HABIT FORMATION AND OLIGOPOLISTIC COMPETITION

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Resumen: Se analiza la competencia en mercados con formación de hábito. Se modela un juego de dos periodos en donde dos firmas entran al mercado secuencialmente. Se encuentra que el producto de la segunda firma es similar al producto original, pero no exactamente igual. El modelo también se aplica a competencia en mercados diferentes que comparten una característica, por ejemplo, el nivel de azúcar es una característica de las bebidas carbonatadas y de los helados, y el consumo de un producto en un mercado afecta las preferencias por productos en el otro mercado. Se observa que las nuevas firmas producen productos con características similares no sólo al producto que entró al mismo mercado, sino a productos que han entrado en mercados con características en común.

Abstract: In this paper I analyze competition in markets with habit formation. I model a two-period game in which two firms enter a market sequentially. I find that the second firm’s product is similar to the original one, but not exactly the same. The model also applies to competition in different markets that share a characteristic; for example, sweetness is a common characteristic of carbonated soft drinks and ice-cream, and consumption of a product in one market affects the preferences for products in the other market. I find that new firms produce products with similar characteristics not only to a product that has entered the same market, but to products that have entered other markets with common characteristics.

Clasificació n JEL/JEL Classification: D03; D43; L13

Palabras clave/keywords: formación de hábito, diferenciación de producto, competencia oligopolística, habit formation, product differentiation, oligopolistic competition.

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1. Introduction

Psychologists have observed that the repeated exposure to a stimulus increases its attractiveness.\(^1\) This applies to many types of food, beverages, music, etc. However, individuals not only increase their preference for a product; they increase their preference for its characteristics as well.

The goal of this paper is to analyze competition in markets where individuals have habit formation. I work with the idea that habits are formed when individuals grow to prefer the characteristics of those products they have consumed previously. This has some advantages. First, it allows us to understand how habit formation affects firms’ design choices over the characteristics of the products. Second, if the design of several products (even if those products are in different markets) is defined over the same characteristic space, then the consumption of a product may affect the preferences for the other products. By modeling habit formation for the characteristics of the products we can analyze this.

In section 2 I introduce my model. By modifying Hotelling’s linear city we can model habit formation for the characteristics of products. Hotelling (1929) models the competition of two stores that are located in a linear city of length one where a number of individuals live and want to buy a product from these stores. However, consumers pay a transportation cost to travel to each store and prefer to buy from the closest one. We can also interpret this model as describing the location of preferences in a characteristic space (e.g. the level of sweetness) where the consumers’ location represents their favorite level of the characteristic, and the firms’ location is the level of this characteristic in their products. This is the interpretation I use in this article, and sometimes I refer to a level of the characteristics as a location in the characteristic space or just as a location and to the disutility of consuming a product with characteristics other than those of the ideal product as transportation cost.

Because habit formation is a fundamentally dynamic phenomenon, it is necessary to extend the classically static Hotelling model to more periods. I work with the simplest possible extension: a two period model. In the first period, I assume that individuals do not have a preference over the characteristics of the products, and they receive a constant utility from consumption. The basic assumption of my model is that in the second period, individuals’ favorite charac-

\(^1\) For a review of the psychological literature see Bornstein (1989).
teristics are the characteristics of the products they consumed in the first period.

Once an individual has consumed a product and learned to enjoy its characteristics, she will prefer a product with the same characteristics, even if it is a product from a different firm. If different individuals grow to prefer the characteristics of the products they consume, and if they consume the same products, then they will grow to prefer the same characteristics as other individuals. Thus habit formation for the characteristics of the products results in individuals’ favorite characteristics being clustered in the characteristics of the products that are available.

In section 2 I analyze the case of two firms that enter a market where individuals have habit formation. I assume that both firms enter the market sequentially: one firm enters in the first period and the other firm enters in the second period. I assume that individuals differ in their willingness to pay for these characteristics (different transportation costs), and that the utility from consumption, gross of transportation cost, is not large. The first firm attracts individuals preferences toward the characteristics of its product, and the second firm steals the low-transportation cost individuals, who prefer the original product but are not willing to pay much more for it. The entrant produces a similar product to the original one, but not exactly the same (to relax competition) and sells it at a lower price. This result contrasts with standard models of product differentiation that normally predict that competition in the design of the product results in maximum differentiation. My model also results in path dependency as preferences shift toward the design of the first product in a market and the design of new products are close to the characteristics of the original product. I find that the firm that enters in the first period has a first mover advantage, as its profits are higher than those of the second firm.

