THE STRUCTURE OF POLITICAL POWER AND REDISTRIBUTION IN ECONOMIES WITH MULTIPLE GOVERNMENTS

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Resumen: Para economías con múltiples gobiernos, el esfuerzo y efectividad de la redistribución pública depende de la estructura política de la federación. El gobierno central determina el grado de redistribución pública; las preferencias y el salario de los votantes que controlan, respectivamente, al gobierno central y gobiernos sub-nacionales determinan si la redistribución del ingreso es efectiva en redistribuir el bienestar. En este artículo identificamos condiciones en las que la interacción entre el gobierno central y gobiernos sub-nacionales conduce a una asignación Pareto superior en la redistribución del ingreso.

Abstract: For economies with multiple governments, the effort and effectiveness of public redistribution policies depend on the political structure of the federation. The central government determines the degree of redistribution and the interaction between the preferences and wages of voters controlling, respectively, the central and sub-national governments determine whether income redistribution can be an effective tool to redistribute welfare. In this paper, we identify conditions in which the interaction between the central government and sub-national governments lead to a Pareto superior allocation in the redistribution of income.

Clasificación JEL/JEL Classification: H23, H21, H7, H30, D72
Palabras clave/keywords: redistribución, eficiencia, federalismo, elecciones, redistribution, efficiency, federalism, elections

Fecha de recepción: 11 XI 2013 Fecha de aceptación: 18 IX 2014

* This article is a product of the research project: Red internacional de análisis y evaluación de políticas económicas y sociales, 2012-2014, financed by PROMEP. Authors would like to thank to two anonymous reviewers whose comments improved this paper, rponce@uacj.mx, ikuho.kochi@uacj.mx, lgtz@uacj.mx

1. Introduction

In modern economies, citizens receive goods from (and pay taxes to) multiple levels of governments that each have their own objectives and implement (possibly) uncoordinated fiscal policies. In this context, the literature has emphasized the fiscal consequences of coordination failures that lead to vertical and horizontal fiscal externalities. For the case of horizontal externalities, the mobility of households and firms induces subnational governments to overestimate the marginal cost of public funds, leading to sub-optimal levels of state taxation and spending (see Wilson, 1999). On the contrary, for the case of vertical fiscal externalities, Johnson (1988), Boadway and Keen (1996), among many others, argue that federal and sub-national governments will underestimate the marginal costs of public funds associated with raising tax revenue leading to too high taxation and spending.

In this paper we analyze a federation with multiple levels of governments implementing uncoordinated policies. The central government implements a linear redistributive policy that seeks to redistribute welfare by redistributing income while sub-national governments provide local public goods (such as health services, local security, parks, bridges, trash recollection, etc.). The objective of this paper is to identify conditions that explain, first, why the central government exerts a high or low level of effort to redistribute income for an economy with multiple governments? Second, what is the role of politics in determining whether the uncoordinated actions of multiple governments lead to a highly effective (or ineffective) redistributive policy of the central government?

To answer these questions, we consider a political economy model with sequential elections and centralized leadership in which parties have fiscal policy preferences (see Wittman 1973, 1983). In our economy, national elections take place and voters elect a party that implements its ideal policy on income redistribution. Moreover, a set of simultaneous elections at the sub-national level leads elected parties to provide local public goods.

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1 In this case tax and spending policies of state governments are too low compared with the optimal levels of taxation and spending.

2 We analyze a federation without horizontal externalities because households have no mobility, but we consider a Stackelberg game with leadership from the central government, hence the central government takes into consideration how redistribution affects local public spending but sub-national governments do not take into account how their policies affect the policy of the central government.
In our economy, the redistributive policy of the central government changes, \( i) \) The distribution of welfare in the society by increasing (reducing) the full income of poor (rich) families and, \( ii) \) The level of spending of sub-national governments. Public redistribution affects local spending through the following channels: first, the provision of local public goods is determined by an elected party that represents the preferences over public spending of a coalition of activist-voters who rule the party. Hence, the party designs public spending to maximize the preferences of the coalition of voters subject to the constraint that local spending is financed by local taxes. If sub-national governments provide an inferior local public good and localities are ruled by parties that represent a coalition of voters with a wage higher than the nationwide average wage then the linear redistributive program of the central government induces a fall in full income and the demand for public goods of the coalition controlling the local government increases.

Second, by redistributing income, the transfer policy of the central government affects the demand of households for private goods and the ability of local governments to raise tax revenue (and the supply of local public goods). Whether a given locality is a net winner or loser on tax revenue as a result of the central government policy depends on the difference between the aggregate transfers from the redistributive program to residents of a given locality and the aggregate tax payments of residents of the locality to the central government. This in turn, depends on the population density of the economy and on the original distribution of income.

In this paper, we contribute to the literature by identifying conditions that map the distribution of political power, the effort, and effectiveness of public redistribution in a federation. Even though in the literature it is well understood that elections and party preferences matter to determine fiscal outcomes (see Reed, 2006; Alt and Lowry, 2000; Caplan, 2001; Rogers and Rogers, 2000; Chernick, 2005; and Fletcher and Murray, 2008), we have little understanding of how the structure of political power in a federation might affect the policies of the central and subnational governments.

In this paper we contribute to filling this gap. In particular, we find that for a large economy, the central government implements a strictly positive universal transfer if the federal government is ruled by a party that represents a coalition of voters with a wage below the nationwide average wage. The effort of the central government in redistributing income will be high (that is the \( \text{per capita} \) transfer is high at the political equilibrium) if the local governments of both
localities are ruled by parties representing voters with wages above the nationwide average wage.

Moreover, we find that if the following conditions are met: (i) Local public goods are inferior, (ii) The central government is ruled by a party that represents a coalition of voters with a wage below the nationwide average wage, (iii) Local governments represent voters with a wage above a certain threshold, (iv) The willingness to pay for local public spending of key activists inside the ruling party of local governments is high, and (v) Localities have sufficiently high densities of population; then the welfare of key voters-activists in the central government is increased above the direct effect of the redistributive program. This outcome implies that the central government is highly effective at redistributing welfare by redistributing income for key voters-activists inside the party in control of the central government. As a result, the strategic interaction among governments leads to a non-cooperative welfare-superior allocation of resources in which the per capita public transfers from the central government are high and the welfare gains associated with public redistribution for key voters-activists in the central government are high as well.

The paper is structured as follows: section 2 reviews the literature on the topic. Section 3, characterizes the politically driven redistribution and the reaction functions of sub-national governments. Section 4, analyzes the structure of political power, the effort to redistribute income and identifies sufficient conditions that lead to the equilibrium with non-cooperative welfare-superior redistribution. Section 5 includes a comparative analysis of our political equilibrium with the Pareto efficient outcome in which the government is ruled by a benevolent social planner. Section 6 concludes.

2. Literature review

In a federation, citizens are represented by a central government and by several sub-national governments who have different preferences and abilities to provide public goods and services. In this context, the literature has emphasized the possibility of coordination failures among governments that lead to vertical and horizontal fiscal externalities and its fiscal consequences. For the case of horizontal externalities, Wildasin (1991) argues that state governments ignore the effect of local taxes on other jurisdictions. Hence, in presence of mobile households and firms, state governments will overestimate the marginal cost of public funds leading to too little sub-national taxes
and spending.\textsuperscript{3} For the case of vertical fiscal externalities, the basic argument by Johnson (1988), Boadway and Keen (1996), Dahlby (1996), Boadway, Marchand and Vigneault (1998), and more recently Rizzo (2008) and Dahlby and Wilson (2003), is that the federal and sub-national governments might not take into account how their policies affect each other. Therefore, these governments will underestimate the marginal costs of public funds associated with raising tax revenue leading to too high taxation.

Another branch of the literature has recognized that, in a Stackelberg game with leadership from the central government, the federal government could eliminate the vertical externality (see Keen, 1997; Dahlby, 1996; Caplan, Cornes and Silva, 2000; Gong and Zou, 2002; Aronsson, Andersson, and Wikström, 2004; and Aronsson and Blomquist, 2008). The theory of coordination failures usually emphasizes how self-interested agents might fail to cooperate and reach a Pareto superior allocation. While there is a large body of literature that studies the case of negative externalities, an issue that has received little attention is the case of uncoordinated policies leading to Pareto superior allocations. However, this topic is highly relevant for policy making. In this paper we study such a case and we call it non-cooperative welfare-superior redistribution. That is, we focus on whether the fiscal policy of the central government is complemented (from the point of view of the underlying objectives of the central government) by uncoordinated policies of sub-national governments leading to a high (or low) effectiveness of the policy of the federal government.

