UNIT ROOTS AND MULTIPLE STRUCTURAL BREAKS IN REAL OUTPUT: HOW LONG DOES AN ECONOMY REMAIN STATIONARY?

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Resumen: Al utilizar los métodos de remuestreo, presentamos evidencia sobre el desempeño de modelos estacionarios en diferencias y en tendencia para describir la producción real de Mexico. Bajo la hipótesis alternativa, la producción fluctua estacionariamente alrededor de una tendencia de largo plazo con un número desconocido de cambios estructurales, identificados a través de métodos de búsqueda globales y secuenciales. Introducimos una nueva regla de detección del número de cambios estructurales y comparamos los resultados con técnicas tradicionales.

Abstract: Utilizing resampling methods, we present evidence on the rejection probabilities for difference-stationary and trend-stationary models for Mexico's real and real per-capita annual gross domestic product. The trend stationary alternative allows for stationary fluctuations around a long-run trend function with endogenously determined multiple structural breaks, via global and sequential search methods. The number of breaks is determined using a unit-root rejection stopping rule and a parameter-constancy stopping rule.

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1. Introduction

The issue of whether annual Gross Domestic Product, GDP, has a unit-root in the long run has been widely investigated over the last fifteen years. One of the main conclusions one can draw from the literature is the great importance of the role of the deterministic trend function: for many countries, the rejection of a unit root in real GDP depends on whether the trend function under the alternative hypothesis is modeled as a linear trend or as a linear trend with structural breaks. Examples can be found in Ben-David and Papell (1995), Duck (1992), Noriega (1992), Noriega and de Alba (1999), Perron (1989, 1992, 1993, 1997), Raj (1992), Zivot and Andrews (1992), Zelhorst and Haan (1995), for the case of a single break, and Lumsdaine and Papell (1997)for the case of two breaks. This research has utilized versions of the so called Dickey-Fuller (1979) unit root test augmented with dummy variables to accommodate a broken-trend function. In this setup, the null hypothesis implies a random walk behavior for GDP, while the alternative hypothesis implies that GDP is generated from an autoregressive process which fluctuates stationarily around a trend with structural breaks in either its level, trend or both. It is well known that if real GDP follows a unit root process, then the traditional separation between (short-term) business cycles and (long-run) trend growth is misleading: any 'cyclical' movement can permanently alter the long-run trend of the variable.

Lumsdaine and Papell (1997), argue that it is far from obvious that a single structural break is a good characteristic of long-term macro series. Clemente, Montañez and Reyes (1998) emphasize the importance of correctly specifying the number of breaks when testing for a unit-root. In the empirical part of their paper, the unit-root null hypothesis is tested against a trend stationary model with an increasing number of breaks, starting from zero. They conclude that the unit-root hypothesis can be rejected only after a double change in the deterministic mean is accomodated in the model. Both papers consider the case of models allowing for two breaks in the trend function. In practice, however, the true number of structural breaks in a time series is unknown, and there are different ways to determine it. Recent literature on multiple structural change (Bai (1997a,b), Bai and Perron (1998a,b)) studies the

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possibility of estimating the number of breaks based on a parameter constancy test in a sequential fashion. In these papers, structural breaks are added to the model until the null hypothesis of parameter constancy is not rejected.[1] However, since there are unlimited ways in which a process could be nonstationary, the stopping ruleof parameter constancy is only one possibility among many. In this paper weexplore the possibility of relating a structural break to unit root behaviour.[2] That is, the stopping rule we examine is: stop adding breaks when the *unit root* form of nonstationarity is rejected, instead of: stop adding breaks when the *parameter variation* form of nonstationarity is rejected. The advantage of this criterion is that it allows the possibility, by construction, of identifying those dates which are responsable for unit root behaviour.[3] Obviously, the higher the number of structual breaks needed to reject the null hypothesis of a unit root the closer the process will be to unit root behaviour. From the empirical literature, however, it can be seen that a few breaks are usually all that is needed in order to reject the unit root null hypothesis.

This paper presents evidence on the nonstationarity properties of production series, once allowance is made for an unknown number of structural breaks in the deterministic trend function of the process, utilizing both stoppingrules to determine the number of breaks. There are no empirical results in the literature on the use of search methods coupled with these two stopping rules in testing for a unit root in economic time series.

The plan of the paper is as follows. Section 2 presents the data and justifies the use of trend break models. In section 3 we present the resampling procedure carried out to test for the presence of a unit root with an unknown number of structural breaks in the trend function,

[1] In Bai (1997a) and Bai and Perron (1998a) it is shown that this stopping rule yields a consistent estimation of the true number of breaks, provided the size of the test slowly converges to zero.

[2] It is well documented by now that structural breaks in the trend function of macro series are responsable for the 'apparent' unit root behaviour which results from ignoring them in the model's specification (see Perron (1989), Rappoport and Reichlin (1989), Reichlin (1989), Chen and Tiao (1990), and Hendry and Neale (1990)).

[3] The criterion used in this paper is based on Monte Carlo experiments which examine the relative performance of these two stopping rules. Preliminary results in Noriega (1999) show advantages of the unit root rejection stopping rule over the parameter constancy stopping rule in its ability to identifying the true number of structural breaks in the trend function.

making use of the Unit Root Rejection Stopping Rule, URR-SR, and the Parameter Constancy Stopping Rule, PC-SR. In section 4, we apply both stopping rules to two observed macro series for which at least two structural breaks are apparent in the sample. In doing this, we allow for the presence of an unknown number of structural breaks estimated from sequential and global procedures. In particular, following Rudebusch's (1992) resampling procedure, we present evidence on the rejection probabilities for difference stationary and trend stationary models for Mexico's long-run annual real GDP and real per capita GDP, using exact critical values based on the Monte Carlo distributions of the Dickey-Fuller type t-statistic. We extend the set of plausible alternative hypotheses to allow for models with structural breaks in the trend function, under different criteria for the estimation of the break dates, and the determination of the number of breaks. To determine this number, we use the two stopping rules mentioned above. Using the first rule, we start by locating a first break point using search methods recently studied in Bai (1997a), Bai and Perron (1998a), and Lumsdaine and Papell (1997). The *number* of breaks is the result of sequentially applying these methods until the null hypothesis of a unit root *can not* be supported by the data *and* the alternative hypothesis of a broken trend stationary model *can* be. Under the second stopping rule, we utilize the sequential method of Bai (1997b), that consistently estimates the number of break points, in which the stopping rule rests on parameter constancy tests. Finally, in the last section we draw some conclusions.

