

DISAGGREGATED ESTIMATES OF OUTPUT SUPPLY AND INPUT DEMAND ELASTICITIES*

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Resumen: Con el modelo de competencia perfecta, supuestos estadísticamente válidos para simplificar modelos y optimización en varias etapas, se especificaron modelos duales agregados para representar relaciones de producción bajo el contexto de producción múltiple en el estado de Texas. Se derivaron elasticidades precio directas y cruzadas a nivel desagregado para medir la respuesta a cambios en los precios, para 25 funciones individuales de oferta de cultivos y de ganado, y 6 funciones de demanda de insumos. Las elasticidades estimadas de producción y de demanda fueron en general inelásticas. Se encontró que los productos fueron principalmente sustitutos, mientras que los insumos fueron en general complementos.

Abstract: Designed for consistency with competitive theory, non-rejected simplifying assumptions and multi-stage choice, aggregate dual models are specified of Texas agricultural production. Disaggregated own- and cross-price elasticities are derived for 25 commodity supplies and six input demands. Estimated supplies and demands are largely inelastic. Outputs are mainly economic substitutes and inputs are economic complements.

* This material is based on work supported by the U.S. Department of Agriculture under agreement No. 58-3AEM-8-00104.

1. Introduction

Responsiveness of output supplies and input demands to changes in the economic and/or political environment is a common concern of much applied research in agricultural economics. Because of the large number of agricultural commodities produced and inputs used and because of the heterogeneity of production in most countries, complete output supply and input demand elasticity matrices can be derived only if estimation models can be simplified. Simplification in model specification is necessary to conserve degrees of freedom in estimation and reduce collinearity.

This study exploits the analytic simplification opportunities permitted when production data exhibit reasonable consistency with homothetic separability and/or nonjointness properties. The objective is to estimate a nearly complete matrix of commodity-level short-run output supply and input demand elasticities for Texas, a state which produces a large number of commercial agricultural products.

2. Empirical Models

2.1. Model Specification

Assuming that the collection of producers in the state behaves like a price-taking, profit maximizing firm with a state-level aggregate production function, the state was modeled as though it were a perfectly competitive firm. Based on the results of functional form tests conducted by Ornelas, Shumway, and Ozuna (1991) using U.S. agricultural data, the aggregate state-level restricted profit function was modeled using the normalized quadratic functional form. The estimation system consisted of the first-derivative output supply and input demand equations obtained by application of Hotelling's lemma:

$$x_i = b_i + \sum_{j=1}^m b_{ij} p_j + \sum_{j=m+1}^n b_{ij} z_j, \quad \text{for } i = 1, \dots, m, \quad (1)$$

where x_1, \dots, x_m are the netput quantities, positively measured for outputs and negatively measured for inputs, p_1, \dots, p_m are the output and variable input prices divided by the price of netput 0, z_{m+1}, \dots, z_n are

fixed input quantities and other non-price exogenous variables, and the numeraire equation:

$$x_0 = b_0 + \sum_{i=m+1}^n b_i z_i - 0.5 \left(\sum_{i=1}^m \sum_{j=1}^m b_{ij} p_i p_j \right) + 0.5 \left(\sum_{i=m+1}^n \sum_{j=m+1}^n b_{ij} z_i z_j \right), \quad (2)$$

which is a quadratic function in normalized prices and fixed inputs.

When the underlying technology is homothetically separable in a partition of variables, data within the partition can be consistently aggregated and consistent multi-stage choice can be conducted. Assuming the same functional form as for the aggregated model, the suboptimization (second stage) model consisted of the system of linear allocation equations:

$$x_{is} = b_{is} + \sum_{j=1}^m b_{ijs} p_{js} + \sum_{j=m+i}^n b_{ijs} z_{js} + c_{is} q_s, \quad \text{for } i = 1, \dots, m, \quad (3)$$

where x_{is}, \dots, x_{ms} are the allocation equations for the suboptimization model and the numeraire equation:

$$x_{0s} = b_{0s} + \sum_{i=m+1}^n b_{is} z_{is} - 0.5 \left(\sum_{i=1}^m \sum_{j=1}^m b_{ijs} p_{is} p_{js} \right) + 0.5 \left(\sum_{i=m+1}^n \sum_{j=m+1}^n b_{ijs} z_{is} z_{js} \right) + c_s q_s + \sum_{i=m+1}^n c_{is} z_{is} q_s + 0.5 (d_s q_s^2), \quad (4)$$

which is a quadratic function in the normalized prices, aggregate index, and other exogenous variables.