In this type of equilibria, public redistribution changes not only the distribution of welfare in the society but also affects local public spending. One likely outcome is that the local public good might be inferior and its provision might increase as a result of the redistributive policy of the central government. In this case, the welfare of poor families could also be increased by more than just the direct effect of the redistributive program. Hence, the redistributive policy of the central government is highly effective to redistribute welfare in favor of pivotal voters-activists by redistributing income. Our analysis complements the study of Kochi and Ponce (2013), in which they analyze the behavioral effects of redistribution on subnational spending. However in their paper there is no policy design by the central government while in our paper we introduce a Stackelberg game to

\textsuperscript{3} For a literature review of horizontal fiscal externalities and tax competition see Wilson (1999).
analyze the strategic interaction between the central and subnational governments.\textsuperscript{4}

Our paper is also related to the theory of partisan politics and fiscal outcomes. This theory is concerned with the consequences of political outcomes and its effects on fiscal policy. A great deal of the literature has focused on the partisan consequences on the US fiscal policy. For instance, Reed (2006), Alt and Lowry (2000), Caplan (2001) and Rogers and Rogers (2000) find evidence that state taxes increase when Democrats have significant control of the executive and legislative bodies of state governments.\textsuperscript{5} Caplan (2001) finds that corporate and income taxes tend to rise under Democratic control of state legislatures and fall with larger Republican majorities. Chernick (2005) finds that party control by Republicans is associated with more regressive state tax structures. Fletcher and Murray (2008) find that party control by the Democrat party is positively associated with higher top income tax rates, higher income threshold for the first bracket of the income tax, and higher earned income tax credits.

Our analysis is also related to the theory of political economy of fiscal federalism. For instance, Lockwood (2002) studies the allocation of public funds by the party with a majority, Besley and Coate (2003) show that the sub-national provision of local public goods with and without inter-regional spillovers is not Pareto efficient in a model with legislatures. Ortúñor-Ortín and Sempere (2006) study the role of politics in determining the degree of fiscally autonomy regions versus fiscal centralization. Finally, Bolton and Roland (1997) study how political conflicts over redistributive policies might either lead nations to breakup or to increased unity.

Even though in the literature it is well understood that elections and party preferences help to determine fiscal outcomes, we have little understanding of how the structure of political power of the central

\textsuperscript{4} In the real world, election calendars are characterized by sequences of elections with interactions between national and local elections. This structure justifies considering the strategic interaction between the central and local governments throughout Stackelberg games in which the central government can be the leader and local governments followers and vice versa. A full characterization of strategic interactions between the central and the system of local governments with repeated games (in which the roles of the leader and followers change) is out of the scope of this paper. Instead, we consider the central government as the leader and local governments as followers as our starting point of our research agenda and we leave the other cases for future research.

\textsuperscript{5} Party’s control of the legislature can be interpreted as an environment in which a majoritarian coalition faces little or imperfect political competition.
and sub-national governments might affect fiscal outcomes in a federation. This paper seeks to contribute to filling this gap since our theory identifies sufficient conditions for the outcome of the national election to determine the level of effort of the central government to redistribute income (whether tax and transfers are high or low) and for the political structure of sub-national governments to determine whether the redistributive policy of the central government is effective or not.

3. Politically driven redistribution and the reaction functions of sub-national governments

In this section we study the role of political competition in determining the design of the central government of a redistributive program for an economy with strategic interaction among multiple governments. Our economy is a federation constituted by a central government and two sub-national governments (associated with localities 1 and 2). The central and sub-national governments have different tasks mandated by the constitution of the country. We take these constitutional mandates as given. Local governments provide local public goods (such as local security, education, bridges, parks, trash recollection, etc.) and the central government redistributes income.6

3.1. Preferences and constraints of residents

The budget constraint and preferences of a resident of locality or local public spending are given by:

\[ v^i (\tau, T, t^i, g^i, n^i) \] (1)

subject to

\[ g^i = t^i \int_{n_i} h^i (n^i) x^{*i} (\tau, T, t^i, n^i) \, dn^i \quad \forall i \] (2)

6 This structure of the responsibilities of the central and sub-national governments has empirical support in many developed and developing countries (see Ter-Minassian, 1997).
Where the indirect utility is
\[ v^i (\tau, T, t^i, g^i, n^i) = \max \{ \mu^i = \ln x^i + \ln (1 - \ell^i) + g^i \} \]
subject to
\[ q^i x^i = n^i \ell^i (1 - \tau) + T \]
and it characterizes the preferences for feasible local public goods of a resident type \( n^i \) of locality \( i \). Direct preferences on private consumption, \( x^i \), leisure \( (1 - \ell^i) \), and the local public good \( g^i \) are defined by \( \mu^i = \ln x^i + \ln (1 - \ell^i) + g^i \).

The individual’s budget constraint is \( q^i x^i = n^i \ell^i (1 - \tau) + T \) where \( q^i = 1 + t^i \) is the consumer’s price of the private good (we have normalized the producer’s price to one) and \( t^i \) is a tax on private consumption imposed by the local government of locality \( i \) on its residents.

The individual’s after-tax labor income is \( n^i \ell^i (1 - \tau) \) where \( \ell^i \) is the supply of labor, the parameter \( n^i \) is exogenous and represents the ability of the individual living in district \( i \) to earn labor income (we assume markets are competitive, so \( n^i \) is a competitive wage for labor services), \( \tau \) is the tax on labor income and \( T \) is a per capita transfer imposed by the central government to a resident of locality \( i \). The parameters \( \tau \) and \( T \) represent a linear redistributive program of the central government. The distribution of labor skills in each locality is determined by the density
\[ h^i (n^i) > 0 : n^i \in [n^i_{\min}, n^i_{\max}] : n^i_{\min} > \left\{ \frac{T^*}{(1 - \tau^*)} \right\} \forall i \]
such that the cumulative density in locality \( i \) is given by
\[ H^i (n^i) = \int_{n^i_{\min}}^{n^i} h^i (n^i) \, dn^i = N^i / N^T \]
where \( N^i \) is the population in locality \( i = \{1, 2\} \) and \( N^T = N^1 + N^2 \).

The budget constraint of the local government in locality \( i \) is characterized by condition (2). Local public spending is financed by a commodity tax rate \( t^i \) on purchases by local residents of the private good. The tax revenue of the local government in locality \( i \) is given by \( R (t^i) = t^i \int_{n^i_{\min}}^{n^i} h^i (n^i) x^i (\tau, T, t^i, n^i) \, dn^i \).

Moreover, in A1 \( \left( t^i, \tau, T, n^i \right) \in \arg\max \left\{ \mu^i (x^i, (1 - \ell^i), g^i) \right\} \text{ subject to } q^i x^i = n^i \ell^i (1 - \tau) + T \).
we assume heterogeneity of wages between residents of localities 1 and 2, that is, without loss of generality, we consider that the average earning ability of residents of locality 1 is higher than that of residents of locality 2.

\[ \int_{n_1} h^1(n^1) n^1 dn^1 > \int_{n_2} h^2(n^2) n^2 dn^2 \quad (A1) \]

3.2. The politico-economic equilibrium

In this economy, fiscal policy is conducted by governments ruled by elected public officials. The basic social choice problem in this economy is that individuals have different wages, which leads to a set of heterogeneous ideal policies of voters for fiscal policy of the central and sub-national governments. The political institution that solves this social choice problem is a sequential set of elections (one national and two local) in which candidates of political parties propose the size of the government’s spending and voters elect a public official to conduct fiscal policy.