In section 3 I extend my model to multiple markets. Products in different markets often share several characteristics. For example, sweetness is a characteristic that is shared by products in many markets. If individuals learn to enjoy the characteristics of the products they consume, then the consumption of one product would affect the preferences for other products that have the same characteristics, even in different markets. For instance, the level of sweetness in carbonated soft drinks will affect the preference for the level of sweetness for ice-creams. I analyze the simplest case: two markets that share one characteristic.

I first analyze the case of a firm that enters one of the markets
in the first period and a second firm that enters the other market in the second period. Individuals that consume the product in the first period are going to prefer its characteristics in both markets in the second period. The firm that enters in the second period will produce with the same level of the characteristics that both markets share. In other words, there is path dependency even in different markets and firms will take into consideration what other firms have produced in other markets that share the same characteristics.

I also analyze the case where two firms enter in the first period, one in each market. If both firms can change the characteristics of their products in the second period, then they do not have to produce products with the same characteristics in the first period. However, if they do not produce products with the same characteristics, in the second period they will change their characteristics to more closely match the characteristics of the product of the other firm. If firms cannot change their characteristics in the second period, they will produce a product with the same characteristics as that of the firm in the other market in both periods.

In the case where two firms enter each market sequentially, and if firms cannot change their location in the second period, the first firm to enter each market has to enter in the same location as the other first mover. In the case of habit formation, what gives a first mover advantage is its ability to move individuals’ preferences toward the characteristics of its own product. In the case of multiple markets, individuals’ favorite characteristics are not only defined by the original product in each market, but by an average of the characteristics of the original products of every market where individuals consume. In this case, a first firm to enter a market does not have first mover advantage unless it produces a product with the same characteristics as the first firm to enter in the other market. If both first movers don’t enter with the same characteristics, they will produce a product that is not the individuals most preferred. In this case entering in the first period without the same characteristics of the other firm can be a disadvantage that new firms can exploit in the second period by producing a product with individual’s most preferred characteristics.

Other authors have analyzed competition when individuals’ favorite characteristics are a function of the characteristics of the products they have previously consumed. Carpenter and Nakamoto (1989, 1996) essentially assume this, although they interpret the shift in individuals’ favorite characteristics not as habit formation, but as an updating of the ideal attributes after the successful trial of a product. They analyze the behavior of a second entrant in a more general set-
Moreno-Okuno (2010) also models competition in markets where individuals have habit formation for the characteristics of the products. However, he is not interested in analyzing the decision of a second firm to enter a market and he assumes that both firms enter in the first period. He finds that if both firms enter in the first period they locate far from each other in order to separate preferences and create the equivalent of a switching cost between their products.

None of these authors analyze how the consumption of one product affects the preferences of other products that share the same characteristics, but are in different markets, or how the availability of one product affects competition in other markets, as I do it in section 3 of this article.

2. Model

Hotelling (1929) models competition in a linear city when consumers are distributed uniformly in this city and there are two firms that sell a product that consumers see as identical, except for the location of the firm where they purchase it. To simplify the analysis it is assumed that each consumer demands at most one unit of the product and prefers to buy it in the store that is closer to his location (there is a transportation cost associated with travel for each unit of distance). Both firms first choose their location and then compete on prices to attract customers. Hotelling’s model can also be used to represent competition in the design of the products, where the location of a product represents the level of a characteristic in a product (for example, the level of sweetness) and the location of an individual represents her favorite level of the characteristic. As Hotelling, I assume that products are defined in one characteristic and the characteristic space of my model is given by $T = [t_1, t_2]$, where $t_2$ is the highest possible level of the characteristic in a product and $t_1$ is the lowest possible level of the characteristic and the size of the characteristic space is $t = t_2 - t_1$.

An assumption of Hotelling’s model is that preferences are fixed and that individuals’ current consumption does not affect their future utility. I modify Hotelling’s linear city model to incorporate habit formation by including a second period. I assume that in the first period individuals do not have a favorite location. In many products, individuals do not have an innate preference for any specific characteristics of the products and sometimes they cannot even recognize
them the first time they consume them. But if individuals have habit formation, they will increase their preference for the characteristics of the products they previously consumed.\footnote{Several studies have documented that an increase (decrease) in the level of salt, sweetness, fat, etc. in many of the products that we consume for a few weeks will increase (decrease) our preferred level of those characteristics, even when these characteristics are found in other products (see Bertino and Beauchamp, 1986; Bertino, Beauchamp and Engelman, 1982 and Blais et al., 1986).}

I assume that in the second period individuals’ preferences are determined by the product they consumed in the first period, and that the transportation cost is quadratic in the distance to their most favorite location.

