PRODUCTIVITY, STRUCTURAL CHANGE IN EMPLOYMENT AND ECONOMIC GROWTH*

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Resumen: Se desarrolla un modelo de crecimiento endógeno con dos sectores, manufacturero y no-manufacturero. El sector manufacturero es la fuente del crecimiento equilibrado de la productividad. Se estudia como la economía responde a cambios en la productividad específica de sector. Así, cuando la productividad específica de sector en el sector manufacturero aumenta, se encuentra que la fracción del trabajo empleado en el sector manufacturero sigue una curva V invertida, y que la tasa de crecimiento aumenta. Así, el modelo aproximadamente captura el patrón documentado de desarrollo para la participación del empleo manufacturero, una forma de campana en el tiempo. Cuando la productividad específica de sector en el sector no-manufacturero aumenta, la tasa de crecimiento permanece sin cambio porque el sector no-manufacturero es el sector de no-aprendizaje.

Abstract: We develop an endogenous growth model with two sectors, manufacturing and non-manufacturing. The manufacturing sector is the source of the balanced productivity growth. We study how the economy responds to shifts in sector-specific productivity. Thus, when the sector-specific productivity in the manufacturing sector increases, we find that the fraction of labor employed in the manufacturing sector follows an inverted V curve, and that the growth rate increases. Thus, the model captures approximately the documented pattern of development for the share of manufacturing employment, a bell shape over time. When the sector-specific productivity in the non-manufacturing sector increases, the growth rate remains unchanged because the non-manufacturing sector is the non-learning sector.

Clasificación JEL: O14, O30, O41

Palabras clave: sector manufacturero, aprendizaje por la práctica, crecimiento, productividad, cambio estructural, manufacturing sector, learning by doing, productivity, structural change, growth

Fecha de recepción: 29 XII 2006 *Fecha de aceptación: 22 VI 2007*

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1. Introduction

Long-term employment shift patterns have been documented for a number of economies. In particular, it has been shown that employment shifts from agricultural to manufacturing and services as income per capita increases. Further, it has been shown that the relationship between the share of manufacturing employment and the level of economic development resembles an inverted U curve.[1] In order to explain these facts, it is often pointed out that agriculture and manufacturing and services have had different rates of labor productivity growth over time. Alternatively, it is argued that the income elasticity of demand for agricultural goods is low, and that for services is high, while that for manufacturing goods is moderately high. Thus, two approaches to explaining this structural pattern emerge: one explains it in terms of differences in productivity growth among sectors (supply side explanation), the other by differences in income elasticity of demand for goods (demand side explanation).

Recently, researchers have developed models of economic growth that generate such structural change in employment. For example, in the growth literature invoking differences in income elasticity among goods, Matsuyama (1992) assumes, in a two sector endogenous growth model, that the income elasticity of demand for the agricultural good is less than unitary and that the manufacturing sector is the only sector with learning. He then shows that, in a closed economy, higher productivity in the agriculture sector releases labor to the industry sector and the aggregate growth rate increases. Foellmi and Zweimüller (2005) present a model where each new good starts out as a luxury with a high income elasticity and ends up as a necessity with a low income elasticity (hierarchy of needs) and in which endogenous growth is driven by R&D. They show that, along the steady path, structural change takes the form of a reallocation of resources from old to new industries, and that the aggregate growth rate is constant.[2]

In the literature on economic growth, invoking differences in productivity growth among sectors, Ngai and Pissarides (2007) show that if demand is price inelastic, employment moves from the sector with the higher exogenous TFP growth rate to the sector with the lower

[1] Kuznets (1973) and Maddison (1980) observe this structural change in employment for advanced economies. The share in manufacturing employment is falling in Latin American economies (see ECLAC 2002).

[2] For demand side explanations, see also Kongsamut, Rebelo and Xie (2001) and Caselli and Coleman II (2001).

TFP growth rate. Moreover, they show that, given an intertemporal elasticity of substitution equal to one, the aggregate growth rate is constant over time (see also Ngai and Pissarides, 2004). Acemoglu and Guerrieri (2006) develop a two-sector model with non-balanced endogenous technological change in which, in the process of capital deepening, the sector with lower capital share attracts capital and labor. They show that the transition path is consistent with the pattern of employment described above, as well as with the Kaldor's facts.[3]

The objective of this paper is to generate the bell shaped intertemporal pattern in the share of manufacturing employment, using a supply-side explanation.

