MARKET EFFICIENCY, PURCHASING POWER PARITY AND COINTEGRATION IN CENTRAL AMERICAN BLACK FOREIGN EXCHANGE MARKETS

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Resumen: La suposición de que la tasa cambiaria en el mercado negro sigue una senda aleatoria, y/o está determinada por la condición de paridad en el poder de compra, es comúnmente impuesta en modelos macroeconómicos. Las pruebas típicas de eficiencia en el mercado negro, en su forma débil, tienden a no ser concluyentes cuando se aplican a series de tiempo no estacionarias. Este artículo las compara con los resultados concluyentes que resultan de aplicar la teoría de la cointegración para estudiar la eficiencia de mercado y la condición de paridad de compra en el caso de Costa Rica, El Salvador y Guatemala.

Abstract: The assumption that black market exchange rates follow a random walk and/or are determined by the purchasing power parity condition is commonly imposed in macroeconomic models. The results from standard tests of black market efficiency, under the weak form, are not conclusive without any ambiguity whenever they are applied to nonstationary time series. This paper compares then with the unambiguous results arising from the application of the theory of cointegration to the study of market efficiency and purchasing power parity in the black market exchange rates of Costa Rica, El Salvador and Guatemala.

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Is the black market exchange rate (BMER) determined in an efficient market, where the past rate (or any other selected variable) is an unbiased predictor of the current rate? The question seems on the surface to be almost superfluous, since BMERs, as any other asset price, should probably react to fundamental macroeconomic variables, to expectations associated to their past, current and future behavior, to the extent of credibility of economic policy and to political events. In a general sense, if the market is called efficient, their participants are assumed to be rational, that is to fully assimilate available information in the current information set on the predictor variable, which already incorporates all available past information, and the only events which alter this path are unpredicted “news” following a random process. Thus, efficiency implies that no supranormal rate of return is obtained by investors, since the probability that the future BMER will be higher or lower than the rate prevailing today is equally likely, thus reflecting the unpredictable nature of efficient markets and facilitating standard traditional testing of market efficiency.

Black markets in developing-country foreign currencies have flourished in the 1980s as a result of quantitative restrictions placed on the foreign sector by macroeconomic imbalances and debt-service constraints. The importance of these markets in sustaining legal and illegal foreign trade operations and capital flight has been increasingly recognized. It may be argued that black markets are not efficient because information about prices and market participants is often imperfect, markets tend to be segmented, the number of participants is currently small and transaction costs, measured by the differential between the black and official rates (premium), are high and sometimes increased by penalty costs. Nevertheless, as Fama (1970) argues, free availability of information, a large number of transactors and zero transaction costs are sufficient, but not necessary, conditions for an efficient price adjustment. Transactions costs not included in price

1 There is no straightforward method of estimation of the extent of black foreign exchange operations. The popular method of estimating the volume of over-/under-invoicing of imports and exports developed by Mansur (1983) does not take into account capital inflows/outflows. As a consequence, Cuddington (1987) develops a methodology to estimate capital flight, while López and Seligson (1990) propose two complementary demand/supply approaches to estimate the amount of a particular source of private capital inflows: Remittances. Another gap concerning quantitative estimates of black foreign exchange operations is the lack of appropriated indicators to measure the extent of leakages between legal and illegal foreign trade operations. Variations in this ratio might become pervasive against attempts by authorities to control speculation against the domestic currency and to forecast their foreign trade tax income.
and inhibiting transactions to take place, a “sufficient” number of investors who has not ready access to available information and non-cooperative behavior among investors about their feedback trading strategies to a given set of information are only potential sources of market inefficiency.

There are three standard versions of the efficient market hypothesis (EMH) applied to exchange rates:

i) The weak form asserts that exchange rates follow a random walk, that is the sequence of past rates contains no information which can be used to forecast the future rate and this sequence is summarized in the current price.

ii) The semi-strong form states that exchange rates adjust to all (or partial) publicly available information. It has been divided by Geweke and Feige (1979) into two further categories:

a) Single market efficiency which takes into account publicly available information which is important for single exchange rate.

b) Multimarket efficiency which also takes into account available information on other relevant exchange rates.

iii) The strong form sustains that public and private information is fully reflected in exchange rate adjustments.

We associate the standard concept of purchasing power parity (PPP) to a particular formulation of single semi-strong market efficiency, for the publicly available information set is summarized in the current price ratio differential between domestic and foreign (bilateral or composite) price index, without taking into account information on other relevant BMERs. We call it “efficient” as PPP assumes that BMERs adjustments to price-ratio changes operate simultaneously and in the same proportion during the period under consideration. Another example of single semi-strong market efficiency consists of replacing the price-ratio by the official exchange rate (E). On the other hand, an example of multimarket semi-strong efficiency is the relationship between bilateral BMERs.

Applied to the black foreign exchange market, the three approaches to the EMH can be better understood by describing their general form through the rational expectation model contained in equation (1):

\[ B_t = \alpha E[B_t | I_{t-1}] + \beta Z_t + u_t, \]

where \( B_t \) is the BMER (expressed in units of domestic currency for one dollar) at time \( t \), \( Z_t \) is an index vector of fundamentals assumed as determinants of the BMER at time \( t \), \( E[B_t | I_{t-1}] \) is the expected BMER at time \( t \) according to the information set available at time \( t - 1 \), \( \alpha \) and \( \beta \) are respectively the coefficient and the vector of coefficients reflecting the responsiveness of the current BMER to changes in the expected BMER and the fundamentals and, for
simplicity, let's assume that \( u_t \) represents the summary of all exogenous unpredicted random factors ("news") occurring at time \( t \).

Weak market efficiency implies \( \alpha = 1, \beta = 0 \) and \( E[B_t \mid I_{t-1}] = B_{t-1} \). Furthermore, it is called the naive solution to the rational expectations problem. The naive solution is also called a random walk.\(^2\) In this model, the information set is composed by past BMERs, \( I_{t-1} = \{B_{t-1}, B_{t-2}, \ldots \} \). In general, a necessary condition for weak market efficiency is that \( u_t \) be white noise.\(^3\)

If multimarket semi-strong efficiency between bilateral BMERs, \( B_i \) and \( B_j \) (for country \( i \neq \) country \( j \)) holds, then \( \alpha = 0, \beta = 1 \) and \( Z_t = B_{jt} \). Besides, if PPP holds, then single market semi-strong efficiency assumes \( \alpha = 0, \beta = 1 \) and \( Z_t = P_t^i / P_t^* \), where \( P_t^i / P_t^* \) are the domestic to foreign price index ratio at time \( t \), reflecting the PPP condition. The same conditions hold for the case where \( P_t^* \) is replaced by the official rate \( E_r \). In this paper, cointegration tests are implemented to analyze particular forms of semi-strong market efficiency, while traditional and unit root tests take care of the EMH in its weak form. No test of the strong form of the EMH is performed at all. The information set for other semi-strong forms should include alternative sets of significant publicly available fundamental variables,\(^4\) while the information set for the strong-form should add private information to it.

From an economic policy point of view, the relevant issue at testing the efficiency of a BMER in its weak form is to determine if a random walk model explain its dynamics. If this is not the case, and unusual profit opportunities are not eliminated, showing that the market exhibit some degree of inefficiency, then there would be a case for Central Banks' intervention in the foreign exchange market and for theory to search for fundamentals in BMER determination. Unfortunately, a frequent problem related to traditional weak-form tests is that they can not discriminate between exchange rate movements caused by speculators and changes resulting form official intervention in the foreign exchange market by monetary authorities. Central Banks' intervention often takes two mechanisms: An increasingly reduced rationing in hard currency availability to external operations or a devaluation of the official exchange rate. In practice, data on the timing and the volume of official interventions, particularly those related to the rationing mechanism, are

\(^2\) Random walks are examples of Martingales. The stochastic process of the variable \( B_t \) is a Martingale if: i) \( E[B_t \mid I_{t-1}] = B_{t-1} \) and ii) \( E[B_{t+1} \mid I_t] = B_t \). For \( j > 0 \). For a formal proof showing that random walks are examples of Martingales, see Broze (1991, p. 9). Notice that if \( \alpha < 1 \) in equation (1), we have an autoregressive (stationary) process; while if \( \alpha > 1 \), we have an explosive process, currently associated to the presence of bubbles.

\(^3\) The sequence \( u_t \) is defined as a white noise if: i) \( E(u_t) = 0 \), for all \( t \); ii) \( E(u_t \mid u_s) = 0 \), for \( t \neq s \); and iii) \( E(u_t^2) = \sigma_u^2 \), for all \( t \).

\(^4\) In so doing, we enter the controversial field of black exchange rate determination.
generally unknown; therefore, it becomes extremely difficult to verify if black market inefficiency arises from destabilising speculation or from corrective actions taken by monetary authorities to move the official exchange rate back to equilibrium in real terms.

Semi-strong market efficiency between bilateral BMERs is a key indicator of the implicit degree of coordination existing between private dealers and, eventually, official authorities who intervene in the black market of each country. If monetary authorities in two countries implicitly or explicitly link their official exchange rate to the black rate and both rates appear to be cointegrated, then it can be said that there is room for facilitating exchange rate policy coordination between two countries.

On the other hand, the importance of testing PPP is multiple: First, it facilitates to evaluate if the BMER, as assumed proxy of the equilibrium real exchange rate, reflects the external competitiveness of a country; second, it assesses the velocity of BMER adjustments to price differential changes and suggests the eventual presence of BMER or domestic price stickiness; third, it contributes to evaluate if it makes sense that the bilateral price ratio remain as key policy indicator of official real exchange rate misalignment on which Central Banks justify their official interventions in the foreign exchange market; and fourth, it questions the use of the PPP ratio as the only variable on which the official exchange rate feedback rule is built in crawling-peg regimes. Since the mid-eighties, Costa Rica has explicitly recognized the use of the PPP criteria in its crawling-peg regime, while El Salvador has explicitly adopted a positive real exchange rate (in PPP terms) as a key component of its trade policy. Furthermore, both policies, implicitly (in the case of Costa Rica) or explicitly (in the case of El Salvador) look for bringing the premium close to a low and constant range. As a result, it becomes necessary to examine if a long-run statistical relationship exists between the official and black rates. Even though official exchange rates do not follow the PPP rule, they might be more successful at following a "constant narrow premium" rule.

Standard traditional testing of market efficiency (weak form) is based on regression functions of the random walk model and on non-parametric tests like runs analysis. Traditional testing of PPP is based on the study of deviations of the exchange rate from the bilateral price ratio. A recently developed method of testing particular forms of semi-strong market efficiency has been developed based on the theory of cointegration. Its application is particularly well suited to address joint issues. For example, BMERs are recognized to be highly sensitive to expectations regarding future events. So, in periods when expectations are

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\[5\] A discussion of specific conditions under which a BMER can be considered as an adequate proxy of the equilibrium real exchange rate is found in Dacy (1986), with an application to the Vietnamese economy.
continually revised, because of persistent flows of "news", exchange rates may be highly volatile; thus resulting in large and unpredicted exchange rate changes which may reinforce the efficient market view in its weak form and, if prices are less responsive to "news" than BMERs are, simultaneously show temporary sharp deviations from PPP.