We assume that parties have different preferences over the size of government’s spending. Wittman (1973, 1983) argues that parties might be controlled by some coalition of voters. Since voters have preferences over economic policies, parties want to design and implement the policy that maximizes the preferences of the coalition of voters-activists controlling the party.\(^8\)

We consider a dynamic game of perfect information with sequential elections determined as follows: a national election takes place followed by a simultaneous set of local elections. The party winning the election by simple majority in the respective election designs and implements the party’s platform on public spending. In this economy, the central government spends on a program of monetary transfers financed by a tax on labor income while local governments provide local public goods (education, bridges, health care, etc.) financed by commodity taxation. For this economy the politico-economic equilibrium is characterized by the sub-game perfect Nash equilibrium

\(^8\) For some applications of this view of the political process to the analysis of public finance see Roemer (1997, 2001), Ponce (2010), and Kochi and Ponce (2013).
shown in definition 1. For this characterization consider a cumulative distribution function

\[ \Omega : \left\{ \chi^i (n^i) \right\}_{\forall n^i} \rightarrow [0, 1] \]

where \( \Omega \) is a non-decreasing function of the sequence

\[ \left\{ \chi^i (n^i) = v^{L_i} (\tau^* L, T^* L, t^i, g^i, n^i) - v^{R_i} (\tau^* R, T^* R, t^i, g^i, n^i) \right\}_{\forall n^i} \]

where \( \chi^i (n^i) \) reflects a rational choice of the vote for individual type \( n^i \) in the national election and

\[ v^{L_i} (\tau^* L, T^* L, t^i, g^i, n^i) \]

is the welfare of individual type \( n^i \) if party \( L \) wins the national election and implements a tax on labor \( \tau^L \) and a per capita transfer \( T^L \) for residents of all localities. A similar interpretation is given to

\[ v^{R_i} (\tau^* R, T^* R, t^i, g^i, n^i) . \]

Hence, if \( \chi^i (n^i) > 0 \) voter type \( n^i \) votes for party \( L \) and if \( \chi^i (n^i) < 0 \) voter type \( n^i \) votes for party \( R \).

\(^9\) Similarly consider the cumulative distribution functions given by

\[ \Omega^i : \left\{ \Theta^i (n^i) \right\}_{\forall n^i} \rightarrow [0, 1], \forall i = 1, 2, \] where \( \Omega^i \) is a non-decreasing function of the sequence

\[ \left\{ \Theta^i (n^i) = v^{L_i} (\tau^* Z, T^* Z, t^* Li, g^* Li, n^i) - v^{R_i} (\tau^* Z, T^* Z, t^* Ri, g^* Ri, n^i) \right\}_{\forall n^i} , \] where \( \Theta^i (n^i) \) reflects a rational choice of the vote for individual type \( n^i \) in the local election of locality \( i \), and

\[ v^{L_i} (\tau^* Z, T^* Z, t^* Li, g^* Li, n^i) \]

is the welfare of individual type \( n^i \) if party \( L \) in locality \( i \) wins the local election and implements policies \( t^* Li, g^* Li \). A similar interpretation is given to

\[ v^{R_i} (\tau^* Z, T^* Z, t^* Ri, g^* Ri, n^i) . \]

\(^9\) If \( \chi^i (n^i) = 0 \) the voter flips a fair coin.
DEFINITION 1. The subgame perfect Nash political equilibrium for an economy with Stackelberg “leadership” from a national election and local elections “as followers” can be characterized as follows: In the first stage “nature” announces the type of coalitions of voters who run parties \( Z = \{L, R\} \) for the national election. \(^{10}\) “Nature’s” move is common knowledge. In the second stage, candidates of parties \( L \) and \( R \) announce tax \( \tau_z \) and transfer \( T_z \) \( \forall Z \) policy-platforms. In the third stage, citizens vote for a party based on the type of spending policies that these parties would implement if they win the election. \(^{11}\) In the fourth stage, the party winning the national election takes control of the government and the policies \( \tau^*_L, T^*_L \) or \( \tau^*_R, T^*_R \) are implemented. After the national election is over, local elections take place with an exact sequence of events as described for the national election. \(^{12}\) Formally the equilibrium is:

1) In the second stage of the national election, parties announce policies that maximize the party’s preferences for redistribution \( \tau^*_Z, T^*_Z \) \( \forall Z = \{L, R\} \):\(^{13,14}\)

\(^{10}\) In this economy there are two parties and nature announces the values of \( n^*_Z = L, R \).

\(^{11}\) In our economy all citizens vote and voting is sincere and sequentially rational.

\(^{12}\) That is, in the first stage of the local election in locality \( i \) “nature” announces the type of coalitions of voters who run parties \( \{Z = L, R\} \) (that is nature announce the level of wages of the relevant coalition in each party) for local elections in localities \( i \) and \( -i \). The nature’s move is common knowledge. In the second stage, candidates of parties \( L \) and \( R \), announce tax and spending policy platforms \( \tau^*_Z, g^*_Z \). In the third stage, citizens vote for a party based on the type of policies that these parties would implement if they win the local election. In the fourth stage, the party winning the election in locality \( i \) takes control of the local government and implements the ideal policy of the party.

\(^{13}\) The condition \( T^*_Z = \tau^*_Z \sum_{n^*_Z} \int h^i(n^*_Z) n^*_Z \tau^*_Z(t^*_Z, n^*_Z, t^*_Z, n^*_Z) dn^i \) is the budget constraint of the central government in which the per capita transfer \( T^*_Z \) is financed by a tax on labor income of residents of all jurisdictions. The tax revenue function, \( TR^*_Z \), is given by \( TR^*_Z = \tau^*_Z \sum_{n^*_Z} \int h^i(n^*_Z) n^*_Z \tau^*_Z(t^*_Z, n^*_Z, t^*_Z, n^*_Z) dn^i \).

Moreover, the constraints \( g^*_Z(n^*_Z) = g^*_Z(t^*_Z, n^*_Z, t^*_Z, n^*_Z) \) are the reaction functions of subnational governments.

\(^{14}\) In our economy there is complete information about the parties’ types. Hence there is no reason for parties to hide their true preferences over feasible local public spending. This means that parties have no incentives to announce the median voter policy in each locality in the second stage while implementing
\[ \tau^Z, T^Z \in \arg\max \Psi^Z = \sum_{\forall i} v^{Zi}(\tau^Z, T^Z, t^i, g^i, n^i) \forall Z, \forall i = 1, 2 \]

subject to

a) \[ T^Z = \tau^Z \sum_{\forall i} \int h^i(n^i) n^i t^{*i}(\tau^Z, T^Z, t^i, g^i, n^i) \, dn^i \]

b) \[ g^{*Zi}(n^{Zi}) = g^{*Zi}(\tau^Z, T^Z, n^{Zi}) \forall Z, \forall i = 1, 2 \]

2) In the third stage of the national election, the voter type \( n^i \) in region \( i \) votes for party \( L \) if

\[
\chi^i(n^i) = v^{Li}(\tau^L, T^L, t^i, g^{*Zi}(n^{Zi}), n^i) - v^{Ri}(\tau^R, T^R, t^i, g^{*Zi}(n^{Zi}), n^i) > 0.
\]

For party \( R \) if \( \chi^i(n^i) < 0 \).

Moreover, \( \Omega \) is a non-decreasing cumulative distribution of the sequence \( \{\chi^i(n^i)\}_{n^i} \). In the fourth stage, if there exists a majority of voters \( n^i \in [n^i_{\text{min}}, n^i_{\text{max}}] : \chi^i(n^i) > 0 \) then the following condition is satisfied

\[ \Omega(\forall n^i, \forall i \in [n^i_{\text{min}}, n^i_{\text{max}}] : \chi^i(n^i) > 0) > 1/2 \]

In this case, party \( L \) wins the national election in the fourth stage and implements \( \tau^L, T^L \). In contrast, if

\[ \Omega(\forall n^i, \forall i \in [n^i_{\text{min}}, n^i_{\text{max}}] : \chi^i(n^i) < 0) > 1/2 \]

Then party \( R \) wins the national election and implements \( \tau^R, T^R \).

the parties’ ideal size of public spending in the fourth stage (this issue is better known as the dynamic inconsistency problem).

\( \) When we consider convenient, and to save space, we denote \( g^{*Zi}(\tau^Z, T^Z, n^{Zi}) \) as follows \( g^{*Zi}(n^{Zi}) \).