A two sector economy (manufacturing and non-manufacturing) is taken to be closed or, alternatively, the economy can be seen as being open but without capital mobility (thus the balance of trade is zero at all times). In the open economy interpretation, the manufacturing sector corresponds to the tradable sector, and the non-manufacturing sector to the non-tradable sector. The relative price of the non-tradable good is interpreted as the real exchange rate.

We propose these alternative interpretations because we want to stress that an open economy with imperfect capital mobility can behave as a closed economy. Klyuev (2005) stresses this equivalence. He emphasizes that the results about structural change and growth for closed economies hold as long as imperfect capital mobility creates room for domestic interest rate movements.

A two sector endogenous growth model with manufacturing and non-manufacturing goods is analyzed. We assume that the two goods are accumulated. There are two sources of productivity: balanced productivity growth that affects both sectors, and sector specific productivities (see Bergin, Glick and Taylor, 2004). With respect to balanced productivity growth, technological knowledge can only be produced in the manufacturing sector through learning by doing. The non-manufacturing (non-learning) sector can use this knowledge. Technological knowledge generates endogenous growth. Shifts in sector specific productivity account for structural change in employment.

We study how the relative price of the non-manufacturing good, employment in each sector and aggregate growth respond to shifts in sector specific productivity parameters. Thus, when the sector specific productivity in the manufacturing sector increases, the marginal productivity of labor in the manufacturing sector increases, and the

[3] For supply-side explanations, see also Oulton (2001). See Barro and Sala-i-Martin (2004) for Kaldors facts.

fraction of labor employed in the manufacturing sector jumps (labor is freely mobile). While the fraction of labor employed in the non-manufacturing sector instantaneously decreases, the non-manufacturing good market finds itself in excess demand. Since the relative price of the non-manufacturing good is flexible, the relative price immediately increases, and the aggregate growth rate immediately increases. While the economy moves to the new steady state, the relative price increases, the fraction of labor employed in the manufacturing (non-manufacturing) decreases (increases), and the aggregate growth rate decreases.[4] In the new steady state, the fraction of labor employed in the manufacturing sector remains the same, the relative price of the non-manufacturing good is higher, and the aggregate growth rate is higher. In the transition, the fraction of labor employed in the manufacturing sector follows an inverted V curve. Thus, the model captures approximately the documented pattern of development for the share of manufacturing employment, a bell shape over time.

When the sector-specific productivity in the non-manufacturing sector increases, the transition is the opposite of the previous case. In the new steady state, the fraction of labor employed in the manufacturing sector is unchanged, the relative price of the non-manufacturing good is lower, and the aggregate growth rate is unchanged as well. The aggregate growth rate remains unchanged because the manufacturing sector is the only source of the balanced productivity growth, and thus, the shift in sector-specific productivity in the non-manufacturing sector does not affect the long term growth rate.

The paper is organized as follows. In section 2, we develop the two-sector endogenous growth model. In section 3, we construct a system of differential equations describing the economy. In section 4, we study the steady state and the dynamics of the model. In sections 5 and 6, we study how the economy responds to shifts in the sector-specific productivity parameter in the manufacturing sector and in the non-manufacturing sector, respectively. In section 7, we present our conclusions.

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[4] The relative prices of services have increased over time, so the share of employment in services has increased, possibly by higher productivity growth in manufacturing than in services, see Baumol (1967) and Obstfeld and Rogoff (1996). For 1980-2001, labor productivity growth has been higher in manufacturing than in services in OECD countries (see Wölfl, 2005).

2. The Economy

We assume that the two goods, manufacturing and non-manufacturing (formed by structure and services), are produced, consumed and accumulated. For simplicity, manufacturing is the only sector that generates technological knowledge through learning by doing. Knowledge produced in the manufacturing sector becomes available to the non-manufacturing sector. Thus, we have two learning externalities in the model. The output in each sector is produced through physical capital, labor and technological knowledge. Labor is freely mobile between the two sectors. The total labor supply is constant.

2.1. *The Production Functions*

We assume that the production function of the manufacturing sector is Cobb-Douglas:

$$Y_M = A_M K_M^{\alpha} L_M^{1-\alpha} E_1 \tag{1}$$

where Y_M is the output in manufacturing, A_M is the sector specific productivity in the sector, K_M is the stock of physical capital accumulated from the manufacturing good, L_M is the quantity of labor employed in the sector, α and $1-\alpha$ are the shares of K_M and L_M respectively and E_1 is a learning externality. The stock of K_M is used only in the manufacturing sector.