2. Data and Justification

As mentioned above, we follow Rudebusch's (1992) procedure to investigate the closeness of the unit root distribution to plausible alternative hypotheses. Due to some features of our data set, the approach extends that of Rudebusch in that we consider Trend Stationary, *TS*, models with a trend function allowing for structural breaks, as plausible alternatives to a difference stationary, *DS*, model. These *TS* hypotheses assume that both the number and location of the breaks is unknown. This will allow us to assess the relevance of the deterministic trend component in testing for a unit root using exact distributions of the test statistics, when plausible (estimated from the data) alternative hypotheses are considered.

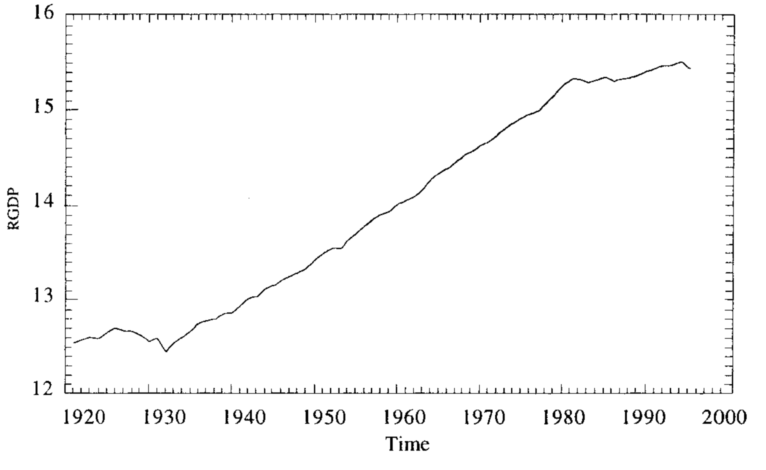
The data consist of annual observations of Mexico's real gross domestic product, RGDP, from 1921 to 1995, and real per capita GDP from 1921 to 1994.[4] Natural logarithms of the data are shown in figure 1 at the end of this paper. As can be seen, the strong upward trend in both series seems to have undergone more than one break inthe sample. There seem to be at least two strong structural breaks which have resulted in multiple growth paths for both series, occurring somewhere at the beginning of the 1930s and of the 1980s. Mexico's internal and external economic environment was particularly interesting around those dates. The years surrounding 1930 brought both external shocks and the internal reorganization of economic activity. During the period 1927-1928, a political conflict over the oil rights of US firms led to a 50% reduction in oil production. The Great Crash of 1929 severely affected Mexican exports: in 1932 they were a third of what they had been in 1929. As a result, government spending dropped by 25% from 1929 to 1932. These factors explain the downward trend in production over the period 1927-1932, see figure 1. On the other hand, internal reforms together with the recovery of the world economy from the Great Crash secured a positive growth path from the beginning of the thirties. By 1931, Mexico abandoned the Gold Standard and the exchange rate was left to fluctuate freely, until there was an 80% devaluation of the peso in 1933. By that year, production and exports increased as US demand stabilized after the crisis. During the 1930s, other important reforms were responsible for the observed growth rates in production: the monetary reform of 1936, which eliminated the relationship between the peso and any metal, and the use of public spending in capital formation.

The break around 1980 produced a slowdown in growth rates for both real GDP and real per capita GDP. In 1979, the government adopted a model based on oil exports, following the oil field discoveries of 1978 and the 150% increase in oil prices the following year. However, this oil-based strategy ended when oil prices fell in 1981, leaving the country with an enormous external debt, which had been contracted to develop the oil industry. The 1982 increases in international interest rates led Mexico into a serious debt crisis.

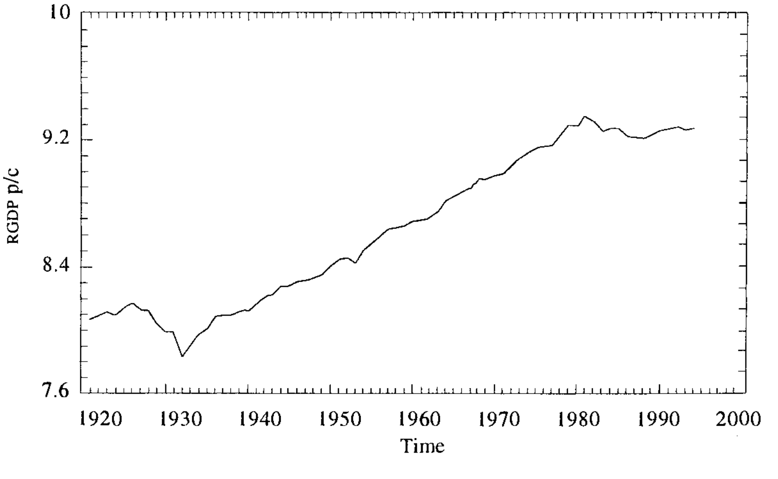
[4] The source for RGDP is the Instituto Nacional de Estadística Geografía e Informática (1996), while that for per-capita RGDP is Alzati (1997).

Figure 1

ln Mexico's Real Gross Domestic Product, 1921-1995
(1980 = 100)



ln Mexico's Real Per-Capita Gross Domestic Product, 1921-1994
(1965 = 100)



Finally, although not as noticeable as in the case of these two breaks, there seems to be a slight but persistent upward change in trend around the middle of the century. Two important devaluations of the Mexican peso took place in 1948 (peso devalued by 42%) and 1949 (26%), followed by an increase of nearly 50% in exports during 1950-1951, due to the Korean War, and the lower price of Mexican exports.[5]

The question of how the cyclical fluctuations are affected when allowing for the endogenous estimation of these breaks is interesting. Diebold and Senhadji (1996) have made conjetures, and our results supply strong empirical evidence. A related question is how long the economy clings to a particular linear trend in such a way as to identify stationary cyclical fluctuations around it. To answer this question, the duration of stationary cyclical fluctuations must be investigated, while allowing economy to 'move' along structurally-different long-run paths. To shed some light on these questions we compute by simulation the sampling distributions of the t-statistic for testing for a unit-root when the data generation process, DGP, is a *DS* model (the null hypothesis), and a *TS* model with $m \geq 0$ breaks (the alternative hypotheses). The *DS* and *TS* DGPs use the parameters from *TS* and *DS* models estimated from the empirical data. The results are thus extracted, as in Diebold and Senhadji (1996), from where the sample estimate of the t-statistic for testing for a unit root ($\hat{t}_{sample}$) is approximately the same as in the empirical data.

