\varepsilon_t \tilde{} N(0,1)$, $e_{S_t} = \mu_z S_t + \delta S_t e_t$, $e_t \tilde{} N(0,1)$ and $S_t = 1, 0$.

Or in matrix notation, and normalizing $\Delta t = 1$:

$$\beta_t = \bar{\mu}_{S_t} + F\beta_{t-1} + Q_{S_t}\upsilon_t \tag{8}$$

with

$$\beta_t = \begin{bmatrix} X_t \\ Y_t \end{bmatrix}, H = [1\ 1], \bar{\mu}_{S_t} = \begin{bmatrix} ka \\ \mu_z S_t \end{bmatrix}, F = \begin{bmatrix} (1-k) & 0 \\ 0 & (1-\alpha) \end{bmatrix}$$

$$Q_{S_t} = \begin{bmatrix} \sigma & 0 \\ 0 & \delta S_t \end{bmatrix} \text{ and } v_t = \begin{bmatrix} \varepsilon_t \\ e_{S_t} \end{bmatrix}$$

The subscripts in $\bar{\mu}_{S_t}$ and Q_{S_t} indicate that these expressions depend on the unobservable switching variable S_t, whose transition probabilities are given by:

$$M = \begin{pmatrix} m_{00} & 1-m_{11} \\ 1-m_{00} & m_{11} \end{pmatrix} \tag{9}$$

where as before $Pr[S_t = 0|S_{t-1} = 0] = m_{00}$, and $Pr[S_t = 1|S_{t-1} = 1] = m_{11}$.

Kim's algorithm is a mixture of Kalman and Hamilton filters, and includes a "collapsing" step to avoid the explosion of possible paths of the state vector due to the transition probability matrix. A complete discussion of the algorithm can be found in Kim (1994) and Kim and Nelson (1999). In this section we summarize the principal equations of the algorithm fitted to our model.

The goal of the filter is to form a forecast of β_t based on the vector of observations available up to $t-1$ (ψ_{t-1}), but also conditional on the random variables S_t and S_{t-1}. In terms of notation we have:

$$\beta_{t|t-1}^{(i,j)} = E[\beta_t|\psi_{t-1}, S_t = j, S_{t-1} = i],$$

and the associated mean squared error matrix is

$$P_{t|t-1}^{(i,j)} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'|\psi_{t-1}, S_t = j, S_{t-1} = i].$$

Conditional on $S_t = j$ and $S_{t-1} = i$, the Kalman filter algorithm follows the next computational steps:

$$\beta_{t|t-1}^{(i,j)} = \bar{\mu}_j + F\beta_{t-1|t-1}^{i},$$

$$P_{t|t-1}^{(i,j)} = FP_{t-1|t-1}^{i}F' + Q_jQ_j^{'},$$

$$\eta_{t|t-1}^{(i,j)} = p_t - H\beta_{t|t-1}^{(i,j)},$$

$$f^{(i,j)}_{t|t-1} = HP^{(i,j)}_{t|t-1}H',$$

$$\beta^{(i,j)}_{t|t} = \beta^{(i,j)}_{t|t-1} + P^{(i,j)}_{t|t-1}H'[f^{(i,j)}_{t|t-1}]^{-1}\eta^{(i,j)}_{t|t-1},$$

$$P^{(i,j)}_{t|t} = (I - P^{(i,j)}_{t|t-1}H'[f^{(i,j)}_{t|t-1}]^{-1}H_j)P^{(i,j)}_{t|t-1},$$

where $\eta^{(i,j)}_{t|t-1}$ is the conditional forecast error of p_t given ψ_{t-1}, $S_t = j$ and $S_{t-1} = i$; and $f^{(i,j)}_{t|t-1}$ is the conditional variance of the forecast error $\eta^{(i,j)}_{t|t-1}$,with $i, j = 0, 1$.