\( ) For simplicity of the analysis we assume that in the event

\[ \Omega(\forall n^i \in [n^i_{\text{min}}, n^i_{\text{max}}] : \chi^i(n^i) < 0) = 1/2 \]

then “nature” flips a coin and the party winning the bet takes control of the government.
Local elections in districts $i$ and $-i$

1) In the second stage of the election in locality $i$, parties announce policies that maximize the party’s preferences for local public spending $t^*Z_i, g^*Z_i$ if $Z = \{L, R\}, \forall i$:

$$t^*Z_i, g^*Z_i(n^Z_i) \in \argmax v^Z_i(t^*Z, T^*Z, t^i, g^i, n^i) \forall Z, \forall i$$

subject to

$$g^*Z_i = t^*Z_i \int \forall n^i h^i(n^i) x^{*i}(t^*Z, T^*Z, t^i, g^i, n^i) dn^i$$

2) In the third stage of the local election, the voter type $n^i$ in locality $i$ votes for party $L$ if

$$\Theta^i(n^i) = v^L_i(t^*Z, T^*Z, t^L_i, g^L_i, n^i)$$

$$-v^R_i(t^*Z, T^*Z, t^R_i, g^R_i, n^i) > 0$$

and for party $R$ if $\Theta^i(n^i) < 0$. 3) Moreover, $\Omega^i$ is a non-decreasing cumulative distribution of the sequence $\{\Theta^i(n^i)\}_{n^i}$. Therefore, in the fourth stage, if there exists a majority of voters in the locality $i$, $n^i \in [n^i_{\min}, n^i_{\max}] : \chi^i(n^i) > 0$ then the following condition is satisfied

$$\Omega^i\left(\forall n^i \in [n^i_{\min}, n^i_{\max}] : \Theta^i(n^i) > 0\right) > 1/2$$

In this case party $L$ wins the local election in locality $i$ in the fourth stage and implements $t^*L_i, g^*L_i$. In contrast, if

$$\Omega^i\left(\forall n^i \in [n^i_{\min}, n^i_{\max}] : \Theta^i(n^i) < 0\right) > 1/2$$

Then party $R$ wins the local election in locality $i$ and implements $t^*R_i, g^*R_i$. 
In the following section, we characterize the optimal labor supply and consumption of private goods for individuals in this economy.

**PROPOSITION 1.** The optimal consumption of the private good and supply of labor for individual type $n^i$ living in locality $i$ are given by\(^{17}\):

$$\ell^* (\tau, T, t^i, n^i) = \frac{1}{2} - \frac{T}{2n^i (1 - \tau)}$$  \hspace{1cm} (3)

and

$$x^* (\tau, T, t^i, n^i) = \frac{n^i (1 - \tau)}{2q^i} + \frac{T}{2q^i}$$  \hspace{1cm} (4)

**PROPOSITION 2.** Public spending of local governments in localities $i$ and $-i$ are given by:

$$g^*Z_i (\tau^Z, T^Z, n^{Z_i}) = \frac{1}{2} \left\{ \int_{\forall n^i} h^i (n^i) n^i dn^i - \frac{n^{Z_i}}{MRS_{y^I, -\alpha^Z_i}} \right\}$$  \hspace{1cm} (5)

$$+ \frac{1}{2} \left\{ T^Z H^i (n^i) - \tau^Z \int_{\forall n^i} h^i (n^i) n^i dn^i - \left\{ \frac{T^Z - \tau^Z n^{Z_i}}{MRS_{y^I, -\alpha^Z_i}} \right\} \right\} \forall Z, \forall i$$

Where

$$\int_{\forall n^i} h^i (n^i) n^i dn^i$$

is the average wage of residents of locality $i$, $n^{Z_i}$ and $MRS_{y^I, -\alpha^Z_i}$ are, correspondingly, the wage and the marginal rate of substitution between the local public good and income of the coalition of voters controlling party $Z$ in locality $i$, $\alpha^{Z_i}$ is the marginal utility of income of the coalition controlling party $Z$ in locality $i$, and

$$H^i (n^i) = \int_{\forall n^i} h^i (n^i) dn^i = N^i / N^T$$

is the density of the population of locality $i$.

\(^{17}\) We omit the proof to save space. The result is easily found by maximizing the individual’s preferences subject to the budget constraint.
PROOF. The problem for party $Z = \{LorR\}$ in locality $i$ is to choose $g^*Z_i$ to

$$\max v^Z_i(t^i, g^i, n^Z_i) \text{ s.t. } g^i = t^i \int h^i(n^i) x^{*i}(\tau, T, t^i, n^i)\, dn^i$$

Define $\gamma^Z_i$ as follows (where $\lambda^Z_i$ is a Lagrange multiplier):

$$\gamma^Z_i = v^Z_i(t^Z_i, g^i, n^Z_i) + \lambda^Z_i$$

$$\left\{ g^Z_i - t^Z_i \int h^i(n^i) x^{*i}(\tau, T, t^i, n^i)\, dn^i \right\}$$

Find the first order conditions of the party’s problem and re-arrange terms to obtain

$$g^*Z_i(\tau^Z, T^Z, n^Z_i) = \frac{1}{2} \left\{ \int h^i(n^i) n^i d n^i - \frac{n^{Z_i}}{MRS_{g^Z_i - \alpha Z_i}} \right\} \quad (8)$$

$$\frac{1}{2} \left\{ TH^i(n^i) - \tau \int h^i(n^i) n^i d n^i - \frac{T - \tau n^{Z_i}}{MRS_{g^Z_i - Z_i}} \right\} \forall Z, \forall i$$

Condition (5) says that the ideal size of the local public good for party $Z$ in district $i$ depends positively on the difference between the average labor income in the locality and a normalized income of the coalition of voters controlling party $Z$, that is

$$\left\{ \int h^i(n^i) n^i d n^i - \frac{n^{Z_i}}{MRS_{g^Z_i - \alpha Z_i}} \right\}.$$  

Condition (5) also says that the size of the local public good in district $i$ also depends on whether the locality is a net winner or loser of the redistributive program of the central government, that is whether the term

$$\left\{ TH^i(n^i) - \tau \int h^i(n^i) n^i d n^i \right\}.$$
is positive or negative, where \( TH^i (n^i) \) represents the aggregate transfers from the redistributive program to residents of locality \( i \) while

\[
\tau \int_{n^i} h^i (n^i) n^i dn^i
\]

is the aggregate tax payment of residents of the district to the central government. Finally, \( g^* Z_i (\tau Z, T Z, n^Z i) \) also depends on whether the coalition that controls party \( Z \) in district \( i \) is a net winner or loser from the redistributive program of the central government. To be more specific, if the coalition of voters that control the party has a net gain from the redistributive program of the central government then \( \{ T - \tau n^Z i \} > 0 \).

In addition, a simple comparative analysis also suggests that for a large economy (which means that \( h^i (n^i) \geq 0 \forall n^i \in [n^i_{\text{min}}, n^i_{\text{max}}] \) but \( h^i (n^i) \) is small), the ideal size of local public spending is a non-increasing function of the voter’s earning ability, that is for all

\[
\bar{n}^i, \hat{n}^i \in [n^i_{\text{min}}, n^i_{\text{max}}] : \bar{n}^i \geq \hat{n}^i
\]

then \( g^* (\bar{n}^i) \leq g^* (\hat{n}^i) \) where \( g^* (\bar{n}^i) \) and \( g^* (\hat{n}^i) \) are the ideal size of local public spending of voters with earning abilities \( \bar{n}^i \) and \( \hat{n}^i \).\(^{18}\)

4. The structure of political power and the effort to redistribute income

The political equilibrium for this economy can produce multiple fiscal outcomes.\(^{19}\) However, in this section we focus on a type of equilibrium we call non-cooperative welfare superior redistribution. In this equilibrium, as we have mentioned before, public redistribution not only changes the distribution of welfare in the society by increasing (reducing) the full income of poor (rich) families but this policy also

\(^{18}\) In proposition 5 we provide a formal proof of this result.

\(^{19}\) The interaction between the central and subnational governments might produce multiple fiscal outcomes that include high (moderate, low) wage income taxes and high (moderate, low) transfers from the central government. Moreover, at the local level we could find equilibria with high (moderate, low) commodity tax rates and local government spending.
affects the spending of sub-national governments. If local governments provide inferior local public goods and the provision of these goods increase as a result of the redistributive policy of the central government, then the welfare of poor families is increased above the direct effect of the redistributive program. In this case, government action to redistribute welfare in favor of key voters-activists by redistributing income is highly effective.