3. Resampling Procedure

In this section we present the resampling procedure carried out to test for the presence of a unit root with an unknown number of structural breaks in the deterministic trend function, making use of both the URR-SR and the PC-SR to determine the number of breaks. Let us start with the no break case, denoting by Y_t the logarithm of the observed output series. The first step is to estimate (by OLS) the following *TS* and *DS* models, respectively:

$$\Delta Y_t = \mu + \beta t + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t \qquad (1)$$

[5] Many of these facts were taken from Alzati (1997), Solís (1970), and Torres-Gaytán (1980).

$$\Delta Y_t = \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t \qquad (2)$$

for $t = 1, 2, \ldots, T$, where T is the sample size and ε_t is an *iid* process. In the *TS* model (1) if $\alpha < 0$, then Y_t generates stationary fluctuations around a linear deterministic trend. If $\alpha = 0$ (the null hypothesis), then Y_t does not generate stationary cycles around this trend. The *DS* specification in (2) represents case where no deterministic trend is considered. The reason for the *DS* specification is that interest centers on the autoregressive parameter and its associated t-statistic estimated from (1), both of which are invariant with respect to the parameters μ and β for any sample.[6] Hence, in simulating the t-statistic on α, the DGP under the *DS* null hypothesis can be stipulated as in (2). In determining the autoregressive order for each model, we first fix an arbitrary maximum order, as in Perron (1997), labeled k max. When the estimated coefficient on the k maxth lag is not significant we estimate again with k max - 1 lags. We continue in this fashion until we find a significant lag.[7]

Next we simulate, as in Rudebusch (1992), the distribution of the t-statistic for the null hypothesis of a unit root ($\alpha = 0$ in (1)), called $\hat{\tau}$, under the hypotheses that the true models are the *TS* model (1) and the *DS* model (2), both estimated from the data. That is, under the *TS* (*DS*) model we use the estimated parameters from (1)((2)), and the first $k + 1$ observations as initial conditions ($\Delta Y_2, \ldots, \Delta Y_{k+1}$) to generate 10,000 samples of ΔY_t, $t = 2, \ldots, T$, with randomly selected residuals (with replacement) for each ΔY_t, $t = k + 2, \ldots, T$ from the estimated *TS* (*DS*) model. For each sample thus generated, regression equation (1) is run and the corresponding 10,000 values of $\hat{\tau}$ are used to construct the empirical density function of this statistic under the *TS* (*DS*) model, labeled $f_{TS}(\hat{\tau})$ ($f_{DS}(\hat{\tau})$).[8]

A *TS* model with m structural breaks in both level and trend can be written as:

[6] The same invariance holds when considering below alternative hypotheses allowing for structural breaks, see for example Perron (1989, p. 1393).

[7] Diebold and Senhadji (1996) report the best-fitting regression for $k = 2, 3, 4$. In Rudebusch (1992), the order of the autoregressions is fixed to two, irrespective of whether we estimate the *DS* or *TS* model. Rudebusch (1993) uses AR orders four, six and eight.

[8] The 10,000 fitted regressions utilize the already estimated value of k, under the *TS* (*DS*) model. All calculations were carried out in GAUSS.

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t \quad (3)$$

where DU_{it} and DT_{it} are dummy variables allowing changes in the trend's level and slope respectively, that is,

$$DU_{it} = \mathbf{1}(t > T_{bi}) \text{ and } DT_{it} = (t - T_{bi})\mathbf{1}(t > T_{bi}),$$

where $\mathbf{1}(\cdot)$ is the indicator function and T_{b_i} is the unknown date of the i^{th} break. This equation is a generalization to m breaks of the Innovational Outlier Model, used by Perron (1989) and others.[9] For reasons explained above, the relevant *DS* model is again, for each value of m, equation (2).

In order to generate, via Monte Carlo, the empirical densities of $\hat{\tau}$ under the estimated *TS* DGP (3) and *DS* DGP (2), $f_{TSm}(\hat{\tau})$ and $f_{DSm}(\hat{\tau})$, (m = 1, 2,...), respectively, we need an estimation of the number (m) and location (T_{b_i}) of breaks. Once these estimates are obtained (see below), the Monte Carlo experiments for generating $f_{TSm}(\hat{\tau})$ and $f_{DSm}(\hat{\tau})$, (m = 1, 2,...), follow the same steps as in the no breaks case.

In order to determine the location of the breaks, we use three different criteria. The first selects the break dates as those years for which the unit root is rejected with the highest probability, *i.e.* when the t-statistic for testing the null of a unit root is minimized, over all possible combinations of break dates. This is done for all values of $k \leq k$ max. Because of perfect-collinearity problems due to the presence of dummy variables in the model, the occurence of a break in either the whole sample or in any subsample, has to be restricted to the following intervals. For $m = 1, k + h \leq T_{b1} \leq T - mh$, for the two breaks case, $k + h \leq T_{b1} \leq T - mh$ and $T_{b1} + h \leq T_{b2} \leq T - (m-1)h$, for the three breaks case, $k + h \leq T_{b1} \leq T - mh$, $T_{b1} + h \leq T_{b2} \leq T - (m-1)h$, and $T_{b2} + h \leq T_{b3} \leq T - (m-2)h$, etc., with $h = 3$. We call this the min $t_{\hat{\alpha}}$ criterion.[10]

The second criterion chooses, among all possible combinations of m break dates, the one which yields the smallest residual sum of squares from (3). Again, this is done for all values of $k \leq k$ max. Under this

[9] The only difference is that (3) does not include a pulse variable called $D(TB)_t$ by Perron (1989). This is also the approach in Zivot and Andrews (1992).