Notice that each iteration of the Kalman filter produces two more cases to consider per estimation of $\beta^{(i,j)}_{t|t}$.[4]

The Hamilton filter component focusses on calculating $Pr[S_t,$ $S_{t-1}|\psi_t]$ and $Pr[S_t|\psi_t]$ as follows:

$$Pr[S_t = j, S_{t-1} = i|\psi_{t-1}] = Pr[S_t = j|S_t = i]$$

$$\cdot Pr[S_{t-1} = i|\psi_{t-1}] = m_{ij} \cdot \pi_i$$

$$Pr[S_t = j, S_{t-1} = i|\psi_t]$$

$$= \frac{f(p_t|S_t = j, S_{t-1} = i, \psi_{t-1}) \cdot Pr[S_t = j, S_{t-1} = i|\psi_{t-1}]}{f(r_t|\psi_{t-1})}$$

$$Pr[S_t = j|\psi_t] = \sum_{i=1}^{2} Pr[S_t = j, S_{t-1} = i|\psi_t]$$

with:

$$p_t|S_t = j, S_{t-1} = i, \psi_{t-1} \sim N(\eta^{(i,j)}_{t|t-1}, f^{(i,j)}_{t|t-1}),$$

$$f(p_t, S_t = j, S_{t-1} = i|\psi_{t-1}) = f(p_t|S_t = j, S_{t-1} = i, \psi_{t-1})$$

$$\cdot Pr[S_t = j, S_{t-1} = i|\psi_{t-1}],$$

[4] The size of the transition probability matrix (M).

$$f(p_t|\psi_{t-1}) = \sum_{j=1}^{2}\sum_{i=1}^{2} f(p_t, S_t = j, S_{t-1} = i|\psi_{t-1}).$$

where π_i is the steady state probability of $S_{t-1} = i$ and m_{ij} is taken from the transition probability matrix (9).

To avoid an explosion in the number of cases to consider, Kim (1994) proposed the following approximation, that collapses the number of terms $\beta_{t|t}^{(i,j)}$ and their corresponding mean squared errors $P_{t|t}^{(i,j)}$ to only two cases (those corresponding to the number of states of S_t):

$$\beta_{t|t}^{i} = \frac{\sum_{j=1}^{2} Pr[S_t = j, S_{t-1} = i|\psi_t] \cdot \beta_{t|t}^{(i,j)}}{Pr[S_t = j|\psi_t]}$$

$$P_{t|t}^{i} = \frac{\sum_{j=1}^{2} Pr[S_t=j, S_{t-1}=i|\psi_t] \cdot \{P_{t|t}^{(i,j)} + (\beta_{t|t}^{i} - \beta_{t|t}^{(i,j)})(\beta_{t|t}^{i} - \beta_{t|t}^{(i,j)})'\}}{Pr[S_t=j|\psi_t]}.$$

With this approximation, $\beta_{t|t}^{i}$ is no longer the linear projection of β_t on ψ_t as in the pure Kalman filter. Now the algorithm is manageable and Kim has showed that the loss in efficiency produced by the approximation is only marginal. In our case, this algorithm is used to identify two stochastic components of electricity prices and therefore it permits us to estimate the parameters of the "normal" electricity process without considering the noise of temporal shocks. What follows is an application of the above model to a competitive wholesale electricity market.

5. Application: Electricity Spot Prices in New South Wales, Australia

In this section we show the results of applying the above model to New South Wales' electricity spot prices.[5] This market began operations as a regional market in 1996. Later, it was integrated into the national grid creating the Australian National Electricity Market (NEM). The NEM operates a supply bidding mechanism that sets electricity prices every half hour. The period studied in this analysis begins with the integration of the national market on January 1999 and ends on May 2002 with a total of 59,835 observations.