Third stage suboptimization models were formulated whenever the suboptimization model included an aggregate index among the normalized prices within the separable subset. These models were constructed following the pattern in (3) and (4).

2.2. Data and Variable Specification

Annual state-level data for the period 1951-1986 were used in this study. Output prices and quantities were obtained from the data set compiled by Evenson (1986) and associates at Yale University for the period 1951-1982, and updated to 1986 by McIntosh (1989a) at the University of Georgia. Pesticide price and quantity data were obtained from McGath

(1989) at the Economic Research Service. Sources on government policy and weather data were from McIntosh (1989b), and Teigen and Singer (1988), respectively.

Based on common non-rejected deterministic and stochastic non-parametric tests of separability using 1956-1982 data for this state (Lim and Shumway, 1992), the data on 25 outputs and six inputs were initially aggregated into four output categories (crops, meat animals, milk-poultry, and other livestock) and three variable input categories (labor-capital, materials, and pesticides). The meat animals category included cows and calves, hogs and pigs, and sheep and lambs. The milk-poultry category included milk, eggs, broilers, and turkeys. The other livestock category included all remaining commercial food animal commodities not included in the meat animal or milk-poultry aggregates. The labor-capital category included hired labor, machinery operating inputs, and capital services. The materials category included fertilizer and miscellaneous variable inputs. The Tornqvist index was utilized in aggregating all prices.

Effective diversion payments and effective support prices were specified following Houck and Ryan (1972). Guided by Lim's (1989) findings, one-year lagged output prices were used as the anticipated output market prices. Using a procedure adapted from Romain (1983), expected prices of farm program commodities were specified as weighted averages of the expected market price and effective support price. Weather variables were monthly averages of temperature and precipitation for critical growing months, weighted by cropland. Exogenous variables included in the models were expected output prices, current variable input prices, quantities of the fixed inputs (family labor and land), time (included as a proxy for disembodied technical change), temperature, precipitation, and effective diversion payments.

Exhaustive parametric tests for short-run non-jointness of output categories and homothetic separability were conducted by Villezca-Becerra (1991) using 1951-1982 data. Based on his non-rejected hypotheses, final aggregate short-run output supply equations for the crops and other livestock categories were specified here as functions only of their own prices, prices of variable inputs, and quantities of the non-price exogenous variables. No justification was found by this author for a higher level of data aggregation than maintained in the initial model design.

Second-stage suboptimization, utilizing corresponding price and quantity disaggregated data, was conducted for crops, meat animals, milk-poultry, materials, and labor-capital categories. Similarly, third-stage suboptimization models were specified for crop categories that had to be aggregated in the second stage due to the large number of individual crops. Non-price exogenous variables included in all suboptimization models were the same as in the aggregate models, except for land and family labor. For the multistage model structure, as in the case of the aggregate models, the data were aggregated into output and variable input categories based on the separability test results obtained by Lim and Shumway (1992). Since neither the weak separability tests conducted by them nor the homothetic separability tests conducted by Villezca-Becerra (1991) on the aggregate models included the non-price exogenous variables of temperature, precipitation, time, or effective diversion payments, these variables were included in all the multistage choice models.

2.3. Estimation Procedure

For the first-stage (aggregate) models, systems of four output supply equations (crops, meat animals, milk-poultry, and other livestock) and two input demand equations (materials and pesticides) were estimated as specified in (1). The capital-labor input price was used to normalize profit and all other output and variable input prices. Because of high collinearity, the quadratic numeraire equation (2) was not estimated as part of this system, but all of its price parameters can be derived from (1) by virtue of shared parameters and homogeneity restrictions.

Systems of output supplies and input demands estimated for the second-stage suboptimization (allocation) models, as specified in (3) and (4), are detailed in table 1. Because of the large number of crops, third-stage suboptimization models were estimated for three crop categories: feed and food grains, vegetables, and oil crops and cotton. Because of high collinearity in several models, parameters on the quadratic terms of the non-price exogenous variables were not estimated in any of the suboptimization models. This exclusion reduced the flexibility of the functional form used for the suboptimization models by imposing cross-equation restrictions on comparative statics among the fixed inputs at the point of approximation.