[10] This criterion is used by Lumsdaine and Papell (1997) for a two breaks model, applied to the Nelson and Plosser (1982) data set.

criterion, the above restrictions are placed over the intervals' length. We call this the min *RSS* criterion.[11]

The third criterion utilizes a sequential algorithm: we start by identifying a first break, T_{b_1}, as that year in which the null hypothesis of no structural break is rejected with the highest probability, that is, when the absolute value of the recursive t-statistic on the change of level or slope parameter is maximized. If a second break is suspected say, in the first subsample ($t = 1,...,\hat{T}_{b_1}$), then this recursive t-statistic is again used to determine a second break date, T_{b2}, running from the beginning of the sample to the already estimated T_{b_1}. Confirmation of the first break is achieved by recomputing again the recursive t-statistic over the subsample running from $\hat{T}_{b2} + 1$ to the end of the sample. If confirmation is achieved, a search for a third break follows the same steps. We call this criterion max $|t_{\hat{\gamma}}|$ for the case of a change in the trend slope, and max $|t_{\hat{\theta}}|$ for the case of changes in the trend level.[12]

Note that the min $t_{\hat{\alpha}}$ and min *RSS* criteria imply simultaneous determination of m breaks via a global search. The max $|t_{\hat{\gamma}}|$ or max $|t_{\hat{\theta}}|$ approach selects one break at a time, yet with a different criterion from the previous two, in such a way that the m breaks found are sequentially confirmed.

In order to determine the number of breaks, we equip the above procedures with both the URR-SR and the PC-SR, which indicate the termination of the search. Under the URR-SR, we proceed sequentially using the first criterion: after we have identified the first break, we verify the presence of a unit root. If it cannot be rejected we allow the procedures to search for two breaks, testing again for the presence of a unit root. We continue in this fashion until we identify as many breaks as necessary in order to reject the unit root null hypothesis. As can be seen, this is a sequential procedure which globally searches for an increasing number of structural breaks. To contrast our results with the URR-SR, we also use the PC-SR, which relies on a sequential procedure based on the min *RSS* criterion coupled with hypothesis testing for parameter constancy as a stopping rule for the determination of the number of breaks. We

[11] This criterion for estimating break points is discussed in Bai (1997a, b), and Bai and Perron (1998a, b).

[12] Here again the above restrictions are placed over the intervals' length,with $h = 3$. This is the criterion used by Noriega and de Alba (1999), and is based on the 'repartition' method studied in Bai (1997a), also called 'refinement' in Bai (1997b).

briefly present here the methods developed in Bai (1997b) for detecting and locating an unknown number of breaks, based on a PC-SR. An interesting aspect of this procedure is that it is possible to estimate both a point and an interval for the location of each break. First, rewrite equation (3) in compact matrix form with $m = 1$ as follows,[13]

$$\Delta Y = X\beta + Z\delta + \varepsilon \tag{4}$$

where $\Delta Y = (\Delta Y_2, \Delta Y_3, ..., \Delta Y_T)$, X is a $(T-1)\times 3$ matrix whose t^{th} row is $(Y_{t-1}, 1, t)$, $\beta = (\alpha, \mu, \beta)'$, Z is a $(T-1)\times 2$ matrix whose t^{th} row is (DU_t, DT_t), $\delta = (\theta, \gamma)'$, and ε is a $(T-1)\times 1$ vector of disturbances obeying very general conditions discussed in Bai (1997b). The sum of squared residuals obtained from applying OLS to (4), denoted $S_T(T_b)$, is the objective function to be minimized in the identification of a break point, that is:

$$\hat{T}_b = \arg \min_{3 \le T_b \le T-3} S_T(T_b)$$

To carry out hypothesis testing of parameter constancy we use the sup-Wald test statistic proposed by Bai (1997b), defined as

$$\sup_{T_b \in [\pi T, (1-\pi)T]} W_T(T_b), \tag{5}$$

where

$$W_T(T_b) = \frac{\hat{\delta}'_{T_b}(Z'MZ)\hat{\delta}_{T_b}}{\hat{\sigma}^2(T_b)},$$

with $M = I - X(X'X)^{-1}X'$, $\hat{\sigma}^2(T_b) = S_T(T_b)/(T-p-q)$, $\pi = \frac{T_b}{T} \in (0, \frac{1}{2})$, $\hat{\delta}_{T_b}$ equal to the OLS estimator of δ from (4), and p and q equal to the number of variables in the matrices X and Z, respectively.

Briefly [see Bai (1997b) for details], the procedure starts by identifying the first break over the entire sample, if the sup-Wald test rejects the null hypothesis of parameter constancy. The sample is then divided into two subsamples separated by the estimated break point, and the sup-Wald statistic is used for testing parameter constancy in *each*

[13] Note that the model allows for a single break since the test of the null hypothesis of parameter constancy is applied sequentially in order to locate one break at a time. Also note that there is no need for augmentation terms. For details see Bai (1997b).

subsample. These two subsamples are to be further segmented whenever additional breaks (if any) are found by the rejection of the parameter constancy null using the sup-Wald statistic (5). The search terminates when the null hypothesis of parameter constancy can not be rejected for all the subsamples separated by significant breaks. When all of the break points are obtained, each one is reestimated if it comes from a sample or subsample containing more than one break. This is called 'refinement' by Bai (1997b). If, as a result of the refinement, the break dates change, then the sup-Wald statistic should be applied over the new subsamples.[14] The procedure stops if there are no additional breaks within each new subsample. If, on the other hand, new breaks are identified within the new subsamples, then a further refinement is required.

Under this stopping rule, the reasoning behind this paper is implemented through the use of critical values calculated via resampling methods. We simulate the distribution of the sup-Wald statistic (5) under the null hypothesis of parameter constancy, estimated from equation (4). In particular, we use the estimated parameters from (4) under the null hypothesis of parameter constancy ($\delta = 0$) and a standard normal variate as the initial condition to generate 1,000 samples of ΔY_t, $t = 2,...,T$, with standard normal errors for each ΔY_t. For each of these samples, the regression equation (4) is run and the corresponding 1,000 values of the estimated sup-Wald statistic (5) are used to construct its empirical density function under the null hypothesis of parameter constancy.

4. Results and Discussion

We begin by testing the null hypothesis of a unit root against the alternative hypotheses of a *TS* specification allowing an increasing number of $m \geq 0$ structural breaks using our unit-root rejection stopping rule.