[5] Data source: http://www.nemmco.com

Secondary markets have traditionally used average daily prices to price futures and other derivatives in electricity markets.[6] Following this practice we based our estimations on average daily prices, which results in a total sample of 1,247 daily observations. With this transformation we also avoid the strong intraday cyclical behavior of the electricity market.

A complete characterization of the stochastic process of electricity prices involves specifying its seasonal component as well as its relationship with other exogenous variables that may determine its trend, such as the average cost of the inputs. In the model we assume that all of these elements are captured by the time varying mean (a_t) of the normal component. However, to focus on the decomposition into "normal" and "jump" components, we estimate a_t with nonparametric techniques instead of explicitly assuming a specific functional form.[7] Once we have estimated this time varying "mean" to which the "normal" component is reverted, we proceed to estimate the diffusion parameters (6) and (7) over the prices' deviations from that mean. Figure 3 shows the original and the transformed data that the estimations of our model are based on.

Using the transformed price series we estimate the parameters of the transition equations (6) and (7) and the probabilities of the transition probability matrix (9). Maximum Likelihood estimates of the parameters are shown in table 1.[8]

By examining the results, it seems clear that the conditional probability of the occurrence of a jump given that there was already a jump in the previous period, is not negligible (m_{11}=0.75). This means that "jumps" are significantly correlated in the NSW market, implying that for modeling purposes such parameters must at least be checked to see if they are different from zero. From the parameters of the Markov transition matrix we can also compute the unconditional probability that the process will be in a "jump" state as follows:

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[6] See Lucia and Schwartz 2000 for a description of the Nordic Power Exchange.

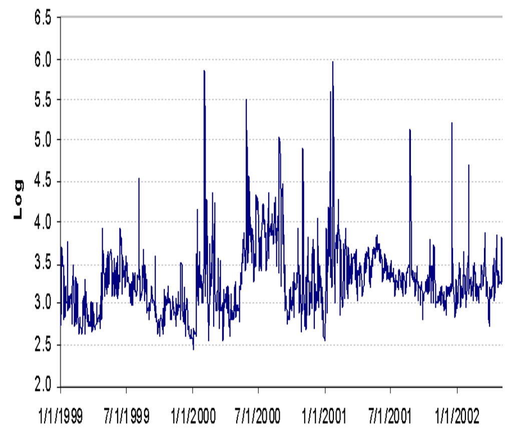
[7] Specifically we follow the next three steps to transform the data: 1) We decompose the original price series into a pseudo "normal" and a pseudo "jump" component using Kim's algorithm (they are called pseudo components because they still capture some movements in a_t), 2) Taking as our data the pseudo "normal" component, we non-parametrically estimate its mean using a normal kernel with the optimal window width $h = 0.25\sigma_t n^{-1/5}$ and $t = 1, 2...n$, 3) Finally we substract the estimated mean from the original prices.

[8] The parameters estimates were obtained using a Bernt, *et al.* (1974) algorithm and the results were robust for different starting values.

$$\hat{\pi}_1 = \Pr[S_0 = 1|\psi_0] = \frac{1 - \hat{m}_{00}}{2 - \hat{m}_{00} - \hat{m}_{11}} = 0.1323$$