The purpose of this section is to identify sufficient conditions to obtain the fiscal outcome described above. On what follows, proposition 3 characterizes the equilibrium level of the redistributive policy of the central government

\[ \tau^*_{\mathcal{Z}}, T^*_{\mathcal{Z}} \mid \mathcal{Z} = \{L, R\}. \]

Proposition 4 identifies how election outcomes in a federation determine the level of effort that parties must spend to redistribute income. Proposition 5 identifies sufficient conditions for an allocation of resources with non-cooperative welfare superior redistribution.

**Proposition 3.** In this economy the government’s per capita transfer \( T^*_{\mathcal{Z}} \mid \mathcal{Z} \) is given by

\[
T^*_{\mathcal{Z}} = \frac{(1 - \tau^*_{\mathcal{Z}})\tau^*_{\mathcal{Z}}}{2} \sum_{\forall i} \int_{\forall n_i} h^i(n^i) n^i dn^i
\]

Where

\[
\sum_{\forall i} \int_{\forall n_i} h^i(n^i) n^i dn^i
\]

is the average wage income in the economy, and

\[
\tau^*_{\mathcal{Z}} = \left\{ \frac{1}{-\varepsilon_{\ell^*_{\mathcal{Z}}} - \tau^*_{\mathcal{Z}}} \right\} \sum_{\forall i} \int_{\forall n_i} h^i(n^i) n^i \ell^{*i} dn^i
\]

\[
-\left\{ \frac{1 - \partial T^{R^Z}/\partial T^{Z}}{-\varepsilon_{\ell^*_{\mathcal{Z}}} - \tau^*_{\mathcal{Z}}} \right\} \left\{ \frac{\sum_{\forall i} \alpha_{\mathcal{Z}i} n^i Z^i \ell^{*i} Z^i - \sum_{\forall i} \mu_{\mathcal{Z}i} \partial g^Z_{\mathcal{Z}i}}{\sum_{\forall i} \alpha_{\mathcal{Z}i} Z^i + \sum_{\forall i} \mu_{\mathcal{Z}i} \partial g^Z_{\mathcal{Z}i}} \right\}
\]
Where \( \mu^Z_i = \partial \mu^{Z_i} / \partial g^{Z_i} \), and the aggregate elasticity of labor supply and the federal tax, \( \varepsilon_{\ell^* - \tau^*} \), is given by\(^{20}\)

\[
-\varepsilon_{\ell^* - \tau^*} = \sum_{\forall i} \int h^i (n^i) n^i \ell^* \frac{\partial \ell^*}{\partial \tau^*} \frac{1}{\ell^*} dn^i > 0 \tag{11}
\]

The term \( \partial TR^Z / \partial T^Z \) is the marginal tax revenue when the government returns $1 to households through transfers, and

\[
TR^Z = \tau^* \sum_{\forall i} \int h^i (n^i) n^i \ell^* (\tau^Z, T^Z, t^i, n^i) dn^i
\]

is the tax revenue from labor income,

\[
\sum_{\forall i} \alpha^Z_i n_i \ell^* Z_i \]

is a weighted average labor income of the coalition controlling the national government, and the response of local governments \( \forall i \) on local public spending to national tax and transfer policies are given by\(^{21,22}\)

\[
\frac{\partial g^{Z_i}}{\partial \tau^* Z_i} = - \int h^i (n^i) n^i dn^i + \frac{n^Z_i}{MRS_{L-Z_i - Z_i}} > 0 \quad \forall Z_i, \forall i \tag{12}
\]

\[
\frac{\partial g^{Z_i}}{\partial T^* Z_i} = - H^i (n^i) - \frac{1}{MRS_{L-g^{Z_i} - Z_i}} > 0 \quad \forall Z_i, \forall i \tag{13}
\]

\(^{20}\) The aggregate elasticity of labor supply and federal taxes, \( \varepsilon_{\ell^* - \tau^*} \) is evaluated at the point where \( \tau^* = 0 \).

\(^{21}\) We should distinguish between the political representatives of the national government and the political representatives of local governments. Thus, the notation \( n^Z_i \) and \( MRS_{L-g^{Z_i} - Z_i} \) is for local politicians while \( n^{Z_i} \) refers to national politicians.

\(^{22}\) Conditions (12) and (13) follow from condition (5) in proposition 2.
PROOF. The problem of policy design for party \( Z = \{L, R\} \) controlling the national government is defined as follows:

\[
\delta^Z (\tau^Z, T^Z, \gamma^Z) = \sum_{\forall i} v^{Zi} (\tau^Z, T^Z, t^i, g^{*Zi} (\tau^Z, T^Z, n^{Zi}), n^{Zi}) \tag{14}
\]

\[
+ \ Y^Z \left\{ T^{*Z} - \tau^{*Z} \sum_{\forall i} \int_{\forall n^i} h^i (n^i) n^i x^{*i} (t^i, \tau, T, n^i) dn^i \right\}
\]

Where \( Y^Z \) is a Lagrange multiplier and \( g^{*Zi} (\tau^Z, T^Z, n^{Zi}) \forall Z, \forall i \) is given by condition 5 and satisfies the following:

\[
g^{*Zi} (\tau^Z, T^Z, n^{Zi}) \in \arg \max v^{Zi} (t^i, g^i, n^{Zi}) \tag{15}
\]

\[
s.t \ g^{*Zi} = t^{*Zi} \int_{\forall n^i} h^i (n^i) x^{*i} (t^{Zi}, \tau, T, n^i) dn^i
\]

Find the first order conditions and re-arrange terms to obtain condition (10). Condition (9) follows from (10) and from the budget constraint of the government. Conditions (12) and (13) follow from the equilibrium condition (5) in proposition 2.

\[\Box\]

PROPOSITION 4. THE STRUCTURE OF POLITICAL POWER AND THE EFFORT TO REDISTRIBUTE INCOME. At the politico-economic equilibrium of a large economy, \( T^{*Z} \) is strictly positive if a party that represents a coalition of voters with a wage \( n^{Zi} \) below the nationwide average wage rules the central government. Moreover, \( T^{*Z} \) also depends on the distribution of political power at sub-national governments: \( T^{*Z} \) is high (low) if local governments of both localities are ruled by parties representing voters with wages \( n^{Zi}_L \) above (below) the nationwide average wage.\(^\text{23}\)

\(^{23}\) This equilibrium is achieved when the median voters of districts \( i \) and \(-i\) vote for party \( R \) because the ideal policy of a majority of voters is closer to the policy proposed by party \( R \) than the policy proposed by party \( L \) in these local elections. Moreover, a majority of voters in the national election votes for party \( L \) because the ideal policy of the nationwide median voter is closer to the policy proposed by party \( L \) than to the policy proposed by party \( R \).
PROOF. To prove this proposition it is sufficient to show that increases of \( n^{Z_i} \) reduce \( T^{*Z} \) and increases of \( n^{L_i} \) increase \( T^{*Z} \). The first part of the proposition follows from the fact that

\[
\frac{\partial T^{*Z}}{\partial n^{Z_i}} \equiv (1 - 2\tau^{*Z}) \frac{\partial \tau^{*Z}}{\partial n^{Z_i}} \sum_{i} \int h^i (n^i) n^i dn^i
\]

(17)

For a large economy \( h^{Zi} (n^{Zi}) > 0 \), but small implies \( h^{Zi} (n^{Zi}) \equiv 0 \) thus

\[
\frac{\partial T^{*Z}}{\partial n^{Z_i}} \equiv (1 - 2\tau^{*Z}) \frac{\partial \tau^{*Z}}{\partial n^{Z_i}} \sum_{i} \int h^i (n^i) n^i dn^i < 0
\]

since

\[
\sum_{i} \int h^i (n^i) n^i dn^i > 0, \quad (1 - 2\tau^{*Z}) > 0 \quad \text{and} \quad \partial \tau^{*Z} / \partial n^{Z_i} < 0.
\]

Note that \( \partial \tau^{*Z} / \partial n^{Z_i} < 0 \) because \( \partial T^{*Z} / \partial n^{Z_i} > 0 \) and \( h^{Zi} (n^{Zi}) \equiv 0 \) imply

\[
\partial \tau^{*Z} / \partial n^{Z_i} \equiv
\]

\[
- \left\{ \frac{1 - \partial T R^Z / \partial T^Z}{-\varepsilon^{\tau^{*Z}}} \right\} \left\{ \frac{\alpha^{Zi} \partial^{\ell^i} + \alpha^{Zi} n^{Zi} \{ \partial \ell^i / \partial n^{Z_i} \}}{\sum_{i} \alpha^{Zi} - \sum_{i} \{ \partial g^{Zi} / \partial T^{*Z} \}} \right\} < 0
\]