[14] For instance, if two breaks have been found, $\hat{T}_{b_1}$ and $\hat{T}_{b_2}$, where , $\hat{T}_{b_1} < \hat{T}_{b_2}$, then the first one has to be 'refined' by reestimating it using the subsample $[1, \hat{T}_{b_2}]$. Call $\hat{T}^*_{b_1}$ the resulting estimate. If $\hat{T}^*_{b_1} = \hat{T}_{b_1}$, then the second break date is refined using the subsample $[\hat{T}_{b_1} + 1, T]$, otherwise we use $[\hat{T}^*_{b_1} + 1, T]$. If as a result of the refinement of the second break $\hat{T}^*_{b_2} = \hat{T}_{b_2}$, then there is no need to refine $\hat{T}_{b_1}$, otherwise the first break date is refined using the subsample $[1, \hat{T}^*_{b_2}]$.

When this is done, the results are contrasted with those obtained from the application of the methods developed in Bai (1997b) for detecting and locating an unknown number of multiple breaks using the *PC-SR*, as outlined in the previous section.

We first discuss the results for real GDP, presented in table 1. The first column indicates the number of breaks allowed in the trend function under the alternative hypothesis, m. The second column refers to the three different procedures for selecting the break dates, explained above. As mentioned earlier, the AR order comes from the first significant lag $k \leq k$ max. In the empirical applications k max is set at 5. The reason for this is that the data show an apparent break very close to the beginning of the sample. Setting k max = 5 allows the procedure to detect this apparent break, while a higher value for k max is chosen (say k max =10) the procedure may not detect it. Column 3 reports the value of the estimated value of k. Columns 4-6 report the break dates resulting from the estimation under each of the three criteria of column 2. Columns 7-9 indicate the type of break allowed in the trend function. The selection of the model for each time series followed the suggestions of Perron (1993). As he argues, although a model allowing changes in both level and slope of trend is the most general one (it encompasses models with breaks in level alone, or with breaks in slope alone), there are power gains by estimating a model without irrelevant regressors. For example, model (3) with $\theta_i = 0$ would be more appropriate if it were apparent from the data that the type of break involved no change in level but only in trend. Columns 10-12 report the p-values for the diagnostic tests AC (the Lagrange Multiplier test of the null hypothesis that the disturbances are serially uncorrelated against the alternative that they are autocorrelated of order one), N (Jarque-Bera's (1980) test of the normality of residuals), and H (a test of the null hypothesis of homoscedasticity). In the second to last column, $\hat{\sigma}_\varepsilon$ stands for the standard error of regression. The last column reports the value of the t-statistic for testing the null hypothesis of a unit root.

Let us begin our analysis for the case of a linear trend with no breaks ($m = 0$). The estimation results for this model are presented in the first row of table 1. To draw an exact inference on the unit root hypothesis through $\hat{t}_{sample}$, we simulate the empirical density of this t-statistic under both a *DS* and a *TS* DGP (with parameter values and disturbances extracted from the corresponding estimated model, as explained above). Then we calculate, under each density, the probability of rejecting the

Table 1

Broken Trend Models for Mexico's Real GDP (1921-1995)

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t$$

m	*Criterion*	$\hat{k}$	Tb_1	Tb_2	Tb_3	*Type of break**			*p-values*			$\hat{\sigma}_\varepsilon$	$\hat{\tau}_{sample}$
									AC	*N*	*H*		
0	...	2							0.252	0.000	0.008	0.43416	-2.01
	Min $t_{\hat{\alpha}}$	4	1981			*L* and *T*			0.015	0.000	0.046	0.037825	-4.05
1	Min *RSS*	0	1932			*L* and *T*			0.527	0.293	0.657	0.035843	1.35
	Max $\lvert t_{\hat{\gamma}} \rvert$	4	1979			*L* and *T*			0.005	0.000	0.050	0.037396	-3.96
	Min $t_{\hat{\alpha}}$	5	1931			*L* and *T*			0.067	0.052	0.712	0.029675	-7.00
				1978			*L* and *T*						
2	Min *RSS*	4	1932			*L* and *T*			0.100	0.441	0.437	0.027929	-2.34
				1981			*L* and *T*						
	Max $\lvert t_{\hat{\gamma}} \rvert$	4	1932			*L* and *T*			0.385	0.453	0.531	0.029158	-3.80
				1977			*L* and *T*						
			1931			*L* and *T*							
	Min $t_{\hat{\alpha}}$	3		1949			*T*		0.293	0.682	0.356	0.023096	-11.21
					1980			*L* and *T*					
			1931			*L* and *T*							
3	Min *RSS*	3		1948			*T*		0.332	0.543	0.490	0.023094	-11.21
					1980			*L* and *T*					
			1931			*L* and *T*							
	Max $\lvert t_{\hat{\gamma}} \rvert$	3		1948			*T*		0.685	0.334	0.547	0.023473	-11.20
					1979			*L* and *T*					

* *T* stands for trend, and *L* stands for level.

DS and the *TS* specification, denoted $\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$, and $\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$, respectively (here $\hat{\tau}_{sample} = -2.01$). These probabilities are presented in the first two rows of table 2, from which we can conclude that it is not possible to discriminate between the two specifications. Finally, the diagnostic tests in table 3 show strong evidence of non-normality and heteroscedasticity in residuals.

For the case of $m = 1$, both the $\min t_{\hat{\alpha}}$ and the $\max|t_{\hat{\gamma}}|$ select $\hat{k} = 4$ and a break date around 1980. The corresponding *t*-statistics for testing a unit root ($\hat{\tau}_{sample}$) are -4.05 and -3.96. The corresponding *p*-values in table 2, show a clear rejection of the *DS* model in favor of the *TS* model with a single structural break in the trend function. However, the diagnostic tests in Table I show significant serial correlation, non-normality and heteroscedasticity. From the min *RSS* criterion, the identified break year is 1932 and the sample test statistic for the hypothesis of a unit root is $\hat{\tau}_{sample} = 1.35$ implying an explosive autoregressive root.

Turning to $m = 2$, the estimated breaks are found at very similar dates. The *p*-values from table 2 show we can reject the *DS* model only under the $\min t_{\hat{\alpha}}$ criterion.[15] However, the diagnostic tests in table 1 show significant serial correlation and non-normality in the residuals.