In this paper, the EMH and the PPP condition are tested on the black foreign exchange markets of three Central American countries: Costa Rica, El Salvador and Guatemala. The paper is organized as follows. Section 2 develops a brief survey of the literature on the EMH and PPP applied to BMERs. Section 3 introduces the notion of cointegration, its formal definition and its relationship with the concept of market efficiency. Section 4 explains the methodology to be applied for testing, where traditional methods are distinguish from recent cointegration methods. Data sources and definitions are provided by Section 5, while empirical results are given by Section 6. Conclusions and further extensions of research are summarized in Section 7.

2. A Brief Review of the Literature

There exists considerable literature on the EMH applied to foreign exchange markets, with most of empirical work supporting weak form efficiency in most of developed country currencies. Nevertheless, literature on the efficiency of black foreign exchange markets is scarce. In a seminal study, Guiddy (1978) examines weekly changes in the black market premium for Brazil, Peru and Israel by using Box-Jenkins techniques in the sequence of first differences of black market premia and find evidence of efficiency in the weak-form sense for all three markets. In the same year, Fishelson (1978) analyzes whether changes in spot rates of the British pound, the German mark and the Swiss franc in the European markets are transmitted as "news" to the Israeli black market for dollars. His results are supportive of perfect arbitrage in the black market, i.e. a one percent increase in the price of the British pound in terms of the U.S. dollar raises with some lag the price of the British pound denominated in Israeli pounds by one percent. The presence of significant first-order serial correlation in these results is explained by Fishelson in terms of the lack of a forward black market for dollars and speculative caution on the part of traders.

6 Perhaps the most comprehensive treatment of this literature is found in Levich (1978, 1979 and 1985) and Frankel and Meese (1987).

7 The essential technique consists in pre-fitting an autoregressive integrated moving-average (ARIMA) process so as to "whiten" the residuals.
Gupta (1981a, b) combines parametric methods (autocorrelation functions and cross-correlation tests) with non-parametric methods (runs analysis and Alexandrian trading filter rules) to test for the weak form market efficiency on the first differences of natural logarithms of the monthly and weekly black exchange rate series of India, South Korea and Taiwan. Only the test using filter rules provides some evidence of market inefficiency, but when account is taken of transaction costs, it also becomes difficult to reject the EMH. More recent work by Culbertson (1989) goes back to traditional tests of the EMH (weak-form) for ten black foreign exchange markets in developing economies, four of them from Latin America, finding evidence that the behavior of BMRs in those countries were consistent with the implications of the EMH. Interestingly, he also shows that BMRs for those countries are largely determined by the same set of economic forces which determine the unobserved official equilibrium nominal exchange rate and that this set of fundamentals is summarized, in part, by the PPP relationship. Therefore, policy and structural changes which affect the official equilibrium rate, also affect the ratio between domestic and foreign prices and if these changes are of a mainly monetary nature, the association between the price ratio (as a proxy for the unobserved equilibrium official exchange rate) and the BMR will be particularly strong.

Koveos and Seifert (1985 and 1986) were the first to explicitly examine BMR deviations from PPP under the efficient hypothesis framework. They analyze the eventual existence of lead-lag relationships in monthly deviations between changes in BMRs and bilateral price ratios for a group of ten Latin American countries, among them El Salvador, during the period of April 1973 to March 1983. Their study of cross autocorrelation functions on pre-whitened identified series of such deviations, through the use of filters prescribed by Box-Jenkins techniques, find evidence supporting that the

8 Notice that in Gupta's test, transaction costs act like corrective tools of market inefficiency.
9 The countries are: Argentina, Bolivia, Brazil, Chile, India, Indonesia, Pakistan, Philippines, Turkey and Yugoslavia.
10 In practice, the unobserved equilibrium nominal official exchange rate is defined as the nominal rate who correspond to the equilibrium real exchange rate (RER) derived by the PPP condition. This is done with respect to a base period (usually a year or a month) where RER = 100. The RER index sometimes takes the following inversed price index ratio (not used here) form: \( RER = E_{t}P_{e} / P_{t} \), where \( E \) is the actual nominal official exchange rate at time \( t \). For a description of common usage attached to both RER definitions, see Edwards (1989, p. 11).
12 The ten countries are: Argentina, Brazil, Bolivia, Chile, Colombia, Ecuador, El Salvador, Mexico, Peru and Venezuela.
current rate of change of the PPP ratio can be considered as a good predictor of the current rate of change of BMERs.

Koveos and Seifert’s results address two central questions previously examined for developed foreign exchange markets. First, the controversy originating from Rogalski and Vinso (1977) between an “efficient” PPP simultaneous adjustment mechanism towards the equilibrium exchange rate and a monetary-PPP adjustment mechanism, which assumes the existence of leads or lags in such an adjustment which are consistent with monetary disequilibria. Rogalski and Vinso’s findings conclude that foreign exchange rates react as rapidly as prices, thus supporting the “efficient” PPP view. Nevertheless, their evidence contrast to Frenkel’s results (1983) who finds that the prices of nondurables and services exhibit some stickiness and are less responsive than exchange rates to news. Therefore, aggregate price indexes are less volatile than exchange rates inducing deviations from purchasing power parity. In addition, the former-ones reflect primarily current and past information while the latter-ones reflect public expectations vis-a-vis future events. Daniel (1986) also presents empirical evidence strongly indicating that “news” is instantaneously incorporated into exchange rates, but only slowly incorporated into prices. The second question addressed by Koveos and Seifert was first suggested by Mussa (1982) who argues that PPP is more likely to be satisfied under a fixed exchange rate regime than under a floating exchange rate regime. Now, it is recognized that the generalized floating of exchange rates has engendered substantial volatility in both nominal and real exchange rates for developed and developing countries, but inflation rates have not shown such degree of volatility.13 In fact, even some Koveos and Seifert’s results show that changes in inflation differentials do not carry any consistent information about movements of deviations of BMERs from PPP and, in most cases, the deviation time-serie leads inflation.14

Koveos and Seifert’s findings contrast to more pessimistic results for PPP testing obtained by Edwards (1989) for 28 developing economies,15 among them again El Salvador, although for quarterly data during the period

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13 Evidence on developing countries for this assertion is found in Brodsky, Helleiner and Sampson (1981) and Rana (1983).
14 See Koveos and Seifert (1986, pp. 321-322). Remember that cross-autocorrelation tests have the disadvantage of giving no indication about the causal direction of the relationship.
15 The 28 countries examined are: Bolivia, Brazil, Chile, Colombia, Cyprus, Dominican Republic, Ecuador, El Salvador, Ethiopia, Greece, India, Israel, Kenya, Korea, Malaysia, Mexico, Pakistan, Paraguay, Peru, Philippines, Singapore, South Africa, Sri Lanka, Thailand, Tunisia, Turkey, Yugoslavia and Zambia.
1965-1985. He searches if official and "parallel" exchange rate series behave accordingly to the absolute version of PPP theory and concludes that they fail miserably.\textsuperscript{16, 17}

The theory of cointegration has early been used to test PPP in official exchange rates of developed countries. Baillie and Selover (1987) estimate by OLS absolute PPP with drift as a cointegrating equation in five countries: United Kingdom, Japan, West Germany, Canada and France. Their analysis uses monthly data from March 1973 to December 1983 and perform Dickey-Fuller (DF) tests for the presence of unit roots in the estimated residuals. Their results show that the null hypothesis of residuals being non stationary can only be rejected for France, implying that no cointegration relationship can be found for the other four countries.\textsuperscript{18} In a similar vein, Layton and Stark (1990) test absolute PPP for the United States and get little empirical support for the cointegration of the U.S. inflation rate and effective exchange rate adjusted inflation series computed from its six major trading partners. Furthermore, in no instance whether the data refer to a fixed or floating exchange rate period, does the cointegration test find any cointegration relationship. Other examples of cointegration tests applied to PPP in developed countries include Johnson (1990), MacNown and Wallace (1990) and Johansen (1992).

Cointegration tests have only recently been applied to black foreign exchange markets in developing countries. In a rare effort, Cáceres and Núñez (1991) develop two tests of the EMH in the black foreign exchange markets of El Salvador and Guatemala. Their period under study is August 1982-December 1989 using monthly observations. In the first test, they find that both markets are weak inefficient, for both rates are reintegrated. Nevertheless, it is now recognized that two exchange rates may be jointly cointegrated and efficient in the semi-strong form, if they satisfy some conditions; in this case

\textsuperscript{16} According to Edwards' absolute version of PPP, if the real exchange rate behaves as suggested by the PPP theory, deviations of the logarithms of the actual real exchange rate from its PPP constant equilibrium level must be characterized by a white noise process, i.e. the logged series of the official or parallel real exchange rate should be completely random.

\textsuperscript{17} Notice that Edwards argues that there is no a priori reason to expect that parallel nominal exchange rates will be positively correlated with official nominal exchange rates and finds in 13 out of 28 countries examined that the coefficient of correlation between both rates is even negative. This result does not mean that BMERs are not good proxies of the official equilibrium nominal exchange rate, but only that BMERs are not necessarily good proxies of the current official nominal exchange rate.

\textsuperscript{18} Despite the early widespread use of the DF distribution for testing the presence of unit roots in the residuals of the cointegrating equation, the most appropriated statistics is not the DF distribution but the Engle and Granger (EG) distribution. See Engle and Granger (1991).
cointegration becomes a necessary, but not sufficient, condition for market efficiency. Thus, their results might only suggest that BMERs for both countries are cointegrated. The second test, which also concludes at weak market inefficiency, is a traditional one which consists at examining if BMERs for both countries follow a random walk with drift. Their conclusion is again debatable. In fact, both intercepts are close to zero and both slopes are close to one, results which support weak EMH; so, their proof is based essentially on the existence of serially autocorrelated estimated residuals. However, as we show in this paper, the presence of non-stationary BMER time series makes their least squares estimates suspect. In this case, regressor coefficients have a Dickey-Fuller distribution and conventional t-ratios to test for their significance, based on the assumption of normality, would not be valid, for they are seriously biased towards rejection of the null hypothesis and hence towards acceptance of a spurious relationship.\footnote{These findings led Granger and Newbold (1974) to suggest that, in the joint presence of a high $R^2$ and low Durbin Watson (DW) statistics (a useful rule of thumb being $R^2 > DW$), regressions should be run in first differences.}

3. Cointegration and Market Efficiency

This section introduces the statistical meaning of cointegration and its economic implications for semi-strong market efficiency. Specific testing procedures are discussed in next section. Cointegration is a new statistical concept first enunciated by Granger (1983) and more formally developed in Engle and Granger (1987 and 1991) and Campbell and Perron (1991). Intuitively, the concept of cointegration can be assimilated to the existence of a long-run equilibrium between two or more economic variables. Let $x_t$ and $y_t$ be a pair of time series measured at time $t$. In general, theoretical models associate the existence of stationary equilibria between $x_t$ and $y_t$, to the presence of economic forces acting like "attractors" when shocks to both time series tend to take them away from their equilibrium path. Therefore, if two economic variables are in long-run equilibrium, they will be kept close one to each other, with economic forces acting like attractors.