Moreover, \( (1 - 2\tau^{*Z}) > 0 \) since \( \partial T^{*Z} / \partial \tau^{*Z} > 0 \). To see this, note that at the equilibrium it must be satisfied that

\[
\partial T^{*Z} / \partial \tau^{*Z} > 0 \Rightarrow (1 - 2\tau^{*Z}) > 0,
\]

otherwise

\[
\exists \tau^Z \leq \tau^{*Z} \land T^Z \geq T^{*Z} \land \Upsilon^Z \neq \Upsilon^{*Z}:
\]

\[
\delta^Z (\tau^Z, T^Z, \Upsilon^Z) \geq \delta^{*Z} (\tau^{*Z}, T^{*Z}, \Upsilon^{*Z})
\]
implying
\[ \tau^* Z, T^* Z, Y^* Z \notin \text{argmax } \delta^Z (\tau^Z, T^Z, Y^Z). \]

Therefore, since
\[ \tau^* Z, T^* Z, Y^* Z \in \text{argmax } \delta^Z (\tau^Z, T^Z, Y^Z) \]
then
\[ \partial T^* Z / \partial \tau^* Z > 0 \land (1 - 2\tau^* Z) > 0 \ \forall Z = \{L, R\}. \]

For the second statement in the proposition, we recognize that
\[ \partial T^* Z / \partial \tau^* Z > 0, \]

hence
\[ \frac{\partial T^* Z}{\partial n^Z_i} = (1 - 2\tau^* Z) \frac{\partial \tau^* Z}{\partial n^Z_i} \]
\[ \sum_{\forall i} \int_{\forall n^Z_i} h^i (n^i) n^i d n^i + \tau^* Z h^Z_i (n^Z_i) n^Z_i > 0 \]
To see this, recall
\[ \sum_{\forall i} \int_{\forall n^Z_i} h^i (n^i) n^i d n^i > 0 \]

Hence,
\[ (1 - 2\tau^* Z) > 0 \land \partial \tau^* Z / \partial n^Z_i > 0 \land \sum_{\forall i} \int_{\forall n^Z_i} h^i (n^i) n^i d n^i > 0 \]
\[ \Rightarrow \partial T^* Z / \partial n^Z_i > 0 \forall Z, \forall i. \]

PROPOSITION 5. NON-COOPERATIVE REDISTRIBUTION WITH WELFARE SUPERIOR GAINS. Consider an economy with:

v.1) Inferior local public goods
v.2) A central government ruled by a party that represents a coalition of voters with a wage below the nationwide average wage.

v.3) Local governments ruled by parties that represent a coalition of voters with a wage \( n^Z_i \) and \( \text{MRS}_{g^Z_i - \alpha^Z_i} > 1 \) (their willingness to pay for local public spending is high enough) such that:

\[
n^Z_L > \text{MRS}_{g^Z_i - \alpha^Z_i} \left\{ \int_\forall n^i \ h^i (n^i) n^i \, dn^i \right\}
\]

v.4) Localities have sufficiently high densities of population such that:

\[
\sum_{\forall i} H^i (n^i) = 1 : H^i (n^i) > \frac{1}{\text{MRS}_{g^Z_i - \alpha^Z_i}} \forall i = 1, 2
\]

In this case, the spending policy of sub-national governments complements the redistributive policy of the central government. As a result, the strategic interaction among governments leads to a non-cooperative allocation of resources in which \( T^*Z \) is high and the welfare gains associated with public redistribution for the pivotal voters-activists of the central government are high as well.

**Proof.** To see that condition v.1) is satisfied, note that we can generalize condition (5) in proposition 2 for any voter in every district. Hence, \( \forall n^i \in [n^i_{\text{min}}, n^i_{\text{max}}] \), the ideal size of local public spending for a voter type \( n^i \) is

\[
g^*Z_i (n^i) = \frac{1}{2}
\]

\[
\left\{ E [n^i] - \frac{n^i}{\text{MRS}_{g^Z_i - \alpha^Z_i}} + T^Z H^i (n^i) - \tau^Z E [n^i] - \left\{ \frac{T^Z - \tau^Z n^i}{\text{MRS}_{g^Z_i - \alpha^Z_i}} \right\} \right\}
\]

where \( E [n^i] = \int_\forall n^i \ h^i (n^i) n^i \, dn^i \). It follows that

\[
\partial g^* (n^i) / \partial n^i = \frac{1}{2} \left\{ 1 - \tau^Z \right\} \left\{ h^i (n^i) - \frac{1}{\text{MRS}_{g^Z_i - \alpha^Z_i}} \right\}.
\]
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Note that \( \{1 - \tau^Z\} > 0 \). For a large economy \( h^i(n^i) > 0 \) but small and \( \text{MRS}_{g-\alpha^Z}^i > 0 \) implies

\[
\left\{ h^i(n^i) - \frac{1}{\text{MRS}_{g-\alpha^Z}^i} \right\} < 0 \text{ hence } \frac{\partial g^* (n^i)}{\partial n^i} < 0
\]

Therefore, the ideal size of local public spending is a decreasing function of the voter’s earning ability.

By proposition 4, \( \partial T^*Z / \partial n^Zi < 0 \) (this is condition v.2). In addition, recall \( \partial T^*Z / \partial \tau^Z > 0 \) and

\[
\tau^*Z = \tau^Z\left( \frac{\partial g^*Zi}{\partial \tau^Z} , \frac{\partial g^*Zi}{\partial T^*Z} , \ldots \right) \cdot \frac{\partial T^*Z}{\partial \tau^*Z} > 0
\]

(see condition 10 in proposition 3). Thus, condition v.3

\[
n^Zi_L > \text{MRS}_{-g^Zi-\alpha^Z} \left\{ \int_{h^i(n^i)} n^i dn^i \right\},
\]

implies that \( \partial g^*Zi / \partial \tau^*Z > 0 \) (see condition 12) and

\[
\partial T^*Z / \partial \left( \frac{\partial g^*Zi}{\partial \tau^*Z} \right) > 0.
\]

Moreover, condition v.4

\[
H^i(n^i) > \frac{1}{\text{MRS}_{-g^Zi-\alpha^Z}}
\]

implies \( \partial g^*Zi / \partial T^*Z > 0 \), and

\[
\partial T^*Z / \partial \left( \frac{\partial g^*Zi}{\partial T^*Z} \right) > 0.
\]

Thus, conditions v.1 to v.4 mean that the effort to redistribute income (i.e. the size of \( T^*Z \)) is high at the equilibrium which means
that politicians can effectively redistribute welfare in favor of politically influential coalitions by redistributing income.

Propositions 4 and 5 say that election outcomes (that is the structure of the identity of the party controlling public office at each level of government) can explain not only the size of public redistribution and sub-national public spending but also whether the policy of the central government is effective or not in redistributing welfare by redistributing income in a federation.\(^\text{24}\)

To see this, we first need to recognize that the linear redistributive program of the central government entails a net positive (negative) transfer to any individual with a wage that is lower (higher) than the nationwide average wage. This explains why parties in control of the central government representing a coalition of voters with a wage lower than the average wage choose a positive level of \(T^*\) (see proposition 4 and condition \(v.2\) in proposition 5).

Moreover, parties seeking to win the national election to form the central government recognize that the gains from the net fiscal exchange associated with public redistribution depend on sub-national election outcomes because the structure of inter-regional political power explains the reaction of local governments to the redistributive policy of the central government.

Public redistribution affects local spending through the following channels: first, in our economy sub-national governments provide an inferior local public good, hence the demand for sub-national public spending increases with a fall in the after tax and transfer income (or full net income). If local governments are ruled by a coalition of voters with a wage higher than the nationwide average wage then the linear redistributive program of the central government induces a fall in full income and the demand for public spending of the coalition controlling the local government increases (this effect is also characterized by conditions \(v.3\) and \(v.4\) in proposition 5).