Table 1
*Output Supply and Input Demand Equations Estimated in Multistage
 Suboptimization Models*

<i>Second-Stage Allocation</i>		<i>Third-Stage Allocation</i>	
<i>Model</i>	<i>Equations</i>	<i>Model</i>	<i>Equations</i>
Crops	Feed and Food Grains (A) Oil Crops (A) Vegetables(A) Oranges Grapefruit Hay Other Crops (R) (N)	Feed-Food-Grains	Wheat Rice Corn Barley Sorghum Oats (N)
Meat Animals	Cattle Hogs Sheep (N)	Vegetables	Onions Lettuce Tomatoes Potatoes (N)
Milk-Poultry	Milk Eggs Broilers Turkeys (N)	Oil Crops-Cotton	Cotton Soybeans Peanuts (N)
Materials	Fertilizer Miscellaneous Variable Inputs (R) (N)		
Labor-Capital	Hired Labor Capital Services Machinery Operating (N)		

Codes: A is an aggregate category for which a higher-level allocation model is estimated. R is a residual aggregated category for which no further allocation can be estimated. N is the numeraire.

Error terms associated with each model were assumed to be additive, and independently and identically distributed with mean zero and a constant contemporaneous covariance matrix. The covariance matrix used to transform the observation matrix was obtained by using the iterative version of Zellner's seemingly unrelated regression (ITSUR).

Using the procedure `SYSNLIN` `ITSUR` in the *SAS* package (1984), the variance-covariance matrix was iterated until it stabilized for each model. Imposition of the non-linear inequality restrictions for maintaining convexity was accomplished by the Cholesky factorization. With the convexity restrictions imposed and using the observation matrix transformed by the iterated covariance matrix, a reduced gradient nonlinear program (Talpez, Alexander, and Shumway, 1989) was employed using the algorithm code *MINOS 5.1* (Murtagh and Saunders, 1983) to obtain least squares estimates that satisfied curvature properties for each system of output supply and input demand equations. Model estimates were obtained subject to homogeneity, symmetry, and convexity in prices and also subject to non-rejected short-run non-jointness hypotheses. Monotonicity, the final property implied by price-taking, profit-maximizing behavior, was not maintained but was checked at each observation.

3. Results

Summary statistics for the aggregate and each suboptimization model are reported in table 2. A .05 level of significance was used throughout this study in drawing conclusions from hypothesis tests. Curvature properties were tested against the non-convex alternative and were not significantly violated for any of the aggregate or suboptimization models. Violations of monotonicity were observed for only two suboptimization models. The five significant monotonicity violations for the oil crops model all occurred early in the data period. This one set of model estimates significantly violated the implications of the competitive theory for individual firms for early observations, but not at the means or at recent observations.

Given the model specification, the number of significant parameter estimates varied from 27 percent in the milk-poultry suboptimization model to 67 percent in the labor-capital suboptimization model. Across all models, the proportion of significant parameter estimates was 40 percent.

Disaggregated price elasticities were computed from these multi-stage model estimates at the most recent observation (1986). The elasticities for individual commodities and inputs were derived from

Table 2
Summary Statistics of the Multistage Model Parameter Estimates

<i>Model</i>	<i>Convexity, F-Statistic</i>	<i>Monotonicity</i>		<i>Percent of Significant Parameters, .05 Level</i>
		<i>Number of Violations^a</i>	<i>χ^2 Statistic</i>	
Aggregate	0.29	0		39.7
Crops	0.57	0		30.5
Meat Animals	0.38	0		38.9
Milk-Poultry	0.11	0		26.9
Materials	^b	0		45.5
Labor-Capital	^b	0		66.7
Feed and				
Food Grains	0.76	1	2.68	37.3
Vegetables	0.04	0		53.8
Oil Crops	0.0002	5	29.49*	52.6

* Significant at .05 level.

^a Number of violations of monotonicity from a possible total of 36 x number of equations estimated in the respective model.