**DEFINITION 1.** An individual has habit formation for the characteristics of the products if in the first period her utility is:

\[
U_1 = v - p_1
\]

if she consumes a product, zero otherwise. In the second period, if an individual consumed a product in the first period, her utility is:

\[
U_2 = S - \theta(l_1 - l_2)^2 - p_2
\]

zero otherwise.

where \(v\) is the surplus from consumption in the first period, \(S\) is the surplus from consumption in the second period (gross of transportation cost and price), \(l_2\) is the location of the product that individuals consume in the second period, \(l_1\) is the location of the product that individuals consumed in the first period and \(\theta\) is a parameter that represents the transportation cost.

This definition implies that in the second period, individuals’ favorite location becomes the location of the product they consumed in the first period, and their utility decreases as the product they consume in the second period differs from this location. Although the assumption that individuals like the characteristics of one product just by consuming it one time may seem extreme, it captures the idea that individuals grow to prefer the characteristics of the products they consume and allows us to model habit formation in a simple way.
Since my objective is studying competition between firms, and not how individuals manage their habit formation, I assume that individuals are not forward looking, that is, they do not take into consideration their utility in the second period when they choose which product to consume in the first period. With respect to the transportation cost, I assume that $\theta$ differs for each individual and is uniformly distributed in the interval $[a, b]$. For simplicity’s sake, I normalize the number of consumers to one and assume that $a = b - 1$. I include these properties in Assumption 1.

**ASSUMPTION 1.**  
\begin{enumerate} 
  \item Individuals are not forward looking; 
  \item $\theta$ is distributed uniformly in the interval $[a, b]$, where $0 < a \leq 1$ and $a = b - 1$.
\end{enumerate}

I assume that firms have no capacity constraint, their marginal cost is zero and they do not discount the second period. Finally, I assume that firms cannot change location from the first to the second period\(^3\) (the motivation for this is that it is costly for firms to design new products). This assumption is not important for my results, however, it eliminates the possibility that in the second period the entrant chooses the same location as the incumbent, forcing the incumbent to change location, which I believe is not consistent with what we observe in reality.

**ASSUMPTION 2.**  
\begin{enumerate} 
  \item The cost of production is zero, there is no capacity constraint and firms do not discount the second period, 
  \item Firms cannot change location from the first to the second period.
\end{enumerate}

Hotelling’s model assumes that individuals’ utility from consumption is large enough with respect to the characteristic space that any individual is willing to consume a product, no matter its characteristics. This assumption is made for simplicity, as this means that firms’ only concern is the competition of the other firm. I relax this assumption and analyze the case where $t$ is large and the case where $t$ is small. In Assumption 3 a), the size of $t$ is large and firms can choose the optimal differentiation. In Assumption 3 b), the size of $t$ is not large enough for firms to choose their optimal differentiation and they choose to locate at the extremes of the characteristic space.

**ASSUMPTION 3.**  
\begin{enumerate} 
  \item $t \geq \sqrt{\frac{3S}{b + 1}}$ 
  \item $t < \sqrt{\frac{3S}{b + 1}}$
\end{enumerate}

I analyze the case of two firms that enter the market sequentially, which means that one firm enters in the first period and another one enters in the second period.

\(^3\) I relax this assumption for Proposition 4.
ASSUMPTION 4.  

a) One firm enters in the first period and another enters in the second period.

The timing of the game is the following,

*First period:*

1. One firm enters the market.
2. After the firm enters, it chooses the characteristics of its product.
3. The firm sets the price of the good.
4. Individuals decide whether to buy or not.
5. The firm produces and sells.

*Second period:*

1. The second firm enters in the second period.
2. Firms choose the characteristics of their products.
3. Firms set prices.
4. Individuals decide whether to buy or not, and if they do, which product to consume.
5. Firms produce and sell.