Formally, two variables $x_t$ and $y_t$ assumed to be nonstationary are said to be in statistical equilibrium if there exists a linear combination of them which is stationary, that is its property of having all mean, variance and covariance constant with respect to time.\footnote{In fact, it can easily be proven that nonstationarity in the univariate case implies that the variance of the time series tends to infinite; while in the bivariate case both variables drift apart without bounds.} A common feature of most
Macroeconomic variables is their nonstationarity. If after taking first differences of both variables, they become stationary, and if there exists a particular linear combination and a constant \( a \neq 0 \), such that

\[
w_t = y_t - ax_t,
\]

where \( w_t \) is called the equilibrium error term and is stationary, i.e. \( w_t \) is said to be integrated of order 0, \( I(0) \). In this special case, \( x_t \) and \( y_t \) are said to be cointegrated of order 1, i.e. have a unit root or are \( I(1) \), \( a \) is called the cointegrating vector and

\[
y_t = ax_t + w_t,
\]

is called the cointegrating, or equilibrium, regression. Given that \( w_t \) is \( I(0) \), both series never drift far apart. In the bivariate case, cointegration requires both series to be integrated of the same order, although the integrating order may be higher than one. Cointegration can also be easily extended to the multivariate case. Except for the bivariate case, there may be several linear combinations which are stationary for the multivariate case; in such cases, the cointegrating vector is not necessarily unique.

Engle and Granger (1987) show that cointegration in the bivariate case can be characterized as an error-correction (EC) model of the form:

\[
Dx_t = k_{10} + k_1 \hat{w}_{t-1} + \sum_{i=1}^{p} \gamma_i DX_{t-i} + \sum_{i=1}^{p} r_i Dy_{t-i} + n_{1t},
\]

\[
Dy_t = k_{20} + k_2 \hat{w}_{t-1} + k_3 Dx_t + \sum_{i=1}^{p} \gamma_i Dx_{t-i} + \sum_{i=1}^{p} r_i Dy_{t-i} + n_{2t},
\]

where \( Dx_t = x_t - x_{t-1} \), i.e. \( D \) represents the first difference operator, \( \hat{w}_{t-1} \) is the estimated lagged error term from equation (2), \( k_{10} \) and \( k_{20} \) are two constant terms, \( k_1, k_2, k_3 \) and eventually some \( \gamma_i, r_i, \) \( s_i, \) and \( u_i \) are \( \neq 0 \), and \( n_{1t} \) and \( n_{2t} \) are white noise residuals.\(^{22}\) Equation (5) explains that the current change in \( y_t \) is due to current change in \( x_t \), eventually to lagged \( Dy \) and \( Dx \) and to an additional lagged error term which represents the long-run equilibrium

\(^{21}\)Engle and Granger (1987) say that it is generally true that \( w_t \) is \( I(1) \). For \( w_t \) being \( I(0) \), the constant \( a \) is such that the bulk of long run components of \( x_t \) and \( y_t \) cancels out, so it acts like the scaling needed in order to achieve stationarity.

\(^{22}\)The number of lags in the EC model (otherwise the choice of \( p \)) is chosen in such a way to ensure residuals to be white noise. Hakkio and Rush (1989) report that using too many lags, reduce the power of the tests of cointegration.
constraint (where the variables are defined in levels). In general, no particular preference should be given to one variable as a priori corresponding to $x_t$ or $y_t$. Thus, the test can be conducted twice with the same variables. Estimating EC models and testing the coefficient of $w_{t-1}$ provides with another test of cointegration hereinafter applied.

Hakkio and Rush (1989) and MacDonald and Taylor (1989) are among the first to have developed the theoretical relationship between cointegration and market efficiency. There exist two steps at testing market efficiency through the theory of cointegration.

The first step is based on Granger’s (1986) demonstration that two prices from a pair of efficient markets cannot be reintegrated. Thus, assuming that both assets are independent, cointegration weak market efficiency and vice versa. This is reflected in Cases I and II of Figure 1 respectively.

Figure 1
Relationship between Cointegration, Market Efficiency and the Assumptions of Dependency between Two Assets

<table>
<thead>
<tr>
<th>Cointegration</th>
<th>No Cointegration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent assets</td>
<td>Case I. Weak market efficiency is not possible.</td>
</tr>
<tr>
<td>Interdependent assets</td>
<td>Case III. Semi-strong ME is possible and joint (or single) WME is possible.</td>
</tr>
</tbody>
</table>

However, this basic argument has an important qualification suggested by Hakkio and Rush (1989): If countries explicitly (and rigidly) fix their exchange rate parity or implicitly coordinate their economic policies while possessing similar production technologies, then their currencies might not be considered as different assets. In this particular case, cointegration becomes a necessary condition for semi-strong market efficiency. Thus, exchange rates from two interdependent countries may be cointegrated even though they are jointly determined in weak efficient markets. This is represented in Case III. Case IV represents two interdependent exchange rates which are not cointegrated, i.e. are not semi-strong efficient, but are individually weak efficient. Furthermore, cointegration of two exchange rates between different

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23 Intuitively, if two exchange rates are cointegrated, it implies that there must be Granger-causality running in at least one direction and, therefore, one exchange rate can be used to forecast the other one.
nations becomes a useful tool to discriminate between countries that conducted independent economic policies and those than did not.  

We use these propositions to test for semi-strong market efficiency in the Guatemalan and Salvadoran black market for dollars, since recent studies suggest that cointegration exists between both markets. This is not a coincidence, Cáceres and Núñez (1990 and 1991) and Meléndez (1990) propose the following arguments as the rationale for supporting strong interdependence between both economies during the 1980s: i) legal and illegal trade between them is important; ii) evidence of some degree of currency substitution between the Guatemalan quetzal and the Salvadoran colon exists; iii) some tests of Granger-causality from monetary disequilibria in Guatemala to domestic output in El Salvador are positive; iv) common trends for BMERs in both countries are observed; v) big speculative traders in black dollars do participate in both markets. Therefore, we expect both markets to be cointegrated and probably efficient in the weak or semi-strong form. Besides, we do not examine cointegration between respectively Guatemalan and Costa Rican, and Salvadoran and Costa Rican black foreign exchange markets; since no such apparent tie exists and expect that bilateral BMERs for these countries are not cointegrated, if they are weak efficient.

The second step is a test of the joint hypothesis of market efficiency (semi-strong form) and purchasing power parity. It implies that the current black market rate and the current bilateral price ratio are "close together" in a long-run equilibrium; so that they are cointegrated. As it was indicated above, cointegration and PPP testing has been performed by Baillie and Selover (1987) and by Layton and Stark (1990). Both efforts do not explicitly consider that market efficiency implies the following additional constraints on equation (3), which provide specific tests of the semi-strong form of the EMH:

a) The cointegration vector, \( \alpha \), must be equal to 1.0.

b) The estimated error term, \( \hat{\epsilon}_t = y_t - \hat{\alpha}_x_t \), in equation (3) must be white noise, while cointegration only requires it to be stationary.  

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24 See Hakkio and Rush (1989, pp. 77,78). For an example of such an approach, see Diebold et al. (1988).

25 The above indicated criteria for the economies of Guatemala and El Salvador seek to avoid defining economic interdependence through some ambiguous definition; in particular, economic interdependence between two countries is not equivalent to belonging to the same economic region. However, we must recognize that any criterion employed is somewhat arbitrary.

26 Hakkio and Rush (1989, p. 77) provide an example where the error term follows an autoregressive process of order 1, \( w_t = \delta w_{t-1} + \varepsilon_t \) for \(-1 < \delta < 1\). In this case, the error term is stationary, but not white noise. So, the predicting variable does not incorporate all available information and the market is inefficient.
c) The third condition uses the fact that cointegration and error-correction specification are equivalent. For simplicity, let’s call the price differential ratio as \( PPP_t = \frac{P_t}{P^*_t} \) and \( ppp_t = \log PPP_t \). Thus, if \( b_t = \log B_t \) and \( ppp_t \) are cointegrated and jointly satisfy a \((p + 1)\)-th order vector autoregression, they can be written as the error-correcting regressions:

\[
Dppp_t = k_1 + k_1 \hat{w}_{i-1} + \sum_{i=1}^{p} v_i Dppp_{t-i} + \sum_{i=1}^{p} r_i Db_{t-i} + n_{1t},
\]

(6)

and

\[
Db_t = k_2 + k_2 \hat{w}_{i-1} + k_2 Dppp_t + \sum_{i=1}^{p} s_i Dppp_{t-i} + \sum_{i=1}^{p} u_i Db_{t-i} + n_{2t},
\]

(7)

PPP requires all lagged coefficients, \( v_i, s_i, u_i \), and for \( i > 0 \) to be equal to 0.0, then \( k_2 = k_3 = a = 1 \), and residual terms \( n_{1t} \) and \( n_{2t} \) to be white noise. If these conditions are met, the current purchasing power parity ratio will be an unbiased predictor of the current black market rate, i.e. \( b_t = ppp_t + n_{2t} \).

4. Methodology

Levich (1985) has argued that all tests of market efficiency (weak form) refer to a joint hypothesis: That investors can set actual prices to conform to their expected values and that the pattern of prices conforms to the pattern produced by the underlying model (although unknown) generating the time series of expected prices. Black foreign exchange markets are no exception to

27 This point can be demonstrated by assuming all lagged coefficients and constant term to be equal to zero and finding the backward solution of equation (7). This is

\[
b_{t-1} - b_{t-1} = k_1 \hat{w}_{t-1} - k_2 ppp_{t-1} + k_3 ppp_{t-1} + k_4 ppp_{t-1} + n_{2t-1}.
\]

But

\[
b_{t-1} = (1 + k_2) b_{t-2} - k_2 ppp_{t-2} + k_3 ppp_{t-1} - k_4 ppp_{t-2} + n_{2t-1};
\]

therefore, in \( b_t = k_3 ppp_t + (1 + k_2)b_{t-1} - (k_3 + k_4)pmp_{t-1} + n_{2t} \) substitution of \((1 + k_2)b_{t-1}\) yields

\[
b_t = k_3 ppp_t + (1 + k_2) b_{t-2} + k_3(k_3 - 2)pmp_{t-1} - k_3(1 + k_2)pmp_{t-2} -
- k_3(k_3 + a)pmp_{t-2} + n_{2t} + (1 + k_2)n_{2t-1};
\]

and by applying the same technique recursively gives

\[
b_t = k_3 ppp_t + k_3(k_3 - a)pmp_{t-1} + (1 + k_2)pmp_{t-2} +
+ (1 + k_2)pmp_{t-3} + ... + n_{2t} + (1 + k_2)n_{2t-1} + ...
\]
There are two standard traditional tests of weak-form market efficiency: Parametric and non-parametric. Before going into further testing through cointegration theory, our interest in using them is to set a baseline case, where it is illustrated how in the presence of nonstationary series, it is difficult to derive unambiguous conclusions about market efficiency.