Table 2

Exact p Values of the Dickey-Fuller Statistic for Mexico's Real GDP (1921-1995)

Criterion	*p-value*			
	$m = 0$	$m = 1$	$m = 2$	$m = 3$
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$	0.881			
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$	0.584			
Min $t_{\hat{\alpha}}$				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.768	0.824	0.885
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.033	0.000	0.000
Min *RSS*				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.225	0.943	0.861
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.996	0.536	0.000
Max$\lvert t_{\hat{\gamma}}\rvert$				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.777	0.840	0.822
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.043	0.104	0.000

[15] The *t*-statistic for the unit root null of oversetwedge $\hat{\tau}_{sample}$ is equal to -7, and is significant at the 5% level according to the critical values supplied in Lumsdaine and Papell (1997) for their *'CC'* model.

For three breaks, $\hat{k}=3$ was selected by all three criteria, and nearly identical dates are found for the three identified breaks. Note that the second breaks is only in the slope of the trend, whilst the first and third breaks are in both level and trend. The diagnostic tests are not significant, as can be seen from table 1. The p-values from table 2 make clear that the sample values of $\hat{\tau}_{sample}$ under each criterion could hardly have been generated from the *DS* model. On the other hand, the p-values under the *TS* model indicate that there is no strong evidence against the *TS* specification.

Regarding real per-capita GDP, the empirical results are similar. For the no breaks case, we see from table 3 evidence of autocorrelation and non-normality in residuals, while table 4 reveals the impossibility of rejecting either the *DS* or the *TS* specifications. Allowing for a single break, it is only under the $\min t_{\hat{\alpha}}$ criterion that we can reject the *DS* model at the 3.7% level, according to the p-values in table 4. Table 3 shows significant autocorrelation and non-normality in the residuals under both the $\min t_{\hat{\alpha}}$ and the $\max|t_{\hat{\gamma}}|$ criteria, and an explosive autoregressive root under the min *RSS* criterion. For the case $m = 2$, the p-values from table 4 indicate strong rejection of the *DS* model, under the $\min t_{\hat{\alpha}}$ and the $\max|t_{\hat{\gamma}}|$ criteria. However, there is also strong evidence of heteroscedasticity in residuals, and *all* the other diagnostic tests are only marginally non-significant. As with real GDP, allowing for three structural breaks results in a sharp rejection of the *DS* model in favor of the *TS* one, whilst the diagnostic tests are much more reasonable. The first and second breaks with real GDP coincide across all criteria and the third one lies around 1980.

Let us now turn to the application of the sup-Wald statistic (5) and the parameter constancy stopping rule on real GDP and real GDP per capita. Results are summarized in tables 5, 6 and 7. Table 5 reports the results of sequentially applying (5), as explained at the end of section 3. As mentioned earlier, perfect collinearity problems due to the presence of dummy variables in the model restrict the occurrence of a (single) break to the sample $k+3 \leq T_{b1} \leq T - 3$, and that is why we select π in (5) such that $\pi \times T_s = 3$, where T_s represents either the sample size, or the size of a subsample. For example, for the full sample of real GDP in the table, 1921-1995, we have 75 observations, and $\pi \times 75 = 3$ implies $\pi = 0.04$. For the subsample 1933-1995, there are 63 observations, which implies $\pi = 0.047$. As can be seen, the search for breaks uses all the available data in the sample or the relevant subsample, which implies that we

Table 3

Broken Trend Models for Mexico's Real Per-Capita GDP (1921-1994)

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU_{it} + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t$$

m	*Criterion*	$\hat{k}$	Tb_1	Tb_2	Tb_3	*Type of break*			*p-values* AC	N	H	$\hat{\sigma}_\varepsilon$	$\hat{\tau}_{sample}$
0	...	3							0.093	0.000	0.197	0.037938	-2.64
	Min $t_{\hat{\alpha}}$	4	1979			T			0.031	0.000	0.776	0.034016	-4.00
1	Min RSS	4	1932			L and T			0.401	0.770	0.152	0.029569	2.98
	Max $\|t_{\hat{\gamma}}\|$	4	1971			L and T			0.055	0.000	0.596	0.034134	-3.69
	Min $t_{\hat{\alpha}}$	3	1931			L and T			0.113	0.120	0.006	0.025250	-8.92
				1977			L and T						
2	Min RSS	4	1932			L and T			0.107	0.662	0.005	0.023986	-2.25
				1981			L and T						
	Max $\|t_{\hat{\gamma}}\|$	3	1931			L and T			0.113	0.120	0.006	0.025250	-8.92
				1977			L and T						
			1931			L and T							
	Min $t_{\hat{\alpha}}$	3		1953			T		0.572	0.782	0.042	0.021537	-11.39
					1978			L and T					
			1931			L and T							
3	Min RSS	3		1953			T		0.269	0.771	0.155	0.021116	-11.84
					1981			L and T					
			1931			L and T							
	Max $\|t_{\hat{\gamma}}\|$	3		1953			T		0.251	0.784	0.182	0.021287	-11.32
					1980			L and T					

Table 4
Exact p Values of the Dickey-Fuller Statistic for Mexico's Real Per-Capita GDP (1921-1994)

Criterion	p-value			
	$m = 0$	$m = 1$	$m = 2$	$m = 3$
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$	0.833			
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$	0.270			
Min $t_{\hat{\alpha}}$				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.734	0.812	0.821
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.037	0.000	0.000
Min *RSS*				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.447	0.790	0.785
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.997	0.558	0.000
Max $\lvert t_{\hat{\gamma}} \rvert$				
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$		0.782	0.812	0.849
$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$		0.126	0.000	0.000

have no prior information on the time of possible structural change, as noted in Andrews (1993).

For real GDP, the upper panel of table 5 shows the results from applying the sup-Wald statistic. Over the entire sample (1921-1995) a significant break (with changes in both level and slope of trend) is identified in 1932. Upon dividing the sample into two subsamples separated by this break, only one additional break is identified in 1981 for the subsample 1933-1995. Looking for further breaks in the resulting three subsamples yields no additional breaks. The table also shows that these dates are actually confirmed by the refinement process. The table shows a tight 95% confidence interval only for the first break date.[16] For real per-capita GDP the lower panel of table 5 shows the four confirmed break dates. Note that the first three break dates are very close to those obtained under the 'unit root rejection' stopping rule (see table 1). Note also that the one found in 1981 has a very large confidence interval.