The equilibrium concept of the game is subgame perfect Nash equilibrium, and I restrict my analysis to pure strategies. The model is similar to the model of quality by Gabszewics and Thiesse (1979), where firms can differentiate in quality without any cost.\(^4\) In my model, the quality would be represented by the closeness of the product to individuals' favorite characteristic, which is given by the characteristic of the product consumed in the first period.

**PROPOSITION 1.** 

a) If individuals have habit formation for the characteristics of products and assumptions 1, 2, 3a and 4a are satisfied, in the first period the incumbent can choose any location as long as it is at a distance greater or equal to \(\sqrt{\frac{3S}{b+1}}\) from any of the extremes of the characteristic space and charges a price of \(v\) for its product. In the second period the entrant will locate at a distance from the incumbent:

\[d_e = \sqrt{\frac{3S}{b+1}}\]

and firms price at \(p_{2i} = S\) and \(p_{2e} = \frac{(2-b)S}{(b+1)}\).

b) If individuals have habit formation for the characteristics of the products and assumptions 1, 2, 3b and 4a are satisfied, in the

\(^4\) With the difference that \(S\), individuals utility, gross of transportation cost and price, is the same for every consumer.
first period the incumbent locates at one of the extremes of the characteristic space and charges a price of \( v \) for its product. In the second period, the entrant locates at the opposite extreme to the incumbent and both firms price at

\[
p_{2t} = \frac{(b + 1) t^2}{3} \quad \text{and} \quad p_{2e} = \frac{(2 - b) t^2}{3}.
\]

All proofs are in the appendix.

The first firm attracts individuals' preferences for its product and the second firm will compete for some of these individuals. The new firm decides how similar its product will be to the product of the original firm. The new firm can eliminate any advantage the incumbent has by duplicating the same characteristics. However, this would increase competition as individuals are indifferent as to which product they consume. If the new firm relaxes competition by selling a product with different characteristics, this will increase the transportation cost for consumers and individuals may not want to consume its product. I find that the new firm will produce a product that is similar to the original product to attract some of its consumers, but that it will not produce the exact same product in order to relax competition. This result seems to capture an important aspect of reality, which is that latecomers to a market produce products that are similar to the original ones, but try to differentiate themselves in some way.

From proposition 1 a) we can see that for small values of \( S \), the difference between the product of the entrant and the original product increases with \( S \). As \( S \) increases, individuals are willing to pay higher prices. Since the entrant has an incentive to differentiate until the incumbent charges the highest possible price, a higher value of \( S \) results in a higher differentiation.

We can also see that the difference between products decreases with \( b \). An increase in \( b \) means that it is costlier for consumers to consume away from their favorite location, so the entrant has to produce a product that is more similar to the original product to attract some of the consumers of the incumbent.

However, when \( t < \sqrt{\frac{4S}{3b + 1}} \), the size of the characteristic space is smaller than the optimal differentiation and the entrant will differentiate as much as possible by locating at the furthest extreme in the characteristic space from the incumbent. Foreseeing this, the incumbent will locate in one of the extremes in the first period to allow maximum differentiation.

The price and profits of the incumbent are higher than those of the entrant. The incumbent has an advantage by entering in the first period in markets with habit formation, since the entrant will differentiate from the incumbent by producing a different product
than the one the consumers have grown to prefer and sell it to a lower price than the original product.

3. Multiple Markets

Up until now I have analyzed habit formation in the characteristic space in only one market. However, products in different markets often share many characteristics. For example, sweetness is a characteristic that is shared by many markets, such as carbonated soft drinks, ice-cream, etc. If individuals learn to enjoy the characteristics of the products they consume, then the consumption of one product will affect the preferences for other products that have the same characteristics, even in different markets. In this section I analyze the case of two markets that share one characteristic; however, the analysis can easily be extended to more markets.

In the case where several markets share one or more characteristics, the value of a product depends not only on where its characteristics are in relation to other products in the same market, but also in relation to products in other markets that share the same characteristics. Individuals want to consume products that minimize their transportation cost to the products they have consumed, even if these products are in different markets. I extend the definition of individuals’ utility function to two markets that share one characteristic.