In the most common parametric test, the logarithm of the current black rate, $BMER_t$, is regressed on its past rate. The equation is set as follows:

$$b_t = c_0 + c_1 b_{t-1} + e_t,$$  \(8\)

Weak efficiency requires $c_0 = 0$, $c_1 = 1$ and $e_t$ to be white noise. The "relative" form of the test, performed in logged first differences, is sometimes preferred to its "absolute" version because eventual non-stationarity of the rate of change of $BMER$ series would make "absolute" regression results suspects. In particular, it would raise doubts about the consistency of its estimated standard errors. However, if $BMER$ series are stationary, then the use of the absolute version of the test is also justified. Hakkio and Rush (1989) also notice that the relative version contained in equation (8) generally yields weaker results than the absolute one: One cannot often reject the hypothesis that the constant is 0.0 and the slope is 1.0, but simultaneously one cannot also reject that the constant is 0.0 and the slope is -1.0.

Non-parametric tests apply procedures widely used in modern finance. In this paper, we use a method known as "runs analysis". Moore (1964) and Fama (1965) were among the first to perform it for the analysis of the randomness of stock prices. A run is defined as a sequence of price (exchange rate) changes of the same sign. The runs from the sample observation are divided into three groups containing positive, negative and zero changes respectively. The expected number of runs of all signs of the $BMER$ series are estimated as having a normal distribution with mean $\mu$ and variance of $\sigma_\mu^2$ as follows:

$$E(\mu) = \left[ n(n+1) - \sum_{i=1}^{3} n_i^2 \right] / n,$$  \(9\)

The latter hypothesis is problematic since it implies that the expected rate of return for holding black currency is zero. This may be true, but as Levich (1979) points out, "there is no general agreement on models for equilibrium pricing or equilibrium rates of return". Two useful extensions of the present work, which are not treated here, would be to provide for a time-varying return or for a time-varying risk.

A special case is the so-called random-walk with a drift parameter model, which is based on the assumption that the expected yield is constant over time, but different from zero, so that $c_0 > 0$ in equation (8).

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and

\[ \text{Var}(\mu) = \left\{ \sum_{i=1}^{3} n_i^2 \left[ \sum_{i=1}^{3} n_i^2 + n(n+1) \right] - 2n \sum_{i=1}^{3} n_i^3 - n^3 \right\} / [n^2(n-1)] , \]  

(10)

where \( n \) is the total number of price changes and \( n_i \) is the number of price changes of each sign, \( \sum n_i = n \). To test whether the difference between the actual and expected number of runs is significantly different from the normal distribution, the following statistic is computed:

\[ K = [(R + 0.5) - \mu] / \sigma \mu \]  

(11)

where \( R \) is the actual number of runs, \( \sigma \mu \) is the standard error, \( \sigma \mu = (\text{Var}(\mu))^{1/2} \), and \( 1/2 \) in the numerator is a discontinuity adjustment factor. \( K \) is assumed to follow a standard normal variable distribution with mean zero and variance one. If \( K \) lies in the interval \([+1.96, -1.96]\), then the null hypothesis of absence of serial correlation (which implies WME) cannot be rejected at the 5 percent level of significance. An important qualification of the test is that it also requires from the tested variables (prices or exchange rates) to be stationary in levels.

Testing for cointegration also crucially hinges on the stationarity of the time-series involved. As a consequence, this procedure requires their previous testing for the presence of a unit root. There exist two main strategies of testing for unit roots: i) Dickey-Fuller (DF) tests, first developed in Dickey and Fuller (1979 and 1981) and complemented by Dickey, Bell and Miller (1986); and Phillips and Perron (PP) tests based on Phillips (1987) and Phillips and Perron (1988).\(^{31}\) Notice that DF test for a unit root is equivalent to test the absolute version of the weak form market efficiency (random walk model).

Dickey-Fuller tests are summarized by the three following models applied on \( b_t \) under the null of having a unit root:\(^{32}\)

**Case a. No drift, no trend.** \( H_0 : c^*_1 = 0; H_1 : c^*_1 < 0 \)

\[ D b_t = c^*_1 b_{t-1} + e_{1t}, \]  

(12)

\(^{31}\) Other unit root tests are also available, but these two procedures seem to be the most frequently used.

\(^{32}\) The same testing procedure must be applied to \( Dppp \). Notice that the models are a slight variation of equation (8) reparametrized in order to directly find the \( t \)-values from a standard computer printout which are necessary to perform inference tests on the coefficient of \( b_{t-1} \). See Dickey, Bell and Miller (1986, p. 16).
Case b. Drift, no trend. $H_0 : \hat{c}_0 = 0; \hat{c}_1 = 0; H_1 : \hat{c}_0 \neq 0, \hat{c}_1 < 0$.

$$Db_t = \hat{c}_0 + \hat{c}_1 y_{t-1} + \epsilon_{2t},$$  \hfill (13)

Case c. Drift and trend. $H_0 : \hat{c}_0 = \hat{c}_2 = \hat{c}_3 = 0; H_1 : \hat{c}_0 \neq 0, \hat{c}_2 \neq 0, \hat{c}_3 < 0$,

$$Db_t = \hat{c}_0 + \hat{c}_2 y_{t-1} + \hat{c}_3 t + \epsilon_{3t},$$  \hfill (14)

where $t$ is a trend variable, $c_0^* = \hat{c}_1 = \hat{c}_2 = \hat{c}_3 - 1$, and $\epsilon_i$ is i.i.d $(0, \sigma_i^2)$. Notice that the standard $t$-statistics associated respectively to $c_0^*, \hat{c}_1$ and $\hat{c}_2$, does not have a Student distribution, for the OLS estimators are not asymptotically normal; instead, they have a Dickey-Fuller distribution.\footnote{The DF distribution is not symmetrical, but skewed to the left and has most of its mass below zero; thus, it reflects the fact that values of $c_i > 1$ could have generated a particular data set with only very low probability.}

Under the null hypothesis, there is a unit root in the time-series involved.

As $e_{it}$, for $i = 1, 2, 3$ may not be white noise, a popular parametric solution is the augmented Dickey-Fuller (ADF) test which consists of augmenting regression (12), (13) or (14) by a sufficient number of lagged terms in $Db_{t-i}$, for $i = 1, \ldots, p$, to whiten residuals. For example, the ADF test on equation (12) examines the $t$-statistics corresponding to the estimated $c_i^*$ in the following regression:

$$Db_t = c_0^* b_{t-1} + \sum_{i=1}^{p} \beta_i Db_{t-i} + \epsilon_{1t},$$  \hfill (15)

where $t$ has the same DF distribution. The number of lags $p$ is chosen by a sequential procedure based on standard LM tests to examine the presence of serial correlation. According to Engle an Granger (1987) the "simple to general" specification search is an easy model building strategy to determine the number of lags $p$.\footnote{Alternatively, Layton and Stark (1989) arbitrarily set $p = 4$ for quarterly regressions and $p = 12$ for monthly regressions and examine if these values are sufficient to dampen residuals in the estimated ADF models. As an increased number of lags $p$ decrease the power or tests, Hakkio and Rush (1989) used as highest value $p = 6$ for some monthly data, whenever it was necessary to remove serial correlation. Unlike both approaches, Engle and Granger (1987) first set $p = 4$ in order to test for the significance of the coefficients of the lagged variables and the presence of serial correlation and finally eliminate the lagged variables which are non significant. Their results make them to advise the simple to general specification search procedure.}

According to Schwert (1991), DF’s parametric correction procedure gives a more accurate test size than the popular non-parametric procedure developed by Phillips and Perron (1988). PP’s correction estimates residuals
from equations (12) (or (13) or (14)) and uses them to correct the t-statistics through one additional parameter: The Newey West estimator which takes a weighted average of the autocorrelations of $e_t$. Another drawback of PP test, suggested by Haldrup (1991), indicates that when time series eventually contain double unit roots, i.e. are $I(2)$, there is nothing in PP test that indicates that differencing is required, so their statistic will tend to favorize the stationary alternative.\footnote{On the other hand, the lack of power of the ADF test in specific circumstances is discussed by Campbell and Perron (1991).}

In this paper, we follow the DF approach to first establish if both series, $b_t$ and $ppp_t$, are individually integrated of order 1.

Engle and Granger’s (EG) two-step procedure of testing for cointegration assumes that both variables, $b_t$ and $ppp_t$, are integrated of order 1, denoted as $I(1)$. The cointegration testing procedure between two (bilateral) black currencies or between official and black rates is similar. For simplicity, we only describe the methodology for the $PPP$ case. In the first step, the parameters of the cointegrated vector are estimated and, in the second step, are used in the EC form. So, the first step propose regressing one of the variables on the other by using OLS on the levels. This equation is called the cointegrating regression:

$$b_t = a_0 + a_1 ppp_t + w_t,$$

where $w_t$ is the residual term. A particular feature of the first step is that no previous choice is made concerning the dependent variable. Engle and Granger (1991) argue that this is irrelevant since when two series move together, the fit is almost perfect (high $R^2$), no matter how strong the simultaneous bias or the serial correlation are. However, the cointegrating regression has two important properties: First, following Stock (1984), the $a_t$ estimator is “superconsistent” but asymptotically not optimal;\footnote{Superconsistency means that it will have a finite sample bias of the order $1/T$ ($T$ being the sample size), while ordinary consistency for an estimator means a higher finite sample bias of $1/T^{1/2}$. However, Hendry (1986) proves that its bias in small sample sizes may be substantial, unless $R^2$ is reasonably large ($R^2 \geq 0.95$).} second, it is not possible to test hypothesis about the coefficient parameters, for serial correlation in $w_t$ and endogeneity of regressors makes them to depend upon nuisance parameters and thus standard t-ratios for inference can not be used.\footnote{Nevertheless, Hoffmaister (1991) argues that if the error term, $\epsilon_t$, is serially uncorrelated and the innovations on the LHS variables are weakly exogenous and do not Granger cause the innovations in the RHS variables, then the asymptotic distribution of the cointegration vector is free of nuisance parameters. In such a case, OLS is asymptotically optimal and standard tests of hypothesis are valid.} Typically, it is supposed that there is a non-zero drift, $a_0 \neq 0$, and also, but not so frequent, a trend.
As long as we cannot test if $\hat{\lambda}_1 = 0$, the second step consists of examining the stationarity of residuals $\hat{w}_t$, condition which must be satisfied if both series are cointegrated, and the properties of coefficients in the EC model. In this paper, four of seven statistics proposed by Engle and Granger (1987) are analyzed: The Durbin Watson (DW), the Dickey-Fuller (DF), the augmented Dickey-Fuller (ADF) and the augmented restricted vector autoregression (ARVAR). While the first three tests examine whether $\hat{w}_t$ is stationary, the last test concludes whether or not $h$ and $ppp_t$ follow an EC model.38

**Test a:** The Durbin-Watson (DW) estimated from equation (16) is tested to see if it is significantly greater than zero, i.e. $DW = 0$ under the null. A low DW value is associated to non-stationarity, i.e. reject of cointegration.

**Test b:** The DF test is implemented on the residuals from the cointegrating equation, $\hat{w}_t$ to check if the parameter $\varphi$ is significant in the following equation:

$$Dw_t = -\varphi \hat{w}_{t-1} + z_{1t}$$  \hspace{1cm} (17)

If the estimated parameter $\varphi$ is found significantly positive (by using its $t$-ratio and the EG distribution), i.e. residuals are stationary, the hypothesis of cointegration is accepted. If the estimated $\varphi$ is not significantly different from zero, under the null hypothesis, then residuals follow a random walk and the hypothesis of cointegration is rejected.