Technological knowledge is created through learning by doing in the manufacturing sector. Thus, knowledge is a by-product of investment. Therefore, E_1 is the external effect of K_M on the production function of the manufacturing sector. In order to generate endogenous growth, we assume that $E_1 = K_M^{1-\alpha}$, so the production function of the manufacturing sector has constant returns with respect to a broad measure of capital (see Romer, 1989).

Now, it is convenient to define the two sources of productivity. Knowledge generated through learning by doing is balanced productivity growth that affects both sectors. This type of productivity growth can not affect relative prices. A shift in A_M only affects the manufacturing sector. This type of productivity can affect relative prices.

We use the manufacturing good as the numeraire ($P_M = 1$). Alternatively, if the economy is open without capital mobility, where trade is balanced, P_M can be interpreted as the world price of the tradable good. We assume that K_M has a zero depreciation rate.

Thus, the rental price of K_M is $R_M = r_M$, where r_M is the rate of return in the manufacturing sector. The firms in the manufacturing sector maximize profits taking the externality as given. The first order conditions are:

$$w_M = A_M K_M (1-\alpha) L_M^{-\alpha} \tag{2}$$

$$r_M = A_M \alpha L_M^{1-\alpha} \tag{3}$$

Equation (2) states that the wage rate is equal to the value of the marginal product of labor in the manufacturing sector. Equation (3) states that the rate of return is equal to the marginal product of K_M.

With respect to the non-manufacturing sector, the production function is Cobb-Douglas:

$$Y_N = A_N K_N^{\beta} L_N^{1-\beta} E_2 \tag{4}$$

where Y_N is the output in non-manufacturing, A_N is the sector specific productivity in the sector, K_N is the stock of physical capital accumulated from the non-manufacturing good, L_N is the quantity of labor employed in the non-manufacturing sector, β and $1-\beta$ the shares of K_N and L_N respectively and E_2 is an externality. The stock of K_N is used only in the non-manufacturing sector.

There are spillover effects of knowledge between the sectors. Thus, E_2 is the contribution of technological knowledge (generated in the manufacturing sector) to the non-manufacturing sector. Moreover, in order to generate endogenous growth, we assume that $E_2 = K_M^{1-\beta}$, so the production function of the non-manufacturing sector has constant returns to a broad measure of capital.

We define p_N as the relative price of the non-manufacturing good in terms of the manufacturing good. Alternatively, if the economy is open without capital mobility, where trade is balanced, p_N can be interpreted as the real exchange rate. We assume that K_N has a zero depreciation rate. Thus, the rental price of K_N is $R_N = (r_N - \dot{p}_N/p_N)$, where r_N is rate of return in the non-manufacturing sector and $\dot{p}_N/p_N$ is the growth rate of p_N. The non-manufacturing firms maximize profits taking the externality as given. The first order conditions are:

$$w_N = p_N A_N K_N^{\beta} K_M^{1-\beta} (1-\beta) L_N^{-\beta} \tag{5}$$

$$r_N = A_N \beta K_N^{\beta-1} K_M^{1-\beta} L_N^{1-\beta} + \frac{\dot{p}_N}{p_N} \tag{6}$$

Equation (5) states that the wage rate is equal to the value of the marginal product of labor in the non-manufacturing sector. Equation (6) is the dynamic equilibrium condition for K_N. It states that rate of return is equal to the marginal product of K_N plus capital gains.

We note that the variables K_M and K_N grow permanently. Thus, in order to solve the model, it will be convenient to define the equations of the model in terms of stationary variables. The characteristic of these variables is that they remain constant in the steady state (see Barro and Sala-i-Martin, 2004). Thus, let $z = K_N/K_M$ be the first stationary variable. Moreover, we assume that the total labor supply, L, is constant and normalized to one. Thus, the labor market equilibrium condition is $L_M + L_N = L = n + (1-n) = 1$, where n is the fraction of labor employed in the manufacturing sector and $(1-n)$ is the fraction of labor employed in the non-manufacturing sector. As n is constant in the steady state, we can use it as the second stationary variable.