**DEFINITION 2.** Individuals experience habit formation for the characteristics of the products in two markets if in the first period, individuals’ utility from consuming a product in market $m$ is given by:

$$U_{1m} = v - p_{1m}$$

zero otherwise. In the second period, if individuals consumed a product in the first period, their utility from consuming a product in market $m$ is the following:

$$U_{2m} = S - \theta(l^*_{2m} - l_{2m})^2 - p_{2m}$$

zero otherwise, and the most preferred location is given by:
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\[ l_{2m}^* = \phi l_{1m} + (1 - \phi) l_{1n} \]

or

\[ l_{2m}^* = l_{1n} \]

if individuals did not consume any product in the first period in market \( m \), or

\[ l_{2m}^* = l_{1m} \]

if individuals did not consume any product in the first period in market \( n \), or zero if individuals did not consume any product in both markets in the first period.

where \( l_{1m} \) and \( l_{2m} \) are the locations of the products that individuals consume in the first and second periods, respectively, in market \( m \); \( l_{1n} \) and \( l_{2n} \) are the locations of the products that individuals consume in the first and second periods, respectively, in market \( n \); where \( m, n = 1, 2 \) and \( m \neq n \). \( \phi \) is a positive constant and \((1 - \phi)\) represents the influence that products in one market have in shaping preferences in the other market. In the case where there is no consumption in one market in the first period, individuals’ preferences in that market will be shaped completely by their consumption in the other market.

The difference between this definition and the definition of a utility function for a single market is that in the second period an individual’s favorite location in market \( m \) is a function of previous consumption not only in market \( m \), but also in market \( n \).

3.1. One Firm Enters Each Market

In this part I analyze the case where only one firm enters each market. The following proposition refers to the case where one firm enters in the first period while the other one enters in the second period.

**PROPOSITION 2.** If individuals have habit formation for the characteristics of the products and if one firm enters one of the markets in the first period and another firm enters the other market in the second period, the second firm will produce in the same location as the
product in the first period in the characteristic they share. The first firm prices at \( v \) in the first period and both firms price at \( S \) in the second period.

Proof. See appendix.

Proposition 2 shows that there is path dependency in different markets in the characteristic they share. If only one firm enters in the first period, it will determine the preferences in both markets for that characteristic. After that, if another firm enters it will produce a product with the same characteristic as the other firm to attract the consumers that have learned to prefer that level of the characteristic.

For example, in a country where individuals eat spicy products, those individuals will grow to prefer spicy food in general. If a firm wants to introduce a new product, it would make sense to produce a product with the same level of spice as the other products that were previously introduced, even in markets other than the one the original product is intended for. For example, we observe that in Mexico some brands of potato chips are hot, while in the United States they have barbecue sauce, in Japan they have seaweed and in India curry. Those products have adapted to the local taste where individuals’ preferences have been shaped by other markets.

Now I analyze the case where two firms enter in the first period, one in each market. If we relax assumption 2 \( b \), allowing firms to change the characteristics of their products, then each firm does not have to produce with the same characteristics in the first period, but as the following proposition shows, if they do not do so, they will change their characteristics in order to produce similar products in the second period.

**PROPOSITION 3.** If individuals have habit formation for the characteristics of the products and if one firm enters each market in the first period and both firms can change their location in the second period, they can produce products with any characteristics in the first period, and in the second period they will produce with characteristics \( \phi_1 m + (1 - \phi) l_1 n \), that is, they will move toward the characteristics of the product consumed in the other market in the first period. Both firms charge \( v \) in the first period and \( S \) in the second period.

Proof. See appendix.

The reason for this is that in the second period individuals’ favorite characteristics are an average of the characteristics of the products they consume in the first period. In the second period firms have an incentive to change their product to match individuals’ preferences.
However, if firms cannot change location in the second period, in the first period they will produce in the same location in the characteristics they share.

**PROPOSITION 4.** If individuals experience habit formation for the characteristics of the products and if one firm enters each market in the first period and if firms cannot change their location from the first period to the second period, then firms can produce in any location; however, this location will be the same for both firms. Both firms charge $v$ in the first period and $S$ in the second period.

Proof. See appendix.

Proposition 4 shows that in the case where one firm enters in the first period in each market and firms cannot change the product specifications, the products will be aligned in the characteristic they share. If a firm produces a product that is not aligned with the location of other products that share the same characteristics, individuals' preferences will not correspond with the specifications of the product and the firm can earn higher profits by moving its product to the same location of the product in the other market.

### 3.2. Two Firms Enter Each Market

Next I analyze the case where two firms enter each market. I assume that firms enter sequentially: one firm enters in the first period and the other firm enters in the second period in each market. I assume that firms cannot change their location from the first to the second period.