**Test c:** The augmented Dickey-Fuller (ADF) test is similar to test (b), but additional lags of $Dw_{t-i}$ for $i \geq 1$, are used to whiten serially correlated residuals.

$$D\hat{w}_t = -\varphi \hat{w}_{t-1} + \sum_i \theta_i Dw_{t-i} + z_{2t}$$  \hspace{1cm} (18)

Again, if the estimated $\varphi$ is not significantly different from zero, the null hypothesis is accepted and cointegration is rejected.

**Test d:** The augmented restricted vector autoregression (ARVAR) test uses the fact that cointegrated variables can be written in EC form. For PPP, it requires estimating equations (6) and (7) and testing if the estimated $k_1$ and

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38 Engle and Granger (1991) indicate that the $t$-ratio for this test has no longer the DF distribution, since the parameter $\hat{a}_1$ has been estimated and makes the residual series appear slightly more stationary than if they were computed at the true $a$. Preliminary critical values are found in Engle and Granger (1987, pp. 269-271) and more precise values are included in MacKinnon (1991). Notice that the critical values for the $t$-ratio from the EC distribution are not the same as the ones from the DF distribution or as the ones from the Student distribution.

39 This can be easily understood by remembering that the DW formula is $d = \sum (\hat{w}_t - \hat{w}_{t-1})^2 / \sum \hat{w}_t^2$. Therefore, if the $w_t$'s follow a random walk, i.e. $Dw_t = z_t$, for $z_t$ being i.i.d $(0, \sigma_Z^2)$, the DW will approach the value of zero.
Las $k_2$ son significativas conjuntamente (diferentes de cero). Las estadísticas de prueba para su significación conjunta es la suma de sus estadísticas $t$ cuadradas. Si esta estadística es significativamente mayor que cero, usando valores críticos de Engle y Granger (1987), la cointegración no se rechaza.

La estimación del modelo EC merece comentarios adicionales. Primero, un procedimiento secuencial "simple to general" de búsqueda de especificaciones es utilizado para determinar el número de retardos $p$, comenzando con el orden más bajo hasta el más alto. Segundo, Engle y Granger (1991) indican que al introducir el valor actual de $Dppp_t$ en la ecuación (7) lo convierte en una "forma estructural" más que una "forma reducida". Consecuentemente, este cambio modifica la interpretación de todos los coeficientes pero el coeficiente de cointegración $k_t$. En efecto, si $Dppp_t$ no se incluye y $a_t$, del regresión de cointegración, es utilizado, Engle y Granger (1987) muestran que la estimación por regresión de ecuaciones no relacionadas (SUR) es equivalente a la estimación por OLS separada en cada ecuación, porque las mismas variables aparecen en el lado derecho del modelo EC (7) como en el modelo (6). Tercero, en casi todos los casos, debería haber un desplazamiento (aunque su valor es casi igual a cero) y, si es necesario, un término lineal puede ser permitido en las ecuaciones EC. DW y DF pruebas parecen apropiadas cuando los residuales son blanco de ruido. Esto no es usualmente el caso, por lo que ADF y ARVAR pruebas se aplicarán actualmente. Excelente potencia y estabilidad de valores críticos los hace sugerir el uso de la ADF prueba. Si se sabe que el sistema es blanco de ruido sin el uso de términos retardados, el término extra no se debería introducir.

Una vez que hemos demostrado que ambos series son cointegrados, llega el momento de probar la eficiencia semiestricta del mercado. Resumiendo, tenemos cinco pruebas secuenciales de eficiencia semiestricta del mercado, siguiendo la teoría de cointegración:

**Necesarias condiciones:**
1. El $b_t$ y $ppp_t$ de cada país tienen raíces unitarias?
2. ¿El $b_t$ y $ppp_t$ cointegrados? ¿O son bilaterales BMERs, $b_{it}$ y $b_{jt}$, para $i \neq j$ cointegrados? ¿O son oficiales y tasas negras, $e_i$ y $b_{it}$ cointegrados?

**Suficientes condiciones:**
3. El coeficiente cointegrador, $a_t$, es igual a 1? Aunque la naturaleza no-stationary de ambas series impide pruebas de este caso.

40 Este es el procedimiento estándar utilizado en pruebas PPP. Adicionalmente, Hakkio and Rush (1989) examinan la adecuación estadística de su modelo EC a través de los siguientes tests: LM tests to determine if residuals are serially non-autocorrelated; ARCH tests and tests of heteroskedasticity for the presence of a non-constant variance; F-test, CUSUM and CUSUMSQ tests for the stability of regressors; and Granger's (1983) procedure to test if residuals from the EC regressions are white noise.

41 Aunque nada impide que el investigador muestre que el coeficiente $a_t$ es cercano (o lejos) de 1. Además, podemos arbitrariamente fijar $a_t = 1$, una condición que es compatible con el EMH, y examinar si los residuales restringidos $w_t$ son estacionarios.
4. Is the equilibrium error term, $w_t$, white noise (and not only stationary)?
5. Are the coefficients of the lagged terms in the EC regression equal to 0 and does $-k_2 = k_3 = a_1 = 1$?

5. Data Sources and Definitions

This study uses logged monthly data on BMERs, average official exchange rates and price differential ratios for three Central American countries: Costa Rica, El Salvador and Guatemala. The period under study varies according to each Central Bank data availability on BMERs: For Costa Rica, it covers from October 1980 to December 1990 and for El Salvador and Guatemala, it covers from August 1982 to December 1990. This period includes two clearly distinguished subperiods in the three countries: First, a short period of multiple official exchange rate regimes followed by gradually reunified (managed "dirty" floating, crawling-set or temporarily fixed) official exchange rates regimes. In fact, the resulting exchange rate regimes, following the short experience with temporary multiple official exchange rates, were not equivalent among three countries. Costa Rica established a reunified crawling-peg regime in November 1983 which lasted up to February 1992, when a managed "dirty" floating regime was established. Apparently, Costa Rican "minidevaluation experience" was explicitly guided by the PPP condition. El Salvador got temporarily back to a unified fixed exchange rate regime from January 1986 to July 1989, when a managed "dirty" floating regime was established. For its part, Guatemala slowly got back to a nearly unified fixed exchange rate regime from June 1986 to June 1990, when a bid-ask managed floating regime was implemented. In so-doing, we are able to examine the hypothesis that reintegration between BMERs and the PPP ratio is more likely during periods of reunified, rather than multiple exchange rates regimes. In the latter case, the black dollar is characterized by increased variability. We are also able to examine cointegration between the official and black rate following the reunification of the exchange rate regimes.

42 In order to avoid potentially biased data, derived from the existence of multiple rates in all three countries, we use as proxy for the official rate the average official rate for effective transactions in the current account of the balance of payments. Data were kindly provided by Central Banks.

43 Most transactions exchanged at the former official rate were transferred to the "new mercado regulado rate". Only a small portion of "basic imports" requiring hard currency were allowed to be exchanged at the former official rate. Both rates disappeared in November 1989. Temporarily, commercial transactions were exchanged at the banking official rate which became the single official exchange rate between November 1989 and June 1990.
The study makes use of the bilateral price ratio with respect to the United States. In this manner, the relationship between each particular country and its most important trade partner, the United States, is analyzed. In 1985, trade with the United States represented more than 40 percent of total trade in these countries. A multilateral price index might also be included, but Edwards (1989) shows that bilateral and multilateral price indexes in many developing countries (including El Salvador and Guatemala) tend to move roughly in the same direction between 1960 and 1985, and whether significant differences in behavior appeared, particularly between 1980 and 1985, this was the result of nominal exchange rate policies who used the U.S. dollar as the reference currency for official exchange rate adjustments, while it appreciated steeply against the other major currencies. These policies were effectively implement by Central American countries and give an additional argument for using bilateral price ratios. Finally, it must be recognized not only that in those countries the dollar is the only foreign currency for which a black market exists, but also that this market is strongly supplied by non commercial transactions in dollars such as remittances and tourism.

The PPP ratio is built on the consumer price index (CPI) in both countries (base 1980 = 100). Thus \( \text{PPP}^{i} = \frac{\text{CPI}_i}{\text{CPI}^{US}} \), for country \( i \). While it is generally recognized that this index includes non-traded goods, such as rents, who have no influence on international trade, alternative price indexes, such as the wholesale price index ignores not only some forms of manufactured goods, but the whole range of services.\(^{44}\) We finally choose CPI as a reasonable practical choice because although it covers only consumption goods, it has historically been the most popular index for PPP analysis, does cover both traded and non-traded goods sectors, is a base-weighted index and, because of its availability in monthly publications, allows for an increased number of observations.

The use of monthly data has the disadvantage of the eventual presence of seasonal effects in the time series. Seasonality in BMERs of El Salvador has tested positive in López and Seligson (1990) during the months of November and December for shorter periods.\(^{45}\) Therefore, we proceed in two steps.

\(^{44}\) A price index based on wage rates is even less desirable, given the poor quality of available data on this indicator. The GDP deflator index has also the disadvantage that is also a current-weighted index rather than a base-weighted index, as is the case for the CPI, so it reflects other factors than purely average price changes.

\(^{45}\) This is called the “Christmas effect” of remittances on the BMER, referring to a perceived inflow of remittances from Salvadorans living in the United States, which during some past years has temporarily appreciated the BMER during the months of November and December.
First, we test for seasonal effects by examining the sample autocorrelation function of each variable through the use of the portmanteau Box-Pierce $Q$-statistics for $T/4$ residual autocorrelations, where $T$ is the number of observations.\textsuperscript{46} Second, we avoid the use of seasonal dummies and consider deseasonalization methods by applying a seasonal difference operator, $D_t D_b = (1 - L^{12})(1 - L)b_t$, on the log difference of each variable,\textsuperscript{47} followed by the test of the Box-Pierce $Q$-statistics. While this measure has the advantages of eliminating seasonality easily and of reducing the substantial amount of noise \textit{vis-a-vis} month to month changes, it has the unfortunate effect of reducing by twelve the number of sample observations and of introducing serial correlation; thus, if residual autocorrelation becomes significant, it is then preferable to work with non seasonally differenced data.

6. Empirical Results

Traditional parametric testing of the EMH (weak form) estimates equation (8) by OLS. Logarithms are used as a standard procedure in dealing with exchange rate time series to avoid the so-called Siegel paradox.\textsuperscript{48} Table 1 reports our results for the three countries. The hypothesis that the constant terms are near zero and the slope coefficients are significant and do not differ from unity are met in the cases of El Salvador and Guatemala. However, in both cases the $F$-statistics equivalent to the LM test statistics for the presence of serial correlation indicates the presence of a significant degree of first order serial correlation among residuals. For both countries, if it is the case that the current BMER fails to adequately incorporate the past history of the exchange rate, then a further lagged value $b_t - 2$ should make a significant contribution at increasing the $R^2$ of the equation. The results in Table 1 show that this is not the case. Notice that the sums of the coefficients of both lagged rates (which are also significant) are close to unity and that

\textsuperscript{46} Poskitt and Tremayne (1981) suggest as a “rule of thumb” choosing the number $m$ of residual autocorrelations at $m = T^{1/2}$. As we are interested in the eventual presence of seasonal effects, $m$ must be increased to $T/4$.