^b Unconstrained estimates satisfied convexity restrictions.

equations (1), (3) and (4) by applying the chain rule of calculus and are reported in table 3 for crop supply elasticities and in table 4 for livestock supply and input demand elasticities. Because of the large number of commercial agricultural outputs produced in this state, the supply elasticities reported here are the most detailed and comprehensive to appear in economic literature. Without the ability to do multistage modeling, it would have been impossible to estimate cross-price elasticities for such a large number of commodities from these data.¹

¹ It would be possible to estimate all cross-price elasticities by a single model if the time series data could be pooled across states. A sufficient condition for pooling the data is identical technologies across the pooled states. Although not tested here, this hypothesis was rejected by Poison and Shumway (1990) for all pairs of states in two contiguous production regions.

Table 3
Disaggregated Crop Supply Elasticities, 1986

Elasticity with Respect to the Price of:

Quantity	Wheat	Rice	Corn	Barley	Sorghum	Oats	Soybeans	Peanuts	Hay	Onions	Let-tuce	Tomatoe	Potatoe	Oran-ges	Grape-fruit	Other Crops	Misc. Inputs	Pesti-cides	Hired Labor	Cap-i-tal Serv.	Mach. Oper.
Wheat	0.190	0.052	0.158	0.002	0.220	0.009	-0.016	-0.058	-0.376	0.023	-0.001	-0.001	-0.010	-0.003	-0.161	0.003	0.001	0.001	0.000	-0.001	-0.002
Rice	0.094	0.187	-0.211	0.002	0.122	0.099	-0.008	-0.027	-0.175	0.011	-0.001	-0.005	-0.014	-0.075	0.003	0.001	0.000	0.000	-0.001	-0.001	-0.001
Corn	0.160	-0.049	0.370	-0.021	0.177	-0.094	-0.014	-0.050	-0.323	0.020	-0.001	-0.009	-0.026	-0.139	0.003	0.003	-0.001	-0.002	-0.001	-0.002	-0.001
Barley	0.178	0.092	-2.065	3.949	-1.939	0.354	-0.015	-0.052	-0.338	0.021	-0.001	-0.001	-0.009	-0.027	-0.145	0.003	-0.001	-0.002	-0.002	-0.002	-0.002
Sorghum	a	0.005	-0.009	-0.016	0.011	0.009															
Oats	0.730	0.953	-1.893	0.092	1.106	1.341	-0.061	-0.214	-1.385	0.086	-0.004	-0.002	-0.038	-0.110	-0.595	0.001	0.011	-0.004	-0.007	-0.006	-0.006
Soybeans	-0.212	-0.056	-0.179	-0.002	-0.245	-0.009	0.059	0.265	0.521	-0.036	-0.038	-0.003	-0.004	-0.018	0.026	0.038	-0.103	0.001	0.010	-0.003	-0.007
Peanuts	-0.033	-0.009	-0.028	-0.039	-0.001	0.039	0.206	-0.111	-0.006	-0.006	-0.001	-0.003	0.004	0.006	-0.016	0.002	-0.001	-0.001	-0.001	-0.001	-0.001
Cotton	-0.162	-0.043	-0.137	-0.002	-0.188	-0.007	0.016	-0.042	0.590	-0.028	-0.029	-0.002	-0.003	-0.014	0.020	0.029	-0.079	0.001	0.007	-0.003	-0.005
Hay	0.098	0.026	0.083	0.001	0.113	0.004	-0.012	-0.042	-0.270	0.015	0.014	0.001	0.001	0.007	-0.008	-0.016	-0.014	0.001	0.005	-0.002	-0.004
Onions	-0.003	-0.001	-0.003	-0.004	-0.004	-0.008	-0.030	-0.191	0.010	0.201	-0.048	-0.004	-0.053	0.001	0.135	0.001	0.135	0.001	0.005	-0.002	-0.004
Let-tuce	-0.010	-0.003	-0.008	0.000	-0.011	-0.024	-0.083	-0.537	0.027	-0.531	0.340	0.141	0.321	-0.001	0.001	0.378	0.001	0.001	0.001	0.001	0.001
Tomatoes																					
Potatoes	-0.005	-0.001	-0.004	-0.006	-0.006	-0.012	-0.043	-0.277	0.014	-0.152	0.061	-0.016	0.246	-0.001	0.001	0.195	0.003	0.033	-0.011	-0.022	-0.018
Oranges	-0.901	-0.240	-0.763	-0.009	-1.042	-0.039	0.178	0.628	4.068	-0.166	-0.014	-0.001	-0.006	0.260	0.201	-2.137	0.003	0.033	-0.011	-0.022	-0.018
Grapefruit	-2.527	-0.674	-2.140	-0.024	-2.924	-0.109	0.246	0.866	5.612	-0.324	0.016	0.001	0.007	0.196	0.415	1.369	0.002	0.017	-0.006	-0.011	-0.009
Other Crops	-0.038	-0.010	-0.032	-0.044	-0.002	-0.002	-0.007	-0.043	-0.001	0.011	0.001	0.001	0.005	-0.006	0.084	0.164	0.001	0.001	-0.001	-0.001	-0.001