Figure 3

Electricity Spot Prices in New South Wales, Australia



Transformed Electricity Spot Prices

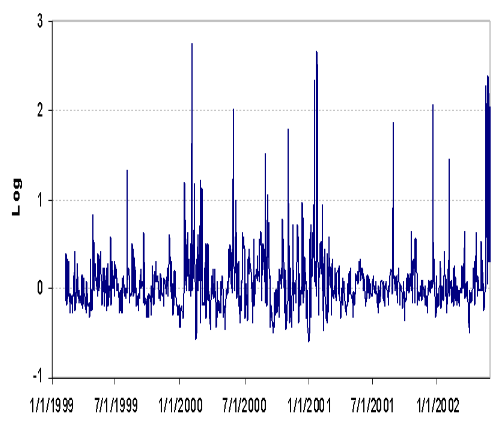


Table 1
Estimation Results of the Model

Parameter	*Estimates*	*Std. err.*	*t values*
Transition Probability Matrix			
m^*_{00}	3.0953 (0.9567*)	0.2157	14.349
m^*_{11}	0.9247 (0.7160*)	0.2821	3.279
"Normal" Component			
k	0.2392	0.0221	10.844
a	-0.0039	0.0195	-0.202
σ	0.1457	0.0050	28.926
"Jump" Component			
α	0.7509	0.0869	8.645
μ_z	0.4811	0.0883	5.451
δ_z	0.6101	0.0436	14.008
ML	0.168604		

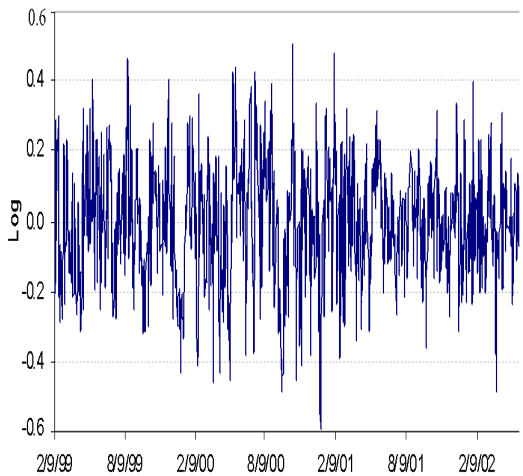
*Probability: $m_{ii} = exp(m^*_{ii})/[1 + exp(m^*_{ii})]$.

On the other hand, it is not surprising that the long-run mean of the "normal" component is not significantly different from zero since we worked with mean adjusted series. The results show that in spite of a high reverting parameter for the "jump" component ($\alpha = 0.75$), prices do not fall back completely on the day after a jump, but follow a gradually decreasing adjustment process. Such a result raises doubts about the assumption that jumps have short-lived effects in other studies (i.e. Huisman and Mahieu (2003)), and suggests the need for considering such gradual adjustment in electricity prices. One explanation of this result may be that after a supply failure or a sudden demand change, the market participants are unsure about the likelihood that such behavior could be repeated in the subsequent periods (observation consistent with the high value of m_{11}). As a consequence, participants adjust prices gradually.

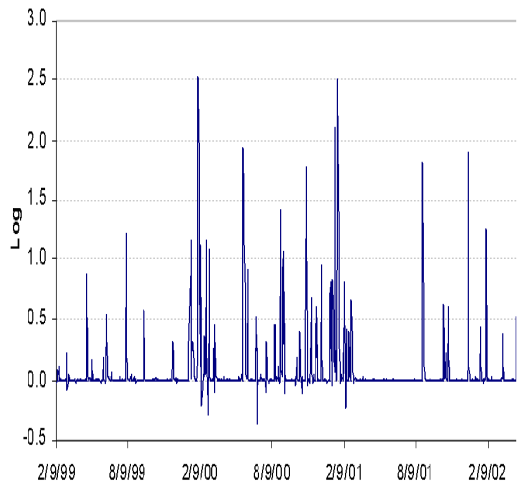
As part of the estimation process, the filtering algorithm also splits electricity prices into two components, $\{X_t\}$ and $\{Y_t\}$, and estimates the unconditional probability that the process will be in a jump state at any period. The decomposition and the probabilities are plotted in figure 4.