**ASSUMPTION 4.** *b*) One firm enters in the first period and another firm enters in the second period in each market.

**PROPOSITION 5.** *a*) If individuals experience habit formation for the characteristics of the products and assumptions 1, 2 and 3a and 4b are satisfied, in the first period the incumbents can choose any location, as long as it is the same location as the incumbent in the other market and it is at a distance greater or equal to $\sqrt{\frac{3S}{v+1}}$ from any of the extremes of the characteristic space and charge a price of $v$ for its products. In the second period the entrant in market $m$ locates at a
distance from the incumbent: \( d_{em} = \sqrt{\frac{2b}{b+1}} \) and firms price at \( p_{2im} = S \) and \( p_{2em} = \frac{2-b}{b+1}S \) for \( m = 1, 2 \).

b) If individuals experience habit formation for the characteristics of the products and assumptions 1, 2, 3b and 4b are satisfied, in the first period both incumbents locate at one of the extremes of the characteristic space and charge a price of \( \nu \) for its product. In the second period, the entrants locate in the opposite extreme to the incumbents and both firms price at \( p_{2im} = (b+1)t^2/3 \) and \( p_{2em} = (2-b)t^2/3 \) for \( m = 1, 2 \).

Proof. See appendix.

In the case of habit formation for the characteristics of the products, what gives a first mover advantage is the ability to move individuals’ preferences toward the characteristics of its product. In the case of multiple markets, an individual’s favorite location is not only defined by the original product in each market, but by an average of the characteristics of the original products of every market where individuals consume. In this case, the first mover does not have a first mover advantage unless it produces with the same characteristics as the incumbent in the other market.

If both incumbents produce with different characteristics, then individuals’ favorite characteristics will be the average of the characteristics of both products. In this case, entering the market in the first period can become a disadvantage (as doing so causes them to produce products that individuals do not like in the second period) and new entrants can exploit this disadvantage by producing products with individual’s most preferred characteristics. This may shed some light into why some first movers succeed when entering a new market while others fail: firms that enter a new market will have a higher probability of succeeding if their products have the same specifications as those in other markets that share the same characteristics.

4. Conclusions

I developed a model of competition in markets with habit formation by working with the idea of habit formation for the characteristics of the products. This approach offers several advantages:

First, it allows us to analyze how firms choose the design of their products to respond to habit formation, and second, it allows us to
analyze how products affect the preferences in other markets that share the same characteristics.

There are numerous ways in which this research can be expanded. The first natural extension is to study markets where consumers are forward looking. In some products, such as wine and cigars, individuals try to “refine” their preference by learning the characteristics of certain products like “fine” wines or “fine” cigars. An extension of my model would analyze competition when individuals are forward looking and how firms compete to establish the characteristics of their products as characteristics of the “fine” products that must be learned.

My model assumes that individuals start without any initial preference for the characteristics of the products. However, if individuals with habit formation have initial preferences that are uniformly distributed in the characteristic space, it is possible that two or more firms enter the first period and split the consumers between them. The results of my model would still apply to new firms that enter after them and compete for the consumers that have grown to prefer the characteristics of each product.

Appendix

Proof of Proposition 1

I solve it using backward induction.

Second Period

First I look for the individual \( \theta^* \) that is indifferent between the original and the new products:

\[
S - p_{2i} = S - \theta^* d_e^2 - p_{2e},
\]

where \( d_e \) is the distance from the location of the original product \( i \) to the product of the entrant \( e \), and \( p_{2i} \) and \( p_{2e} \) are the prices of the incumbent and entrant respectively. Solving for \( \theta^* \) we get

\[
\theta^* = \frac{p_{2i} - p_{2e}}{d_e^2}.
\]
Individuals with transportation costs higher than $\theta^*$ consume the product of the incumbent, and individuals with lower transportation costs consume the product of the entrant.

The profits for the incumbent are: $\pi_{2i} = (b - \theta^*)p_{2i}$, and the profits for the entrant are: $p_{2e} = (\theta^* - a)p_{2e}$.