Table 6 reports the estimated regressions accounting for the identified breaks, together with diagnostic tests, while table 7 reports the p-values resulting from the simulation of the distribution of the t-statistic for the null hypothesis of a unit root ($\alpha = 0$ in (3)) under the hypotheses that the true models are the *TS* model (3) and the *DS* model (2), both

[16] See Bai (1997b) for details on the construction of the confidence intervals.

Table 5
Test Statistics, Break Points, and Confidence Intervals

Serie	*Type of break**	*Sample*	*Sup-Wald*	$\hat{T}_b$	π	*Critical Value*	*95% Confidence Interval*
RGDP	*L* and *T*	1921-1995	42.57	1932	0.04	24.59	
	T	1921-1932	20.75	1927	0.25	24.66	
	L and *T*	1933-1995	32.77	1981	0.05	15.94	
	L and *T*	1933-1981	12.92	1953	0.06	22.56	
	L	1982-1995	4.88	1989	0.21	15.10	
			With Refinement				
	L and *T*	1921-1981	33.14	1932	0.05	23.69	[1931, 1933]
	L and *T*	1933-1995	32.77	1981	0.05	15.94	[1978, 1984]
			With Refinement				
RGDP p/c	Land *T*	1921-1953	36.05	1932	0.09	24.40	[1931, 1933]
	T	1933-1981	15.27	1953	0.06	11.50	[1951, 1955]
	T	1954-1985	23.69	1981	0.09	16.61	[1972, 1990]
	L	1982-1994	22.63	1985	0.23	16.88	[1984, 1986]

* *L* stands form level and *T* for trend. Critical values from a bootstrap experiment.

estimated from the data. As can be seen from table 7, it is only for real per-capita GDP that the unit root is (strongly) rejected, whilst the *TS* alternative (allowing for 4 breaks) can not be rejected. From table 6, the resulting estimated regressions show a marginal rejection of autocorrelation for real GDP, and strong evidence of non-normality and heteroscedasticity in the residuals for real GDP per-capita.

For real GDP, a comparison of the estimated standard errors in tables 1 (under the min *RSS* criterion, $m = 3$) and 6, reveals an increment of 21% from the former to the latter. Also, the four breaks model of table 6 for real per capita GDP reports a higher estimated standard error of regression than the 3 breaks model of table 3.

5. Conclusions

Our results support the view that the structure of the trend function determines whether cycles fluctuate stationarily or not. In particular, the empirical results based on the unit-root rejection stopping rule indicate that it is possible to separate a stationary cycle for Mexico's real and real per-capita production from a long-run trend with 3 structural

Table 6

Regression Results of Using the Sup-Wald Statistic

$$\Delta Y_t = \mu + \beta t + \sum_{i=1}^{m} \theta_i DU + \sum_{i=1}^{m} \gamma_i DT_{it} + \alpha Y_{t-1} + \sum_{i=1}^{k} a_i \Delta Y_{t-i} + \varepsilon_t$$

Serie	*m*	$\hat{k}$	T_{b_1}	T_{b_2}	T_{b_3}	T_{b_4}	*Type of break**	*p-values* *AC*	*N*	*H*	$\hat{\sigma}_\varepsilon$	$\hat{\tau}_{sample}$
RGDP	2	4										
			1932				*L* and *T*	0.100	0.441	0.437	0.027929	-2.34
				1981			*L* and *T*					
RGDP p/c	4	3										
			1932				*T*					
				1953			*T*	0.236	0.000	0.000	0.023056	-10.98
					1981		*T*					
						1985	*L*					

* *T* and *L* stands for level.

Table 7

Exact p Values of the Dickey-Fuller Statistic

Serie	*m*		*p value*
RGDP	2		
		$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$	0.943
		$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$	0.536
RGDP p/c	4		
		$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{TS}(\hat{\tau})]$	0.625
		$\Pr[\hat{\tau} \leq \hat{\tau}_{sample} \mid f_{DS}(\hat{\tau})]$	0.000

breaks. In other words, Mexico's real output has fluctuated stationarily around a 75 year long-run trend perturbed by three major events in or around 1931, 1950 and 1980. Specifically, the inclusion of an appropriate number of breaks makes it possible to end up with a separable stationary cycle. This implies that the appropriate answer to the question of whether the economy is stationary would be to ask for how long.

On the other hand, our results indicate that the application of the sup-Wald statistic coupled with the parameter-constancy stopping rule allows us to identify a stationary cycle only for real per-capita GDP allowing for four structural breaks. On the other hand, the two breaks identified for real GDP do not seem to be adequate. Furthermore, regression results from the application of this procedure are dominated by those obtained from the unit-root rejection stopping rule, which produce a better fit for each series. We present in figure 2 graphs of the natural logarithm of the series, together with the resulting identified broken trend functions, under the URR-SR, and the min *RSS* criterion.[17] The resulting cycles, shown in figures 3, are by construction stationary, and are the input for the relevant business cycle theory.

It has been argued that "subsequent literature should focus on model selection, in determining both the number of breaks and also the type of break" (Lumsdaine and Papell (1997), p. 218). We certainly support this vision as long as it is firmly assumed that this is only an empirical matter. However, we tend to think that economic theory should also have something to say about this. For instance, what sort of alterations in economic agents' parameters would produce (or would be responsible for) a growth path in real output subject to (infrequent) abrupt changes, and what type of parameters are these? This would represent an interesting and productive interaction between growth theory and time-series econometrics. Stock (1994) recognizes an undeveloped link between trend-break models and economic theory. This point was also raised by Perron (1989) and Rappoport and Reichlin (1989). Ina recent paper, Lau (1997) explores the time series properties of both exogenous and endogenous growth models, and finds that changes in economic fundamentals lead to the phenomenon of breaks in trends.