Figure 4
Decomposition of Electricity Prices and Unconditional Probabilities

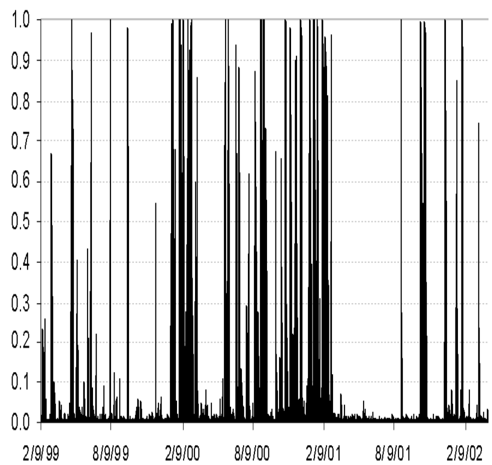
"Normal" Component of Electricity Prices



"Jump" Component of Electricity Prices



Probability that the Process is in Jump State



A direct application of the decomposition of electricity prices is the estimation of the "jump" component contribution to the average electricity price during a certain period of time. Since financial instruments are traded according to their price per kilowatt-hour and the amount of electricity delivered in a certain period of time, knowledge about the contribution of the "jump" switching process provides a cost estimate of having a market mechanism that allows a certain frequency, size and persistence of the jumps.

As an illustrative example we consider the contribution of each component to the average monthly price of the sample over the last four months and the average weekly price over the last two months. The results of this decomposition are given in table 2.

Table 2
Estimated Decomposition of the Average Electricity Price for the Year 2002 (Australian Dollars)

Period	*Observed Prices*	*Normal Component*	*Jump Component*
January	25.50	24.91	0.59
February	29.53	26.61	2.92

Table 2
(*continued*)

Period	*Observed Prices*	*Normal Component*	*Jump Component*
March	25.89	25.25	0.64
April	26.59	25.74	0.84
May	74.94	28.02	46.91
April/06-April/12	25.68	25.63	0.05
April/13-April/19	25.73	25.69	0.04
April/20-April/26	30.87	27.37	3.49
April/27-May/03	26.97	26.94	0.03
May/04-May/10	30.91	27.60	3.32
May/11-May/17	25.72	25.73	-0.001
May/18-May/24	114.75	29.19	85.56
May/25-May/31	149.27	30.39	118.89

6. Bootstrap Simulation

We use the bootstrap method to simulate electricity prices and estimate their expected value in the future. In particular we are interested in knowing the price component that is attributable to the "jump" state in comparison with the contribution of the "normal" state in the industry. This price decomposition may be used to evaluate the benefits of reducing the size or frequency of such "jumps". Also, based on the previous model, the bootstrap technique provides an alternative method to estimate the price of futures and other derivatives in the electricity market.

The bootstrap method is used mainly for estimating test statistics or the distribution of an estimator through simulation techniques that resample the real data set. Here we use the method to simulate the electricity price pattern from decomposing the contribution of the "normal" and "jump" components.[9] From the simulations we may

[9] The discussion does not attempt to provide a detailed description of the bootstrap method. For a comprehensive description of the method see, for example, Horowitz (1999).

calculate the expected value of the average price for future weeks or months, the periods of time in which electricity flows are generally traded.

The simulation is based on the Euler approximation (6) and (7). Notice that the parametric model of the "normal" component of electricity prices (6)reduces its data generation process to a transformation of the independent random variable ε_t. Then a bootstrap sample of $\{X^*\}$ can be directly generated by random sampling of the residuals from the fitted model. That is, we estimate:

$$X_t^* = \hat{k}\hat{a} + (1 - \hat{k})X_{t-1}^* + \hat{\sigma}\varepsilon_t^*,$$

where $\hat{k}$, $\hat{a}$ and $\hat{\sigma}$ are the Maximum Likelihood (ML) estimates of the parameters of (6) and $\{\varepsilon_t^*\}$ is a random sample of the normalized estimated residual $\{\hat{\varepsilon}_t\}$.

In a similar way we can generate a bootstrap sample of $\{Y^*\}$ taking into account that the parametric model (7) does not produce independent errors because of the first order Markov-switching process that governs S_t. Such a bootstrap sample can be generated using the following relationship:

$$Y_t^* = (1 - \hat{\alpha})Y_{t-1}^* + e_{S_t}^*,$$

where $\hat{\alpha}$ is the ML estimate of α and $\{e_{S_t}^*\}$ is a conditional sample of the estimated residual $\{\hat{e}_{St}\}$.