Second, by redistributing income, the transfer policy of the central government affects the demand of households for private goods and the ability of local governments to raise both tax revenue and the provision of public goods (this is also characterized by conditions \(v.3\)

\(^{24}\) Other factors that explain the effort to redistribute income are (see conditions 9 and 10 in proposition 3): the ability of the central government to raise tax revenue from labor income and the negative effects of tax and the public transfer on the supply of labor of households.
and \(v.4\) in proposition 5). Whether the locality \(i\) is a net winner or loser on tax revenues as a result of the redistributive program of the central government depends on the difference between the aggregate transfers from the redistributive program to residents of locality \(i\) and the aggregate tax payments of residents of the district to the central government. This in turn, depends on the density of the population and the original distribution of income (see condition 5 in proposition 2).

The lack of quid pro quo of the fiscal exchange from public redistribution requires that the effects of taxation and transfers be analyzed separately. An increase in taxes on wage income tends to reduce local spending by reducing the available resources in localities for the supply of local public goods and simultaneously tends to increase the supply of local public goods since taxes reduce full income and the demand for public spending of the coalition of voters controlling the local government increases.

Condition \(v.3\) says that if there is a level of income which is high enough so that the latter effect dominates then \(\partial g^{Zi}/\partial \tau^Z > 0\) which in turn means that the size of public transfers is higher at the political equilibrium because

\[
\partial T^Z/\partial \left( \frac{\partial g^{Zi}}{\partial \tau^Z} \right) > 0.
\]

An increase of public transfers tends to increase the supply of local public goods through a positive local tax revenue effect (the higher the density of the population of the district the higher is the share of resources of the redistributive program that is allocated into the district, this in turn increases the demand of residents for private goods, the district’s tax collection, and the provision of the local public good). An increase of public transfers also reduces local public spending because the demand of the coalition of voters controlling the local government falls. Condition \(v.3\) characterizes a sufficient condition in which the first effect dominates implying that \(\partial g^{Zi}/\partial T^Z > 0\) and, because

\[
\partial T^Z/\partial \left( \frac{\partial g^{Zi}}{\partial T^Z} \right) > 0,
\]

the size of public transfers is higher at the political equilibrium.
5. The political equilibrium versus the Pareto efficient outcome

It is interesting to compare our political equilibrium with a cooperative Pareto efficient outcome. If all governments cooperate to design policies to maximize the wellbeing of their citizens, then these coordinated policies will lead to a Pareto efficient outcome maximizing the sum of utilities of all members involved in the agreement. In our case, this outcome is equivalent to a policy that maximizes the sum of utilities of all individuals in the economy. Therefore, the efficient outcome can be characterized by solving an optimization problem with a benevolent social planner in which the central government provides local public goods in all districts, and determines the optimal size of public redistribution, as well as the different forms of taxation.

In this case, the government’s problem of policy design for redistribution ($T$), local public goods, ($g^i, g^{−i}$), labor income taxation ($\tau$), and commodity taxation ($t^i$) is characterized as follows:

\[
\begin{align*}
\text{Max } \Xi &= \sum_{\forall i} \int h^i (n^i) v^i (\tau, T, t^i, g^i, n^i) \, dn^i \\
\text{subject to } g^i + g^{−i} + T &= \tau \sum_{\forall i} \int h^i (n^i) n^i \ell^i (\tau, T, t^i, n^i) \, dn^i \\
&+ \sum_{\forall i} t^i \int h^i (n^i) x^i (\tau, T, t^i, n^i) \, dn^i
\end{align*}
\]

Where $\Xi$ is a nationwide social welfare function and condition (23) is the government’s budget constraint where total tax revenue depends on nationwide collections of public revenue from labor and commodity taxation.

**Proposition 6.** Optimal tax and spending policies are given by

\[ T^{**}, \tau^{**}, g^{**i}, t^{**i|\forall i} ; \]

For $T^{**}$:

\[
\sum_{\forall i} \int h^i (n^i) \frac{\partial v^i}{\partial T} dn^i = \lambda_\Xi - \lambda_\Xi \left\{ \xi_{T} + \sum_{\forall i} \xi_{x^i-T} \right\}
\]
Where
\[
\sum \int h^i(n^i) \frac{\partial v^i}{\partial T} dn^i
\]
is the social marginal utility of income, \(\lambda \Xi\) is the social marginal cost of raising tax revenue from taxation, \(\xi_{\ell-T}\) is an aggregate weighted elasticity of the supply of labor with respect income transfers from the government
\[
\xi_{\ell-T} = \sum \int h^i(n^i) \frac{\partial \ell^{*i} T^{**} n^i \ell^{*i}}{\partial T} \frac{T^{**}}{\ell^{*i}} dn^i
\]
and
\[
\xi_{x-T} = \sum \int h^i(n^i) \frac{\partial x^{*i} T^{**} n^i x^{*i}}{\partial T} \frac{T^{**}}{x^{*i}} dn^i
\]
is an aggregate weighted elasticity of private consumption with respect income transfers from the government. For \(\tau^{**}\):
\[
\tau^{**} = \left\{ \frac{1}{-\xi_{\ell-T}} \right\} \left\{ \sum \int h^i(n^i) n^i \ell^{*i} dn^i + \sum \xi_{x-T} \right\} - \left\{ \frac{1}{-\xi_{\ell-T} \lambda \Xi} \right\} \left\{ \sum \int h^i(n^i) \alpha^i \ell^{*i} dn^i \right\}. \quad (25)
\]
Where
\[
\xi_{\ell-T} = \sum \int h^i(n^i) \frac{\partial \ell^{*i} \tau^{**} n^i \ell^{*i}}{\partial \tau} \frac{\tau^{**}}{\ell^{*i}} dn^i
\]
is an aggregate weighted elasticity of the supply of labor with respect labor income taxes and
\[
\xi_{x-T} = \sum \int h^i(n^i) \frac{\partial x^{*i} \tau^{**} n^i x^{*i}}{\partial \tau} \frac{\tau^{**}}{x^{*i}} dn^i
\]
is an aggregate weighted elasticity of private consumption with respect labor income taxes.
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For $y^{**i}$ for $i$

\[ \int_{\forall n_i} h_i(n_i) \frac{\partial v_i}{\partial g_i} dn_i = \lambda \Xi \quad (26) \]

Where

\[ \int_{\forall n_i} h_i(n_i) \frac{\partial v_i}{\partial g_i} dn_i \]

is the social marginal benefit of local public good in district $i$.

For $t^{**i}$ for $i$

\[ t^{**i} = \left\{ \frac{1}{1 - \xi x_i - t_i} \right\} \]

\[ \left\{ \sum_{\forall i} \int_{\forall n_i} h_i(n_i) x_i^{**i} dn_i - \frac{1}{\lambda \Xi} \sum_{\forall i} \int_{\forall n_i} h_i(n_i) \alpha^i x_i^{**i} dn_i \right\}. \]

Where

\[ \xi x_i - t_i = \sum_{\forall i} \int_{\forall n_i} h_i(n_i) \frac{\partial x_i^{**i}}{\partial t_i} \frac{1}{x_i^{**i}} dn_i \]

is an aggregate weighted elasticity of private consumption with respect commodity tax $t_i$.

PROOF. State the problem of policy design by considering the Lagrangian $\delta \Xi$ as follows:

\[ \delta \Xi = \sum_{\forall i} \int_{\forall n_i} h_i(n_i) v_i(\tau, T, t_i, g_i, n_i) dn_i + \]

\[ \lambda \Xi \left\{ \sum_{\forall i} \int_{\forall n_i} h_i(n_i) n_i^{t^{**i}}(\tau, T, t_i, n_i) dn_i + \right\} - \lambda \Xi \left\{ g^i + g^{-i} + T \right\}. \]
Where $\lambda_\Xi$ is a Lagrange multiplier. The first order conditions are:

$$\frac{\partial \delta_\Xi}{\partial T} = \sum_{\forall i} \int h^i(n^i) \frac{\partial v^i}{\partial T} dn^i - \lambda_\Xi$$  \hspace{1cm} (29)$$

$$\lambda_\Xi \left\{ \tau \sum_{\forall i} \int h^i(n^i) n^i \frac{\partial \ell^*}{\partial T} dn^i + \sum_{\forall i} t^i \int h^i(n^i) \frac{\partial x^i}{\partial T} dn^i \right\}$$

$$= 0 \ \forall \ T^{**} > 0.$$  

Define

$$\xi_{\ell-T} = \sum_{\forall i} \int h^i(n^i) \frac{\partial \ell^*}{\partial T} T^{**} n^i \frac{\tau^{**}}{T^{**}} dn^i$$

and

$$\xi_{x^i-T} = \sum_{\forall i} \int h^i(n^i) \frac{\partial x^i}{\partial T} T^{**} t^{**i} x^i \frac{x^i}{T^{**}} dn^i.$$  