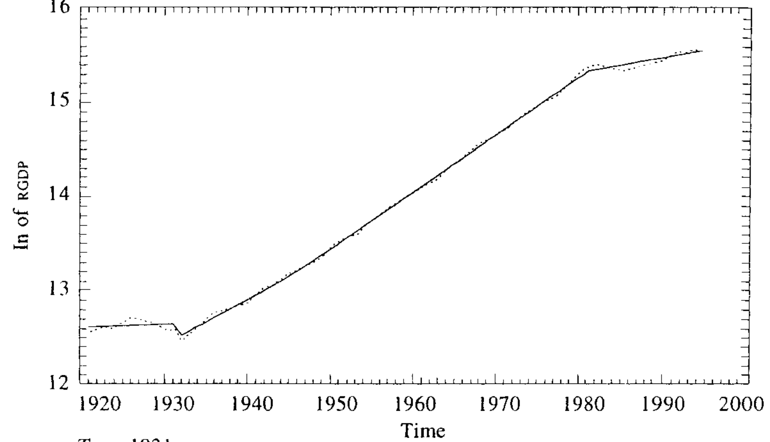
Finally, if by the inclusion of an appropriate number of breaks it were possible to reject the unit root hypothesis at a high level of confidence for any time series, then there would be no need for cointegration analysis. This means that a group of unit root-nonstationary variables do not move together in the long-run, but that all variables are stationary around broken trends, and that what is probably left to do is to test for co-trending, in order to identify long-term affinities among groups of stationary variables subject to individual, or most probably interrelated, infrequent, structural breaks.[18]

[17] Since the min *RSS* criterion ensures the best fitting model, we recomend this criterion over the other two. Furthermore, the min *RSS* criterion does not bias the results towards either rejecting the unit root (as the min $t_{\hat{\alpha}}$ does), or accepting a significant break point (as the max $|t_{\hat{\theta}}|$ or the max $|t_{\hat{\gamma}}|$ do).

[18] Campos, Ericsson and Hendry (1996) investigate the power of several cointegration tests when emph{one of the variables in the cointegratin relationship contains a structuralmedskip break.

Figure 2

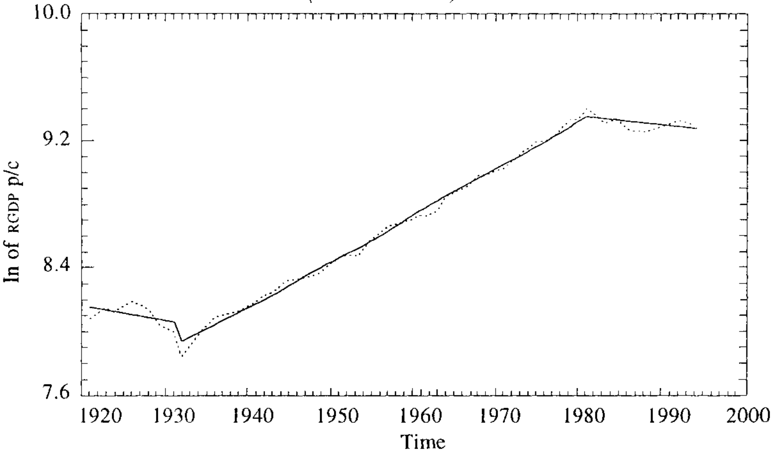
ln Mexico's Real Gross Domestic Product, 1921-1995
(1980 = 100)



T_{b_1} = 1931
T_{b_2} = 1948
T_{b_3} = 1980

ln of RGDP
Fitted trend

ln Mexico's Real Per-Capita Gross Domestic Product, 1921-1994
(1965 = 100)

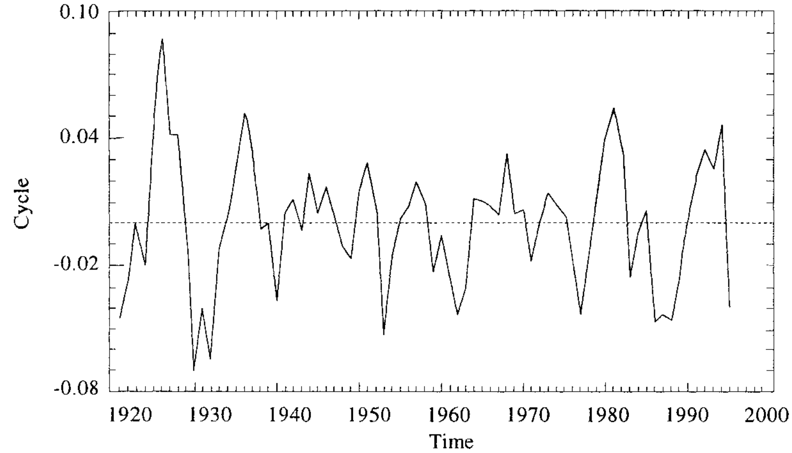


T_{b_1} = 1931
T_{b_2} = 1953
T_{b_3} = 1981

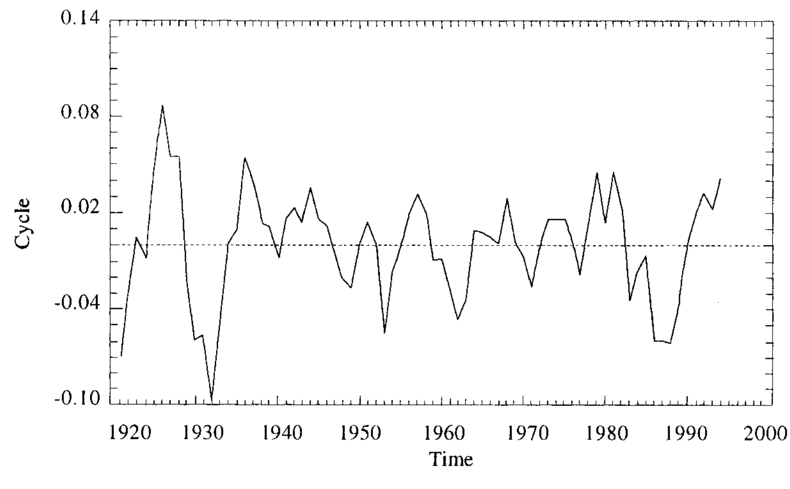
ln of RGDP
Fitted trend

Figure 3

Stationary Cycle of Mexico's Real Gross Domestic Product, 1921-1995



Stationary Cycle of Mexico's Real Per-Capita Gross Domestic Product, 1921-1994



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