\textsuperscript{47} Ghysel and Perron (1990) show that seasonal adjustment filters on tests for a unit root do create biased estimates who tend to accept the null hypothesis of not rejecting the presence of a unit root. However, Dickey, Bell and Miller (1986) prove that removal of seasonal means by an additive seasonal adjustment has no effect on the limit distribution of DF unit root test statistics.

\textsuperscript{48} MacDonald and Taylor (1992) review the importance of working with logarithms in exchange rates in order to avoid Jensen’s inequality, also known as Siegel’s paradox, where $[1/E(\text{colones}/\$)] \neq E(1/(\text{colones}/\$))$ and $E(\cdot)$ represents the mathematical conditional expectation operator.
residuals become white noise. Thus, the preliminary conclusion from these results should be that the current BMER of El Salvador and Guatemala incorporates all available information relevant to the predict the future rate, behavior which is in agreement with weak market efficiency. Nevertheless, this conclusion is wrong. Dickey-Fuller $t$-statistics are non-significant, which indicates the presence of two non-stationary time series. Under such conditions, and as discussed by Meese and Singleton (1982), consistency of the estimated standard errors from equation (8) is suspect. For its part, Costa Rica has apparently a non-zero constant term, an slope close to one and no serial correlation in a stationary serie. This result would suggest the presence of a random walk model with drift, but this conclusion still deserves further examination.

Table 1

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<th>Market Efficiency (Weak Form) of Black Market for Dollars in Costa Rica, El Salvador and Guatemala (Monthly Data)</th>
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</tr>
<tr>
<td>Costa Rica</td>
<td></td>
</tr>
<tr>
<td>(10:80-12:90)</td>
<td>0.19*</td>
</tr>
<tr>
<td>El Salvador</td>
<td></td>
</tr>
<tr>
<td>(8:82-12:90)</td>
<td>0.29</td>
</tr>
<tr>
<td>Guatemala</td>
<td></td>
</tr>
<tr>
<td>(8:82-12:90)</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Note: All variables are expressed in logarithms, $r$-values of the estimates are in parentheses, while the $t$-statistics corresponding to the DF test with drift is set in the sixth column. $F(1)$ corresponds to the $F$-statistics equivalent to the Lagrange Multiplier (LM) test for first order serial correlation. * Indicates significance at the 5 percent level.

Sources: Data provided by Central Banks.

Non parametric testing by runs analysis is shown in Table 2 where the total observed and expected number of runs are compared. The test must be

49 Only the $F$-statistics equivalent to the LM test for first order serial correlation is reported. Nevertheless, all LM tests performed on this article examined at least the eventual presence of autocorrelation of order 1, 2, 3, 4, 6 and 12.
performed on log-differences of BMERs. Results indicate for all three rates that the difference between the observed and expected number of runs is negative and significant at the 5 percent level, indicating positive serial correlation. Therefore, these results would tend to reject weak market efficiency for all three rates. However, these results should not be unexpected for the BMER series which are nonstationary.

Table 2

<table>
<thead>
<tr>
<th>Runs Length in Months</th>
<th>Costa Rica</th>
<th>El Salvador</th>
<th>Guatemala</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>16 10 8</td>
<td>7 11 9</td>
<td>10 6 8</td>
</tr>
<tr>
<td>2</td>
<td>2 7 3</td>
<td>3 - 1</td>
<td>3 6 1</td>
</tr>
<tr>
<td>3</td>
<td>- 2 1</td>
<td>4 4 -</td>
<td>2 4 -</td>
</tr>
<tr>
<td>4</td>
<td>- 2 1</td>
<td>1 1 -</td>
<td>2 2 -</td>
</tr>
<tr>
<td>5</td>
<td>- - 1</td>
<td>- - -</td>
<td>1 - -</td>
</tr>
<tr>
<td>6</td>
<td>- - -</td>
<td>- - -</td>
<td>- 1 -</td>
</tr>
<tr>
<td>11</td>
<td>- - -</td>
<td>- - -</td>
<td>- 1 -</td>
</tr>
<tr>
<td>12</td>
<td>1 - -</td>
<td>- 1 -</td>
<td>- - -</td>
</tr>
<tr>
<td>15</td>
<td>- 1 -</td>
<td>- - -</td>
<td>- - -</td>
</tr>
<tr>
<td>TOTAL</td>
<td>19 25 13</td>
<td>16 19 10</td>
<td>18 20 9</td>
</tr>
</tbody>
</table>

Total Runs

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>CR colon (n = 122)</td>
<td>57</td>
<td>72.0</td>
<td>4.71</td>
<td>-3.07*</td>
</tr>
<tr>
<td>ES colon (n = 100)</td>
<td>45</td>
<td>58.0</td>
<td>4.44</td>
<td>-2.93*</td>
</tr>
<tr>
<td>G quetzal (n = 100)</td>
<td>47</td>
<td>57.5</td>
<td>4.47</td>
<td>-2.35*</td>
</tr>
</tbody>
</table>

Note: * Indicates positive serial correlation at the 5 percent level of significance.
Sources: Data provided by Central Banks.

Cointegration analysis previously requires to examine the presence of a unit root in all time series involved in estimation. The identification techniques based on examination of plots and autocorrelation functions are a previous useful tool at determining the order of differencing and the eventual need for seasonal filters. Plotting time series has the additional advantage of quickly revealing the effects of external influences on data. Logged monthly BMERs and PPP ratios for each country are plotted in Figures 2a, 3a, and 4a.
Their first difference is plotted in Figures 2b, 3b, and 4b. All three time series in levels are clearly characterized by an upward trend and undergo a once-for-all shift in level as a result of the transition period from multiple to reunified official exchange rates. This happens in December 1982 in Costa Rica, in February 1986 for El Salvador and in June 1986 for Guatemala. Curiously, this once-for-all shift is preceded by a typical BMER overshooting (upward movement followed by a downward counterclockwise movement) explained by speculative trading strategies nourished by economic policy mismanagement and by public expectations on future regime changes. While it is difficult to leave little doubt about the need to difference those series, the time series plot of first differenced series suggests possible nonstationarity and more evident is the increased variability of the black rate during the multiple official exchange rate regime.

Figure 2a
Costa Rica
Black Rate and Bilateral Prices Ratio
(In Logs)

Sample Period is 1980(10) - 1990(12)

50 For simplicity, we do not show the official exchange rate series, although they closely follow the path described by BMER series.
51 In reality, Costa Rica got back to a unified official exchange rate regime in November 1983; but the stabilization program agreed with the IMF by the new Monge government, program which gave previous credibility to the official exchange rate reunification, started in December 1982.
Figure 2b
Costa Rica
Black Rate and Bilateral Prices Ratio
(Log-Differenced Values)

Sample Period is 1980(11) - 1990(12)

Figure 3a
Guatemala
Black Rate and Bilateral Prices Ratio
(In Logs)

Sample Period is 1982(8) - 1990(12)
Figure 3b
Guatemala
Black Rate and Bilateral Prices Ratio
(Log-Differenced Values)

Figure 4a
El Salvador
Black Rate and Bilateral Prices Ratio
(In Logs)
The sample autocorrelation functions (SACF) of the first differences for all three series are given in Table 3 for the first 12 lags. Table 3 also includes the Box-Pierce $Q$-statistics for 24 autocorrelations in the case of Costa Rica and 15 autocorrelations in the cases of El Salvador and Guatemala. Notice that the Table is built for the second period where clearer and more significant results are derived. First differences were performed, since autocorrelations in levels are generally large and do not die with increasing lags (the decay is not clearly exponential).

The $Q$-statistics for all first-differenced series, but bilateral prices of Costa Rica and Guatemala and official rates of Costa Rica, show absence of serially autocorrelated residuals. We also second- and twelfth-differenced all series and in all cases, but two (the bilateral prices ratio of El Salvador for second differences and the black rate of Guatemala for twelve differences) the $Q$-statistics rise to significant values rejecting the hypothesis that second or twelve-differences provide with white noise residuals. This result strongly

52 In fact, with the exception of El Salvador (all series) and Guatemala (official rate), during the all sample period the $Q$-statistics for first-differenced data indicated a significant degree of remaining serial correlation.
suggests that almost all variables are stationary after being differenced once\(^{53}\) and that we do not need to use such a seasonal filter.

Table 3
Sample Autocorrelation Functions of First and Twelfth Differences of BMERs and PPP Ratio in Costa Rica, El Salvador and Guatemala
(Monthly Data)

<table>
<thead>
<tr>
<th>Lag</th>
<th>Costa Rica</th>
<th></th>
<th>El Salvador</th>
<th></th>
<th>Guatemala</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- .12</td>
<td>.39</td>
<td>.39</td>
<td>.34</td>
<td>.07</td>
</tr>
<tr>
<td>2</td>
<td>.02</td>
<td>.23</td>
<td>.20</td>
<td>.13</td>
<td>-.14</td>
</tr>
<tr>
<td>3</td>
<td>-.02</td>
<td>.19</td>
<td>.08</td>
<td>-.04</td>
<td>-.07</td>
</tr>
<tr>
<td>4</td>
<td>.03</td>
<td>.19</td>
<td>.08</td>
<td>.14</td>
<td>.07</td>
</tr>
<tr>
<td>5</td>
<td>-.17</td>
<td>.22</td>
<td>.04</td>
<td>.06</td>
<td>.16</td>
</tr>
<tr>
<td>6</td>
<td>-.03</td>
<td>-.06</td>
<td>-.16</td>
<td>.02</td>
<td>-.02</td>
</tr>
<tr>
<td>7</td>
<td>-.03</td>
<td>.01</td>
<td>-.07</td>
<td>.19</td>
<td>.01</td>
</tr>
<tr>
<td>8</td>
<td>-.01</td>
<td>-.01</td>
<td>-.10</td>
<td>.23</td>
<td>-.00</td>
</tr>
<tr>
<td>9</td>
<td>.27</td>
<td>.03</td>
<td>-.12</td>
<td>.17</td>
<td>-.03</td>
</tr>
<tr>
<td>10</td>
<td>.10</td>
<td>.04</td>
<td>.00</td>
<td>-.11</td>
<td>-.14</td>
</tr>
<tr>
<td>11</td>
<td>-.09</td>
<td>.04</td>
<td>.01</td>
<td>.13</td>
<td>-.03</td>
</tr>
<tr>
<td>12</td>
<td>-.02</td>
<td>-.02</td>
<td>-.02</td>
<td>.07</td>
<td>.22</td>
</tr>
<tr>
<td>Q</td>
<td>23</td>
<td>68*</td>
<td>41*</td>
<td>21</td>
<td>10</td>
</tr>
</tbody>
</table>

Note: Remember that lag-operators are commutative: \(Q(k)\) is the box-Pierce statistics for \(k\) autocorrelations; for Costa Rica \(k = 24\) and for El Salvador and Guatemala \(k = 15\). * Means that the value is significant at the 5 percent level. \(Q(24)\) and \(Q(15)\) critical values at the 5 percent level are 36.4 and 25.0 respectively.