Hence

$$\sum_{\forall i} \int h^i(n^i) \frac{\partial v^i}{\partial T} dn^i = \lambda_\Xi - \lambda_\Xi \left\{ \xi_{\ell-T} + \sum_{\forall i} \xi_{x^i-T} \right\}$$  \hspace{1cm} (30)$$

And for $\forall \ \tau^{**} > 0$

$$\frac{\partial \delta_\Xi}{\partial \tau} = \sum_{\forall i} \int h^i(n^i) \frac{\partial v^i}{\partial \tau} dn^i + \lambda_\Xi \left\{ \sum_{\forall i} t^i \int h^i(n^i) \frac{\partial x^i}{\partial \tau} dn^i \right\}$$  \hspace{1cm} (31)$$

$$+ \lambda_\Xi \left\{ \sum_{\forall i} \int h^i(n^i) n^i \frac{\tau^{**}}{T^{**}} dn^i + \tau^{**} \sum_{\forall i} \int h^i(n^i) n^i \frac{\ell^*}{T^{**}} dn^i \right\} = 0.$$
Define \( \xi_{\ell} - \tau = \sum_{\forall i} \int_{\forall n^i} h^i (n^i) \frac{\partial \ell^{*i}}{\partial \tau} \frac{\tau^{*i} n^i}{\tau^{*i}} d\tau \)

and \( \xi_{x^i} - \tau = \sum_{\forall i} \int_{\forall n^i} h^i (n^i) \frac{\partial x^{*i}}{\partial \tau} \frac{\tau^{*i} x^{*i}}{\tau^{*i}} d\tau \).

Hence

\[
\tau^{*i} = \left\{ \frac{1}{-\xi_{\ell} - \tau} \right\} \left\{ \sum_{\forall i} \int_{\forall n^i} h^i (n^i) n^i d\tau + \sum_{\forall i} \xi_{x^i} - \tau \right\} \]

(32)

Moreover

\[
\frac{\partial \delta_{\Xi}}{\partial g^i} = 0 \iff \int_{\forall n^i} h^i (n^i) \frac{\partial v^i}{\partial g^i} d\tau = \lambda_{\Xi} \quad \forall g^{*i} > 0 \quad \forall i \]

(33)

And

\[
\frac{\partial \delta_{\Xi}}{\partial t^i} = \sum_{\forall i} \int_{\forall n^i} h^i (n^i) \frac{\partial v^i}{\partial t^i} d\tau + \lambda_{\Xi} \left\{ \sum_{\forall i} \int_{\forall n^i} h^i (n^i) x^{*i} d\tau + t^i \sum_{\forall i} \int_{\forall n^i} h^i (n^i) \frac{\partial x^{*i}}{\partial t^i} d\tau \right\} \]

\[
= 0 \quad \forall t^{*i} > 0.
\]

Define \( \xi_{x^i} - t^i = \sum_{\forall i} \int_{\forall n^i} h^i (n^i) \frac{\partial x^{*i}}{\partial t^i} \frac{1}{x^{*i} \frac{1}{x^{*i}}} d\tau \) to show that \( t^{*i} \quad \forall i \).
Proposition 6 says that a central government controlled by a benevolent social planner designs a redistributive policy and provides local public goods where the nationwide social marginal utility of public spending is equal to the social marginal costs of raising public revenue through labor income and commodity taxation. A benevolent social planner also takes into consideration the inefficiency costs of tax and spending policies in the households’ decisions of supply of labor and private consumption.

Our comparative analysis of the Pareto efficient outcome with our political equilibrium of section 4 suggests that the benefits of transfers and local public goods and the costs of commodity and labor income taxes are aggregated differently in each model. In a cooperative Pareto efficient outcome, policy makers have incentives to provide higher transfers than in our political equilibrium because the social marginal benefits of transfers are at least as high as the corresponding political benefits in our model due to the fact that parties are ideological and are interested in maximizing the well-being of only a minority of voters (as opposed to a benevolent government that considers the marginal benefits of transfers for the society as a whole). This effect causes optimal public transfers and provision of local public goods to be higher in the cooperative outcome than the politically optimal transfers and local public goods of our model.

With respect to taxation, in our political equilibrium parties competing in national elections might face lower political costs associated with labor income taxes compared with the social marginal costs faced by a benevolent social planner. This might be the case because, again, parties in national elections only consider the negative impact of taxes on the welfare of a minority coalition of voters while benevolent planners consider the nationwide distribution of welfare costs associated with labor taxes. This effect tends to produce
lower taxes and lower public spending (both transfers and local public goods) in the cooperative outcome compared with public spending at our political equilibrium. Therefore, the net effect of the differences (between the Pareto efficient outcome and our political equilibrium) of the process of aggregation of benefits and costs of spending and taxation on the overall size of government is ambiguous.

Moreover, in a cooperative efficient outcome the central government, first, recognizes the different available tax bases and designs a diversified tax structure that includes labor and commodity taxation (which means that a benevolent government equalizes the social marginal costs of commodity and labor taxation) and, second, it avoids vertical and horizontal tax externalities. In our political model there are no horizontal tax externalities because households have no mobility and there is no vertical tax externality induced by the central government toward sub-national governments since parties in the national election take into account how labor income taxes affect the ability of local governments to raise local commodity tax revenue. However, there is still a vertical tax externality from subnational governments to the central government because local governments do not take into account how commodity taxes affect the ability of the central government to raise tax revenue from labor income. This vertical tax externality tends to make commodity taxes higher in our political equilibrium compared with the optimal commodity taxes in a cooperative Pareto efficient outcome.

Another issue is that, because of the economies of scale associated with broader definitions of labor and commodity taxes for the case of the efficient outcome, it is also reasonable to consider that commodity tax rates could be lower in the efficient outcome compared with our political model.

There are also significant differences between the two models with respect to the provision of local public goods. Subnational governments have incentives to equalize the marginal local political costs of commodity taxation with the narrow definition of the marginal local political benefits of public goods for the minority local coalition that these parties seek to benefit. In contrast, a Pareto efficient policy equalizes the nationwide marginal benefits of local public spending with the nationwide marginal costs of the tax structure of the central government.

If the issue of the aggregation of benefits from public spending dominates the issue of the aggregation of costs of taxation then socially optimal public transfers and local public goods are higher than the transfers and the provision of local public goods of our political
model. Moreover, the vertical tax externality discussed above implies that commodity income taxes are lower in the Pareto efficient outcome than they are in our political equilibrium. In this case, a benevolent central government might be forced to set higher labor income taxes than those characterized in our political equilibrium.

6. Conclusions

The evidence of the last decades shows that politics matter for determining economic outcomes. However, we still have little understanding of how the structure of political power in a federation (that is, how the policy preferences of parties controlling public office at each level of government) affects fiscal outcomes. This paper helps to fill this gap. In a model of partisan politics we show that the elected party at the national election selects the size of public transfers while elected parties at local elections are partially responsible for determining whether income redistribution can be an effective tool for redistributing welfare.

In particular, we show that for a large economy, the size of the public transfers is strictly positive if the federal government is ruled by a party that represents a coalition of voters-activists with a wage below the nationwide average wage and there is a majority of voters with incomes below the nationwide average wage. The size of public transfers also depends on the distribution of political power in the sub-national governments: public transfers are high (low) at the political equilibrium if all sub-national governments are ruled by parties representing voters with wages above (below) the nationwide average wage.

We also find that if local public goods are inferior, the central government is ruled by a party that represents a coalition of voters with a wage below the nationwide average wage, the decisive coalition of voters controlling subnational governments has a sufficiently high willingness to pay for local public spending and their wages are above a certain threshold value, and localities have sufficiently high densities of population, then the welfare of key voter-activists is increased above the direct effect of the redistributive program of the central government. As a result, the strategic interaction among governments leads to a non-cooperative welfare superior allocation of resources in which the per capita transfers from the central government are high and the welfare gains associated with public redistribution for key voter-activists inside the ruling party in the national government are high as well.
References


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