I first solve for the first order conditions with respect to prices and obtain that the optimal prices and profits are the following:

\begin{align*}
p_{2i} &= \frac{1}{3}(b + 1)(d_e^2), \\
p_{2e} &= \frac{1}{3}(2 - b)(d_e^2), \\
\pi_{2i} &= \frac{1}{9}(b + 1)^2(d_e^2), \\
\pi_{2e} &= \frac{1}{9}(2 - b)^2(d_e^2).
\end{align*}

While the entrant has an incentive to differentiate itself as much as possible to relax competition and increase prices, there is a limit to this, as prices cannot go higher than individual’s total valuation for a product ($S$). At the distance

\[d_e = \sqrt{\frac{3S}{b + 1}},\]

the price of the incumbent reaches $S$. If the entrant differentiates beyond the point, the incumbent will not raise prices and the profits for the entrant are given by the function:

\[\pi_{2e} = \frac{1}{4d_e^2}(S - ad_e^2)^2.\]

We can see that the profits for the entrant decrease if it differentiates beyond this point. If the characteristic space is bigger than

\[\sqrt{\frac{3S}{b + 1}},\]

the entrant will be able to differentiate until the price of the incumbent reaches $S$, but will not differentiate more. At this distance the prices are: $p_{2i} = S$, and

\[p_{2e} = \frac{2 - b}{b + 1}S.\]
However, if $t$ is smaller than

$$\sqrt{\frac{3S}{b+1}}$$

there is a limit to how much the entrant can differentiate and it will choose to locate at the furthest extreme of the characteristic space from the incumbent.

**First Period**

Given that individuals are not forward looking, consumers are only willing to pay the utility they receive for the product in the first period and therefore the firm will price at $\nu$.

Since profits for the incumbent in the second period increase with the distance from the entrant, the incumbent will choose any location as long as it allows the entrant to differentiate as much as possible in the second period.

In the case that $t$ is smaller than

$$\sqrt{\frac{3S}{b+1}}$$

the incumbent chooses one of the extremes of the characteristic space.

In the second period, given that both firms choose opposite extremes of the characteristic space, firms charge

$$p_{2i} = \frac{1}{3}(b + 1)(t^2) \quad \text{and} \quad p_{2e} = \frac{1}{3}(2 - b)(t^2).$$

**Proof of Proposition 2**

If individuals only consume in market $n$ in the first period, then by the definition of utility function in multiple markets, individuals' favorite location in market $m$ in the second period is given by the product they consume in market $n$ in the first period:

$$l_{2m}^* = l_{1n}.$$
That is, individuals’ utility from consuming the product of a new firm in market $m$ in the second period is:

$$U_{2m} = S - \theta (l_{2m} - l_{1n})^2 - p_{2m}$$

The location that maximizes the profits for the new firm in market $m$ is the individuals’ favorite location, which is the same location as the product individuals consume in the first period in market $n$.

$$l_{2m} = l_{2m}^*$$

Firms charge the highest possible price, which is $v$ in the first period and $S$ in the second period.

**Proof of Proposition 3**

Both firms can produce in any location in period one and still charge price $v$ since individuals do not care about any characteristics in this period. In the second period individuals’ favorite location is an average of the location of the products in both markets:

$$l_{2m}^* = \phi l_{1m} + (1 - \phi) l_{1n}$$

If firms can change location from the first to the second period, both firms would move to this location as it is the only location where individuals are willing to pay $S$. Therefore, the characteristics of both products in the second period will be an average of the characteristics of the products in the first period.

**Proof of Proposition 4**

If firms cannot change location from the first to the second period and if both firms do not produce in the same location as each other in the first period, in the second period individuals' favorite characteristics won’t coincide with the design of the products of each market. The distance between individuals’ favorite design and the design of the product in market $m$ is given by

$$d_{2m} = |l_{2m}^* - l_{2m}| = |(1 - \phi)(l_{2m} - l_{2n})|.$$
Individuals will be willing to pay at most
\[ S - \theta \cdot d_{2m}^2 \]
for the product. It is easy to see that firm \( m \) maximizes the price it can charge and its profits by choosing the same design as firm \( n \).

**Proof of Proposition 5**

We solve it by backward induction, first we solve for the design chosen by a new entrant in the second period and then we solve for the design chosen by the incumbent in the first period.

**Second period**

If the incumbent chooses a design that doesn’t coincide with individuals’ favorite design, the entrant can choose to differentiate even more than the incumbent and attract the individuals with low transportation costs, or it can choose a design closer to individuals’ favorite design and attract the individuals with high transportation costs.

If the incumbent chooses a design close to the design of the other incumbent, then its product is going to be close to individuals’ preferences and a new entrant will choose a design further from individuals’ preferences (I analyze this case in part \( a \) below).