To deal with the dependence of the fitted errors we performed a conditional bootstrap sample of the residuals as follows. First, we identify a jump in any particular period using the estimated unconditional probabilities of observing a jump obtained from Kim's smoothing algorithm (see figure 4). If this probability is greater than that deduced from the estimated parameters, we consider that there was a jump in that particular period. In terms of notation there is a jump if the following condition applies:

$$\hat{P}(S_t = 1|\Psi_t) > \hat{\pi}_1 = \frac{1 - \hat{m}_{00}}{2 - \hat{m}_{00} - \hat{m}_{11}}$$

where $\hat{P}$ is obtained from the smoothing algorithm and $\hat{m}_{00}$ and $\hat{m}_{11}$ are the ML estimates of m_{00} and m_{11} respectively.

Once we identified the periods in which a jump in prices occurred, we classified the estimated residuals in two subsamples: one that collects all the residuals that follow a jump, a sub-sample called J_j, and another that collects all the residuals that do not follow a jump, a sub-sample called J_n. Then we generated bootstrap samples of $\{e^*_{S_t}\}$ by randomly sampling these two sets conditional on the state at $t-1$ ($S_{t-1}=0$ in the "normal" state or $S_{t-1}=1$ in the "jump" state) as follows:

$$e^*_{S_t} = \begin{cases} e^*_t \in J_n & \text{if} \quad S_{t-1}=0, \\ e^*_t \in J_j & \text{if} \quad S_{t-1}=1. \end{cases}$$

For comparative and illustrative purposes we estimated by bootstrap simulation the expected average price of electricity on October 1st, 2001, for energy to be delivered during the same periods as in table 2. As before, we assumed that the mean of the normal component is given exogenously and computed the decomposition of electricity prices based on deviation from that mean. To estimate the expected monthly and weekly average prices, we simulated 10,000 paths of electricity prices for the period between October, 2001 and May, 2002. The expected average price by component and the probability interval for the maximum average price is reported in table 3. From the same simulations we computed the probability density of the average electricity prices for a specific month and week, the result of which is shown in figure 5.

Figure 5

Comparison of the Estimated Probability Density of the Monthly and Weekly Average Electricity Prices on October 2001

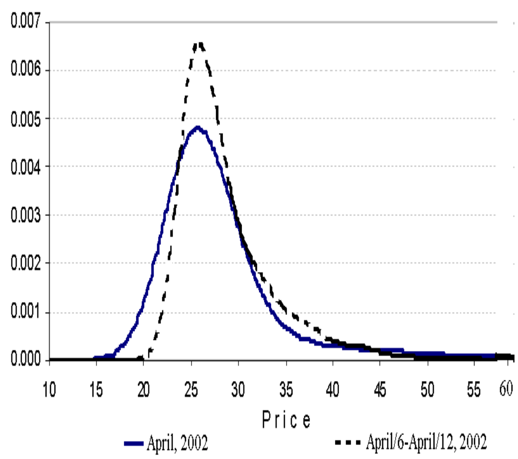


Table 3

Estimated Decomposition of Expected Average Prices
for the Year 2002 on October 2001
(Australian Dollars)