Next, we performed DF and ADF tests for the presence of unit roots. Table 4 reports the results of the tests performed with a drift and a time-trend variable. The test do allow us to accept the presence of a unit root in all time series of El Salvador and Guatemala and the second period official rate and bilateral prices series of Costa Rica, since their \(r\)-coefficients of the estimated \(c_1\) in equation (14) are not significant. Conversely, the hypothesis that we should difference BMER series (with trend) of Costa Rica is rejected at the 5 percent, when we compare its \(r\)-values with the 5 percent critical value of a DF distribution with approximately \(T = 100\). We must notice that in most time series, but bilateral prices of Guatemala and Costa Rica (second period), the trend coefficient is significant at the 5 percent level (although in most of

\(^{53}\) This feature precludes the existence of bubbles in these variables for the whole period under consideration.
the remaining cases, the \( F \)-test on all coefficients --trend included-- is highly significant). LM tests were performed for 1, 2, 3, 4, 6 and 12 lags. ADF tests were performed using a single to general specification search procedure and only in the cases of the bilateral prices ratio of Costa Rica and Guatemala (all sample period), two lags were needed to whiten residuals. A curious result for Costa Rican BMER series is that if we take out the time trend variable, the serie becomes nonstationary.\(^{54}\) Thus, Costa Rican BMER series are stationary around a trend.

\begin{table}[h]
\centering
\caption{Dickey-Fuller Test with Drift and Trend for a Unit Root in BMERs and PPP ratios of Costa Rica, El Salvador and Guatemala (Monthly Data)}
\begin{tabular}{lcccccc}
\hline
Dep. Var./Period & DF Test (1) & ADF Test (2) & Crit. val. (3) & \( t \)-trend (4) & \( F \)-stat. (5) \\
\hline
\textbf{Costa Rica} & & & & & & \\
\( D_{b_t} \) & 10:80-12:90 & -4.68* (0.20) & - & -3.45 & 2.76* & 15.48* \\
\( D_{b_t} \) & 12:82-12:90 & -4.49* (1.18) & - & -3.45 & 4.70* & 11.73* \\
\( D_{e_t} \) & 10:80-12:90 & -5.27* (3.01)* & -3.91 (15.65)* & -3.45 & 3.38* & 5.90* \\
\( D_{e_t} \) & 12:82-12:90 & -2.35 (13.14) & -2.55 (0.63) & -3.45 & 2.76* & 9.51* \\
\( D_{\text{ppp}_t} \) & 10:80-12:90 & -3.69* (3.57)* & -4.53* (1.85) & -3.45 & 2.25* & 12.47* \\
\( D_{\text{ppp}_t} \) & 12:82-12:90 & 0.16 (12.39)* & -1.20 (3.69) & -3.45 & 1.60 & 8.64* \\
\textbf{El Salvador} & & & & & & \\
\( D_{b_t} \) & 8:82-12:90 & -1.53 (11.90)* & -2.15 (0.12) & -3.45 & 1.96 & 4.83* \\
\( D_{b_t} \) & 2:86-12:90 & -1.67 (2.95) & - & -3.49 & 3.39* & 6.27* \\
\( D_{e_t} \) & 8:82-12:90 & -1.59 (4.00)* & -2.03 (0.40) & -3.45 & 2.17* & 2.64 \\
\( D_{e_t} \) & 2:86-12:90 & -1.44 (0.72) & - & -3.49 & 1.38 & 1.08 \\
\( D_{\text{ppp}_t} \) & 8:82-12:90 & -1.39 (8.73)* & -1.87 (0.37) & -3.45 & 1.92 & 3.98* \\
\( D_{\text{ppp}_t} \) & 2:86-12:90 & -3.27 (2.04) & - & -3.49 & 2.97* & 7.69* \\
\hline
\end{tabular}
\end{table}

\(^{54}\) Taking out the trend becomes critical to facilitate the cointegration test between black and official exchange rates of Costa Rica.
Table 4
Dickey-Fuller Test with Drift and Trend for a Unit Root in BMERs and PPP Ratios of Costa Rica, El Salvador and Guatemala
(Monthly Data)

<table>
<thead>
<tr>
<th>Dep. Var./Period</th>
<th>DF Test (1)</th>
<th>ADF Test (2)</th>
<th>Crit. val. (3)</th>
<th>t-trend (4)</th>
<th>F-stat. (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guatemala</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Db_t$</td>
<td>8:82-12:90</td>
<td>-1.30</td>
<td>-1.99</td>
<td>-3.45</td>
<td>1.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(16.38)*</td>
<td>(1.90)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Db_t$</td>
<td>6:86-12:90</td>
<td>-1.01</td>
<td>-3.50</td>
<td>2.95*</td>
<td>5.35*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.21)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$De_t$</td>
<td>8:82-12:90</td>
<td>-1.59</td>
<td>-3.45</td>
<td>1.90</td>
<td>2.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.65)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$De_t$</td>
<td>6:86-12:90</td>
<td>-1.01</td>
<td>-3.50</td>
<td>1.50</td>
<td>1.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Dppp_t$</td>
<td>8:82-12:90</td>
<td>0.36</td>
<td>-0.59</td>
<td>3.45</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(18.30)*</td>
<td>(2.26)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Dppp_t$</td>
<td>6:86-12:90</td>
<td>3.88</td>
<td>-3.50</td>
<td>-1.20</td>
<td>21.34*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.63)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Columns (1) and (2) show the value of the t-statistic of $b_{t-1}$ for the DF and ADF tests. Between parentheses it is shown the value of the F-statistic equivalent to the LM test for first order serial correlation (except for $Dppp$ of Costa Rica and Guatemala, and $De$ of Costa Rica in the all sample period where we needed two lags). Critical values are shown in column (3). Column (4) shows the value of the t-stat. for the trend coefficient and column (5) show the F-stat. for the last regression. * Indicates significance at the 5 percent.

For Salvadoran and Guatemalan time series, the presence of a second unit root should be tested by performing a DF test with drift (but no trend) on the first-differenced series. The $t$-values for Salvadoran BMERs, official exchange rates and bilateral prices ratio on the overall period are respectively -7.28, -8.39 and -7.48, for Guatemalan series are respectively -6.79, -8.38 and -5.50 and for Costa Rican official exchange rate series (second period) is -7.52. These values are significant and indicate that the four series are integrated of order 1, i.e. are $I(1)$.

An additional, but more important, result that we obtain is that since no autocorrelation is found in residuals from the DF test for the black rates of El Salvador and Guatemala during the second period, all four series follow a random walk. This conclusion is supportive of weak market efficiency and requires to examine more closely the following results from equation (14):

\[
Db_t = -0.05Db_{t-1} + 0.001t + 0.02
\]

\[(-1.67) \quad (3.338) \quad (-.43)\]
where only the time trend coefficient is significant at the 5 percent and residuals are white noise, i.e. the BMER follow a random walk around a linear time trend, and

Guatemala (6.86-12.90)

\[ D_{b_t} = -0.04b_{t-1} + 0.001t - 0.08^* \]

\[ (-1.01) \quad (2.95) \quad (-2.46) \]

where both the time trend and drift coefficient are significant and residuals are white noise, i.e. the BMER follow a random walk with drift and trend.

For their part, the stationarity of the Costa Rican BMER time series allow us to examine more carefully results from equation (8) for both periods.

Costa Rica (10:80-12:90)

\[ b_t = 0.91b_{t-1} + 0.34t - 0.001^* \]

\[ (46.9) \quad (5.28) \quad (2.76) \]

where the slope coefficient is close to one and the constant term is close to zero, but the \( F(4,114) = 3.92 \) statistics of White’s test on heteroskedasticity is significant, indicating that it has residuals with a nonconstant variance and the OLS estimators are inefficient.

Costa Rica (12:82-12:90)

\[ b_t = 0.16b_{t-1} + 0.82t - 0.002^* \]

\[ (14.3) \quad (4.47) \quad (4.70) \]

Unlike the overall sample results, during the second period we have white noise residuals and the \( F(4,89) = 2.22 \) statistics of White’s test is not significant at the 5 percent. Thus, these results are supportive of an AR(1) process with drift and trend.

Furthermore, we tested the PPP condition for Costa Rica. For both periods, the coefficient of the bilateral prices ratio is far from one (0.86 and 0.75), the constant term is significantly different from zero and residuals are serially correlated. Thus, PPP is not supported by data for the case of Costa Rica.

\[ \text{This result is clearly reflected by Figure 2b.} \]
\[ \text{Correcting for serial correlation did not change these conclusions.} \]
Having demonstrated that Salvadoran and Guatemalan series are \( I(1) \), we can then proceed to EG two-step test for cointegration. From the discussion of Section 4, the first step requires the estimation by OLS of the cointegrating regressions on the level series. This implies to examine: First, the relationship between bilateral BMERs of El Salvador and Guatemala, see Table 5; second, the PPP condition contained in equation (16) for El Salvador and Guatemala, see Table 6 and third, the cointegration relationship between black and official rates, see Table 7.

In order to compare those relationships under different periods, the cointegrating regressions are estimated for all sample and for the second sub-period with drift and a time trend (Costa Rican case excepted). Including a time trend in the cointegrating regression is equivalent to detrending the series first. In the EG second step, all four test statistics, based upon examination of residuals and coefficient of \( \hat{w}_t \) of the ECM, are performed. Simulated critical values are given by MacKinnon (1991) and by Engle and Granger (1987).

### Table 5

Cointegrating Regressions between Bilateral BMERs
(Monthly Data)

<table>
<thead>
<tr>
<th>Test</th>
<th>All Sample</th>
<th>Critical Values</th>
<th>Second Period *</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. DW</td>
<td>0.30</td>
<td>0.39</td>
<td>0.37</td>
<td>0.39</td>
</tr>
<tr>
<td>2. DF</td>
<td>2.80</td>
<td>3.87</td>
<td>2.59</td>
<td>3.96</td>
</tr>
<tr>
<td>3. ADF</td>
<td>2.64</td>
<td>3.88</td>
<td>2.72</td>
<td>3.96</td>
</tr>
<tr>
<td>4. ARVAR</td>
<td>6.74</td>
<td>11.80</td>
<td>9.33</td>
<td>11.80</td>
</tr>
</tbody>
</table>

Cointegrating regressions:

**All Sample**

\[
b^{ES}_t = 1.26 + 0.0011 + 0.45b^G_t, \quad R^2 = 0.89;
\]

**Second Period**

\[
b^{ES}_t = 1.01 + 0.0031 + 0.48b^G_t, \quad R^2 = 0.91.
\]

*The second period runs from 6:86 to 12:90 (n = 55).*
Table 6
Cointegrating Regressions for Testing the PPP Hypothesis (Monthly Data)

<table>
<thead>
<tr>
<th>Test</th>
<th>El Salvador - BMER Rate on PPP ratio</th>
<th>Critical Values</th>
<th>Second Period *</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. DW</td>
<td>0.11</td>
<td>0.39</td>
<td>0.09</td>
<td>0.39</td>
</tr>
<tr>
<td>2. DF</td>
<td>1.53</td>
<td>3.87</td>
<td>1.52</td>
<td>3.95</td>
</tr>
<tr>
<td>3. ADF</td>
<td>2.15</td>
<td>3.89</td>
<td>1.60</td>
<td>3.95</td>
</tr>
<tr>
<td>4. ARVAR</td>
<td>8.58</td>
<td>11.80</td>
<td>3.02</td>
<td>11.80</td>
</tr>
</tbody>
</table>