^a Blank if elasticity was zero to third decimal place. Column omitted if all crop elasticities were zero to third decimal place.

Table 4
Disaggregated Input Demand and Livestock Supply Elasticities, 1986

Elasticity with Respect to the Price of:

Quantity	Wheat	Rice	Corn	Sorghum	Peanuts	Cotton	Hay	Onions	Crops	Other	Cattle	Hogs	Sheep	Milk	Eggs	Broilers	Turkeys	Fertilizer	Misc. Inputs	Pesticides	Hired Labor	Capital	Machinery	
Cattle											-0.10	0.13	-0.003	-0.001	-0.001	-0.001		0.001	0.005	0.001	-0.001	-0.001	-0.002	-0.002
Hogs											0.002	-0.004	0.001											
Sheep																								
Milk											-0.02													
Eggs											-0.002													
Broilers											-0.010													
Turkeys																								
Other																								
Livestock																								
Fertilizer											-0.04													
Misc. Inputs											-0.005													
Pesticides											-0.004	-0.004	-0.005	-0.001	-0.001	-0.001	-0.001	-0.009	-0.001	-0.001	-0.001	-0.001	-0.001	-0.001
Hired Labor											0.001													
Capital																								
Services																								
Machinery																								
Operating																								

^a Blank if elasticity was zero to third decimal place. Column omitted if all livestock and input elasticities were zero to third decimal place.

Nearly all own-price supply and input demand elasticities were inelastic. The only exceptions were for two minor crops, barley and oats, and for hired labor and machinery operating inputs. More than half had elasticities smaller than 0.2. With very few exceptions, cross-price output supply elasticities were also highly inelastic. The finding of largely inelastic short-run output supply and input demand elasticities was consistent with much prior literature at the state, regional, and national levels (e.g., Shumway, 1983; Antle, 1984; Vasavada and Chambers, 1986; Shumway and Alexander, 1988, and Huffman and Evenson, 1989. Few (e.g., Weaver, 1983, and Ball, 1988), have estimated elastic short-run responses for a large portion of agricultural output supplies and input demands. Nevertheless, these estimates are striking in comparison to earlier work by the degree of inelasticity.

Also consistent with much prior literature, the signs of the cross-price elasticities indicated a wide range of competitive and complementary production relationships. Output supplies exhibited far more competitive than complementary relationships. Two-thirds of the crop interrelationships were competitive, and four-fifths of the livestock interrelationships were competitive. Complementary crop relationships were most evident among the feed and food grains; among the oil crops and cotton; and between fruits, vegetables, and a variety of other crops. Complementary livestock relationships were evident only between cattle and sheep, milk and turkeys, eggs and broilers, and eggs and turkeys.

While outputs were largely gross substitutes, inputs exhibited mainly economic complementarity. Only the relationships between pesticides and fertilizer and pesticides and miscellaneous inputs were competitive.

4. Conclusions

Disaggregated parameter estimates for multiple-output production relationships in Texas were derived from dual models consistent with competitive theory, non-rejected analytic simplifying assumptions (non-jointness), and multi-stage choice (homothetic separability). Linear homogeneity, symmetry, and convexity restrictions were maintained in the estimation. Monotonicity was checked at every observation and was

significantly violated by only one of the nine models estimated. Convexity was also tested and was not rejected by any model.

The multistage parameter estimates were utilized to derive a full matrix of disaggregated elasticities. By exploiting the ability to perform multistage modeling, these elasticities were computed at the most detailed and comprehensive level to appear in economic literature.

A wide diversity among output supply and input demand elasticities was observed. Most elasticities were inelastic, and many were very small in absolute magnitude. More than two-thirds of the outputs exhibited a competitive economic interrelationship while seven-eighths of the inputs were economic complements.

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