However, if the incumbent chooses a design far from the design of the other incumbent, then its product is going to be far from individuals’ preferences and a new entrant will choose to enter with the design that individuals prefer (I analyze this case in part \( b \) below).

\( a \) If the incumbent in market \( m \) chooses in the first period a design closer than \( d_{im}/(1 - \phi) \) to the design of the other incumbent, then its product is going to be closer than \( d_{im} \) from individuals’ favorite location, where \( d_{im} \) is given by:

\[
\overline{d_{im}} \leq \sqrt{\frac{3S(b + 1)}{4b^2 - b + 4}}
\]

In this case the best response from a new entrant in the second period is to attract the individuals with low transportation costs by
producing a product even further from the one individuals prefer. Solving for the first order conditions for the price for each firm I obtain that the profits for the incumbent and the entrant are given by the following equations:

\[ \pi_{im} = \frac{1}{9} (b + 1)^2 (d_{em}^2 - d_{im}^2) \quad (1) \]

\[ \pi_{em} = \frac{1}{9} (2 - b)^2 (d_{em}^2 - d_{im}^2) \quad (2) \]

where \( d_{im} \) and \( d_{em} \) are the distances from the incumbent and entrant in market \( m \) to the individuals’ favorite location, that is, to

\[ l_{2m}^* = \phi l_{1m} + (1 - \phi) l_{1n}. \]

The entrant has an incentive to differentiate as much as possible to relax competition and increase prices. However, the incumbent won’t increase its prices beyond \( S - b \cdot d_{im}^2 \). When the price of the incumbent reaches this price, the entrant won’t differentiate any further as its profits start decreasing. Therefore, the entrant will differentiate just until the price of the incumbent reaches \( S - b \cdot d_{im}^2 \). At this point, the profits for the incumbent are the following:

\[ \pi_{im} = \frac{1}{3} (b + 1)^2 \left( \frac{S - bd_{im}^2}{b + 1} \right) \quad (3) \]

We can see from equation (3) that the profits for the incumbent increase as the characteristics of the incumbent’s product get closer to individuals favorite design.

b) If the design of the incumbent is greater than \( d_{im} \) then a new entrant will choose to attract the individuals with high transportation costs by producing a product with a design closer to individuals’ favorite characteristics than the design of the incumbent. The problem becomes the same as part a), with the entrant attracting the individuals with high transportation costs instead of the incumbent. In this case the entrant profits are given by the following equation:

\[ \pi_{em} = \frac{1}{9} (b + 1)^2 (d_{im}^2 - d_{em}^2) \]
We can see that the best response of a new entrant is to choose $d_{nm} = 0$, which is to produce with individuals’ favorite design. In this case, the incumbent gives away its first mover advantage and attracts the individuals with low transportation costs by selling a less attractive product at a lower price. Given that a new entrant chooses $d_{em} = 0$ the problem is similar to than of section 2 and the highest possible profits for the firm that attracts the individuals with low transportation costs (now the incumbent) are given by equation (2) which are lower than the profits it would get by locating in the same location as the first mover in the other market and attracting the individuals with high transportation costs, which are given by equation (1).

If $t$ is smaller than $\sqrt{\frac{3S}{b+1}}$, there is a limit to how much both entrants can differentiate from the incumbents and they are going to choose to locate at the furthest extreme from the incumbents.

First Period

The design for the incumbent that maximizes its profits is the one that coincides with individuals’ favorite design ($d_{im} = 0$) and for this to be true, the design of its product must coincide with the design of the incumbent in the other market ($l_{1m} = l_{1n}$).

As both incumbents choose the same design, their designs coincide with individual preferences in the second period and the solution is the same as section 2, with both incumbents charging the highest possible price, which is $v$ in the first period and in the second period, if

$$t \geq \sqrt{\frac{3S}{b+1}},$$

the entrants choose to differentiate at distance from individuals’ favorite location

$$d_{em} = \sqrt{\frac{3S}{b+1}}$$

and price

$$p_{2em} = \frac{2-b}{b+1}S$$

for $m = 1, 2$. If

$$t < \sqrt{\frac{3S}{b+1}},$$
firms locate at opposite extremes of the characteristic space and charge

\[ p_{2im} = \frac{1}{3}(b + 1)(t^2) \]

and

\[ p_{2em} = \frac{1}{3}(2 - b)(t^2) \]

for \( m = 1, 2 \) in the second period.

References