Cointegrating regressions:
All Sample
\[ b_1 = 1.3 + 0.006t + 0.004\text{PPP}_t, \quad R^2 = 0.67; \]
Second Period
\[ b_1 = 1.1 + 0.02t - 0.84\text{PPP}_t, \quad R^2 = 0.58 \]

<table>
<thead>
<tr>
<th>Test</th>
<th>Guatemala - BMER Rate on PPP ratio</th>
<th>Critical Values</th>
<th>Second Period *</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Sample</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. DW</td>
<td>0.08</td>
<td>0.39</td>
<td>0.52*</td>
<td>0.39</td>
</tr>
<tr>
<td>2. DF</td>
<td>1.55</td>
<td>3.87</td>
<td>1.89</td>
<td>3.96</td>
</tr>
<tr>
<td>3. ADF</td>
<td>2.19</td>
<td>3.88</td>
<td>1.52</td>
<td>3.98</td>
</tr>
<tr>
<td>4. ARVAR</td>
<td>12.53*</td>
<td>11.80</td>
<td>4.23</td>
<td>11.80</td>
</tr>
</tbody>
</table>

Cointegrating regressions:
All Sample
\[ b_1 = 0.29 + 0.003t + 0.75\text{PPP}_t, \quad R^2 = 0.73; \]
Second Period
\[ b_1 = 0.61 - 0.007t + 1.81\text{PPP}_t, \quad R^2 = 0.93. \]

*The second period runs from February 1986 to December 1990 for El Salvador and from June 1986 to December 1990 in Guatemala. *Means that the coefficient is significant at the 5 percent level, that is to accept the hypothesis that the series are cointegrated. Critical values are taken from Engle and Granger (1987) and McKinnon (1991). In the EC regressions for the ARVAR test, the value of \( p = 1 \) was necessary to whiten residuals, except for the overall sample \( \text{PPP} \) estimation of El Salvador where \( p = 2 \) was used.
Table 7
Cointegrating Regressions between Black and Official Rates
(Monthly Data)

<table>
<thead>
<tr>
<th>Test</th>
<th>Costa Rica - BMER Rate on the Official Rate</th>
<th>El Salvador - BMER Rate on the Official Rate</th>
<th>Guatemala - BMER Rate on the Official Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All Sample</td>
<td>Critical Values</td>
<td>Second Period</td>
</tr>
<tr>
<td>1. DW</td>
<td>-</td>
<td>-</td>
<td>0.70*</td>
</tr>
<tr>
<td>2. DF</td>
<td>-</td>
<td>-</td>
<td>4.76*</td>
</tr>
<tr>
<td>3. ADF</td>
<td>-</td>
<td>-</td>
<td>4.32*</td>
</tr>
<tr>
<td>4. ARVAR</td>
<td>-</td>
<td>-</td>
<td>14.10*</td>
</tr>
</tbody>
</table>

Cointegrating regressions:

Second Period

- Costa Rica: $b_t = 0.1 + 1.00e_t; \quad R^2 = 0.99$

- El Salvador: $b_t = 0.86 - 0.002t + 0.66e_t; \quad R^2 = 0.76$

- Guatemala: $b_t = 0.70 - 0.01r + 1.28e_t; \quad R^2 = 0.90$

* The second period runs from December 1982 to December 1990 for Costa Rica, from February 1986 to December 1990 for El Salvador and from June 1986 to December 1990 in Guatemala. * Means that the coefficient is significant at the 5 percent level, that is to accept the hypothesis that the series are cointegrated. Critical values are taken from Engle and Granger (1987) and McKinnon (1991). In the EC regressions for the ARVAR test, the value of $p = 1$ was enough to whiten residuals, except for the overall period estimation of El Salvador and the second period estimation of Costa Rica where $p = 4$ was used (although in the latter case, serial correlation was not totally removed).
As can be seen from Table 5, and contrary to results from Cáceres and Núñez (1991), the evidence of four t-test statistics fails to reject the hypothesis that bilateral black rates of El Salvador and Guatemala are not cointegrated, which is consistent with single weak market efficiency in both markets. LM tests to determine if residuals are serially independent for up to fourth- and twelfth-order correlation were implemented. White noise residuals are found at lag 2, for the overall period, and at lag 1, for the second sample period, in the ADF test. The fact that the estimated cointegrating vector $a_l = 0.45$ (or 0.48) is not close to one suggests that this relationship is far from being close to a one-to-one correspondence required by semi-strong market efficiency. It is apparent that results are similar for both periods under study.57

In the same vein, and contrary to Koveos and Seifert's (1986) findings for the case of El Salvador, all four test statistics prove that there is not a long-run statistical equilibrium supporting the existence of a PPP relationship between bilateral prices ratios and BMERs of El Salvador and Guatemala. As exceptions, only the ARVAR value in the all sample period and the DW value in the second period of Guatemala appear significant at the 5 percent level. However, it must be noticed that when extra lags are required, the DW critical value is very sensitive to the particular parameters under the null and the ADF test should be preferred. As all residuals from the DF test show first degree of autocorrelation, the use of ADF tests is justified and in most cases one lag is enough to whiten residuals, except in the case of the bilateral prices ratio of Guatemala, where four lags were needed. Comparing results from both countries, PPP relationship appears as extremely weaker in the second period (particularly in El Salvador where the cointegrating vector is even negative).

57 Close examination of Cáceres and Núñez's ADF results provides with some insights about the probable sources of their wrong conclusions. First, they only use the Box-Pierce Q-statistics to test for residual autocorrelation, whereas according to Hendry (1989, p. 54) this statistics is biased towards zero when lagged dependent variables are included. Instead, an appropriated test is the LM test under the F-form suggested by Harvey (1981), because it is better behaved in small samples. Thus their ADF test might be unnecessarily overparametrised or still might not have white noise residuals. Second, besides their ADF tests, the t-statistics from their DF test is not significant and their DW value is only weakly significant at the 10% level. Third, the typical cointegration test runs both the cointegrating regression and the corresponding ADF tests on its residuals with an intercept (although this be almost identically zero). The non-inclusion of the intercept alters the distribution of the test statistics (and its critical values). Cáceres and Núñez only include the intercept in the cointegrating regression but not in their "no drift, no trend" ADF test. We replicated their test and got for the "with drift", "no-trend" case -3.30 as t-value. This value is less that the EG critical value of -3.41 for 85 observations with drift at the 5% level of significance. Thus, their t-statistic is not significant and cointegration is rejected!
Surprisingly, black and official rates in the cases of Costa Rica and El Salvador for the second period do appear to be cointegrated. This conclusion is stronger in the case of Costa Rica than in the case of El Salvador. In the latter case, both DF tests did not test positive, but others tests did so; given the limited number of observations, we accepted cointegration. Then, for both particular cases, it is possible to test sufficient conditions for semi-strong market efficiency.\textsuperscript{58} Although the coefficient $\hat{\delta}_1$ is equal or close to one, the nonstationary nature of both series makes this result conclusive. Instead, residuals from both cointegrating regressions do not appear to be white noise\textsuperscript{59} and the estimated $\hat{k}_t$ coefficients, 0.18 for Costa Rica and 0.35 for El Salvador, do appear to be far from one. Thus, semi-strong market efficiency between official and black rates is rejected.

All error correction equations were estimated sequentially testing for significance and residual autocorrelation in a single to general specification search procedure. We tested up to sixth- and twelfth-order autocorrelation by using LM tests. Except for all sample estimate in the bilateral BMERs relationship between El Salvador and Guatemala (where $p = 4$ was necessary), in the all sample PPP estimation for El Salvador (where $p = 2$ was required) and in the cointegrating black to official estimation for the all sample period of El Salvador and the second period of Costa Rica (where $p = 4$ was used), $p = 1$ was enough to whiten residuals in the remaining equations.

7. Conclusions

The purpose of this paper has been to show the advantages of using the theory of cointegration to define without any ambiguity if a market is efficient in the weak or semi-strong form. Before applying it to the BMERs of three Central American countries, it is clearly illustrated how the presence of nonstationary series affects the estimation and interpretation of traditional testing of market efficiency.

Empirical findings from unit root tests report mixed results. For all sample tests on all BMERs, there is no evidence of weak market efficiency. However, during the second period under examination, where the official exchange rate market has been reunified, the black rates of El Salvador and Guatemala satisfy weak market efficiency, i.e. follow a random walk with trend in the case of El Salvador and with drift and trend in the case of Guatemala. This result suggests that by eliminating multiple official

\textsuperscript{58} See Section 4.  
\textsuperscript{59} $Q(24) = 99.7$ for Costa Rica and $Q(15) = 107.8$ for El Salvador. Therefore, residuals from both cointegrating equations are serially correlated, i.e. are not white noise.
exchange rates, government authorities do reach at least some success at eliminating market inefficiencies which had favorized previous speculative pressures against the black and official rates. In the reunified regime, as the market becomes efficient, investors who do not obtain supranormal returns on positions taken on black currency anymore lose interest at their participation on it, consequently reduce their volume of operations and the premium decreases. In the case of the BMER of Costa Rica, it is shown that it follows an AR(1) process with drift and trend, so that the current BMER does not summarize all relevant information to forecast the future BMER.

Consistent with weak market efficiency, and contrary to what we expected, no cointegration is found between the BMERs of El Salvador and Guatemala. Previous research supporting this hypothesis is carefully shown to be flawed. Therefore, even though both markets have relevant close ties, cointegration test show some power at determining that both black rates behaved independently one to each other during both periods.

The paper also provides little empirical support for the cointegration of the bilateral prices ratio between each one of the three Central American countries with respect to the United States, and their BMERs. In no instance, and for a monthly sampling frequency, does the cointegration test find support for cointegration. Consequently, there is little justification for regarding the BMERs of all three countries as a reliable measure of their external trade competitiveness, particularly with respect to the United States.

On the other hand, black and official rates are cointegrated during the reunified managed dirty floating and crawling-peg set regimes of El Salvador and Costa Rica respectively. This result strongly justify the implementation of exchange rate rules which explicitly link the official rate to the black rate, thus keeping the premium close to a constant predetermined low range. However, the implicit assumption that by following this rule, the official rate will remain close to the equilibrium real exchange rate predetermined by PPP is not necessarily true. Furthermore, the hypothesis of semi-strong market efficiency between both black and official rates finds poor empirical support.

Although these results might be affected by several modifications regarding the sample frequency (for example, the use of quarterly data instead of monthly data), the definition of the "predictor" variable (for example, the official exchange rate in another semi-strong form of the EMH) or the sampling size, possible explanations for the lack of empirical support for cointegration found here are: Price stickeness, changing internal supply, and demand

60 The existence of a significant nonzero drift or time trend coefficient respectively means that the rate of change is not equal to zero in this market and that this rate is steadily increasing through time.
conditions on the black currency, the existence of fundamental variables omitted from estimation and the presence of speculative feedback strategies affecting the foreign exchange market. Unfortunately, our test is unable to determine which factor (or factors) causes these rejections.

References