Period	*Expected Average Prices*	*Normal Component*	*Jump Component*	*Participation Jump Compon.*	*Maximum Price at 95% confidence*	
	(A)	(B)	(C)	(C/A)	(D)	(D/A)
January	28.658	25.114	3.544	0.141	39.080	1.364
February	29.672	25.986	3.686	0.142	40.880	1.378
March	29.342	25.653	3.689	0.144	39.820	1.357
April	29.823	26.029	3.794	0.146	41.140	1.379
May	32.114	28.056	4.058	0.145	43.740	1.362
Average	29.922	26.168	3.754	0.143	40.932	1.368
April/06-April/12	29.380	25.643	3.736	0.147	49.728	1.693
April/13-April/19	29.666	25.986	3.680	0.142	49.832	1.680
April/20-April/26	30.221	26.368	3.852	0.146	50.976	1.687
April/27-May/03	30.765	26.815	3.951	0.147	51.964	1.689
May/04-May/10	31.170	27.332	3.838	0.140	51.288	1.645
May/11-May/17	32.190	27.952	4.238	0.152	54.980	1.708
May/18-May/24	32.512	28.463	4.049	0.142	54.720	1.683

Table 3
(*continued*)

Period	*Expected Average Prices*	*Normal Component*	*Jump Component*	*Participation Jump Compon.*	*Maximum Price at 95% confidence*	
	(A)	(B)	(C)	(C/A)	(D)	(D/A)
May/25-May/31	33.087	28.946	4.141	0.143	55.604	1.681
Average	31.124	27.188	3.936	0.144	52.387	1.683

Note: The mean is assumed to be given.

According to the simulation results, the percentage of the expected price attributable to the "jump" component is around 14% for both monthly and weekly average prices. This average contribution appears low compared to "jumps" that rise up to 10 times the average price in a single day. Also, if we compare the expected average price with the observed average in table 2, we notice that May 2002 was an exceptionally high price period, with the highest prices primarily concentrated in the last two weeks of the month. Looking at the estimated confidence interval of the average prices, we find that as an average there is a 95% probability that average prices will not increase by more than 37% of the expected monthly value and by not more than 68% of the expected weekly average. These expected average prices appear to be maintained regardless of the starting date of the simulation, provided that there is enough time to dissipate the initial shock in prices.

7. Conclusion

This study highlights the necessity of decomposing the movement of electricity prices into two components: one driven by normal market conditions and the other that captures the effect of supply failures and/or constraints, or sudden increases in demand. The proposed model treats the stochastic process of each component as independent from the other, each one with its own mean reverting parameter. This study also maintains that considering the jumps and spikes in prices as a jump switching process in which the effects do not disappear quickly, bears certain advantages, since, technologically speaking, it is natural to consider two states, the failure and the normal state, in electricity markets. Technically this approach overcomes identification problems by capturing the big jumps with low frequency instead of the small jumps with high frequency, as is usually the case in jump-diffusion type processes.

We applied the model to New South Wales' electricity spot prices and found all the parameters of the model to be statistically significant. One of the most important results is that the estimated mean reverting parameter of the jump component does not completely eliminate the effect of a jump in the next period. There is also evidence that jumps are not independent but correlated in this market. These results contrast with the assumptions of other studies, suggesting a need to explicitly test the mean reverting speed of the jumps and their independence. With respect to the decomposition of the observed average electricity prices in the NSW electricity market, we found that in May 2002 the jump component rose up to 70% above the average of the normal component, and up to 300% in the last week of the same month.

The bootstrap simulation technique was also implemented to estimate the expected average price over a future month or week. It was found that, as an average, the expected contribution of the "jump" component in the expected average price is around 14%. On the other hand it was estimated that there is a 95% probability that average prices will not increase by more than 37% of the expected monthly value and by no more than 68% of the expected weekly average.

Finally, although the model deals with the identification of the "normal" and "jump" components in prices, seasonality is another factor that is not treated explicitly in this study. The assumption from which we construct our decomposition is that prices do not follow a time varying mean to which they revert, but that there is a fixed long-run mean in the "normal" component. This assumption is obviously not true for markets with strong seasonality or in situations in which exogenous variables play an important role in the price determination, such as the price of natural gas. However the model can be extended to explicitly include such components, allowing functional specification of the time varying mean of the "normal" component. In the same way the model can be extended in several directions, for example including time dependent probabilities or a stochastic volatility specification.

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