Thus, we can rewrite the production function of the manufacturing sector in terms of the stationary variables as:

$$Y_M = A_M K_M n^{1-\alpha} \tag{7}$$

and the marginal conditions can be rewritten in term of stationary variables as:

$$w_M = A_M K_M (1-\alpha) n^{-\alpha} \tag{8}$$

$$r_M = A_M \alpha n^{1-\alpha} \tag{9}$$

Likewise, the production function of the non-manufacturing sector can be rewritten in terms of the stationary variables as:

$$Y_N = A_N K_M z^{\beta} (1-n)^{1-\beta} \tag{10}$$

and the marginal conditions in term of stationary variables as:

$$w_N = p_N A_N z^{\beta} K_M (1-\beta)(1-n)^{-\beta} \tag{11}$$

$$r_N = A_N \beta \mathrm{z}^{\beta-1} (1-n)^{1-\beta} + \frac{\dot{p}_N}{p_N} \tag{12}$$

We assume that $\alpha > \beta$, so the manufacturing sector is more intensive in capital than the non-manufacturing sector (see Obstfeld and Rogoff, 1996). Turnovsky (2000) analyzes this type of assumption.

2.2. *Individuals*

The flow budget constraint of individuals is:

$$Y_M + p_N Y_N = C_M + p_N C_N + I_M + p_N I_N \tag{13}$$

where $Y_M + p_N Y_N = Y$ is total output or total income, C_M is consumption of the manufacturing good, C_N is consumption of the non-manufacturing good, I_M is investment in K_M and I_N is investment in K_N.

For simplicity, there is no intertemporal choice, so the saving rate is always constant. Therefore, we assume that the total expenditure on consumption is a fixed fraction of the total income:

$$p_C C = \hat{c}(Y_M + p_N Y_N) \tag{14}$$

where p_C is the consumer's relative price index, C is aggregate real consumption and $\hat{c}$ is the propensity to consume, which is constant $(0 < \hat{c} < 1)$.

Given the total consumption level, equation (14), the consumption basket, divided among manufacturing and non-manufacturing goods, is determined by a static utility maximization. Thus, the individual maximizes:

$$u = BC_M^{\gamma} C_N^{1-\gamma} \tag{15}$$

subject to the total expenditure on consumption $p_C C = C_M + p_N C_N$, where $B = 1/[\gamma^{\gamma}(1-\gamma)^{1-\gamma}]$ is a parameter, γ and $1-\gamma$ are the shares of C_M and C_N with respect to total expenditure on consumption, respectively. The consumer relative price index can be defined as $p_C = p_N^{1-\gamma}$. Then, the demand of C_M is:

$$C_M = \gamma p_C C \tag{16}$$

and the demand of C_N is:

$$C_N = (1-\gamma)\frac{p_C C}{p_N} \tag{17}$$

2.3. *Equilibrium in Markets*

The price of the non-manufacturing good is flexible, ensuring that this market is always balanced. The equilibrium condition for the non-manufacturing good market is:

$$p_N Y_N = p_N C_N + p_N I_N \tag{18}$$

With equations (13) and (18), we obtain the equilibrium condition for the manufacturing good market:

$$Y_M = C_M + I_M \tag{19}$$

3. The Dynamic System

We can form a system of differential equations for the two stationary variables. Thus, we can obtain a dynamic system:

$$\begin{aligned} \dot{z} &= f_1(z, n) \\ \dot{n} &= f_2(z, n) \end{aligned} \tag{20}$$

where f_1 and f_2 will be nonlinear functions. We now proceed to deduce the first differential equation of (20). Using the definition of z, the growth rate of z is:

$$\frac{\dot{z}}{z} = \frac{\dot{K}_N}{K_N} - \frac{\dot{K}_M}{K_M} \tag{21}$$

where $\dot{K}_N/K_N$ is the growth rate of K_N and $\dot{K}_M/K_M$ is the growth rate of K_M.

In order to obtain the growth rate of K_N, we use the equilibrium condition for the non-manufacturing good market, (18), with the production function of the non-manufacturing sector, (10), the level of C_N, (17), and the identity $I_N = \dot{K}_N$, and we get:

$$\frac{\dot{K}_N}{K_N} = \frac{A_N(1-n)^{1-\beta}}{z^{1-\beta}} - \frac{(1-\gamma)}{p_N}\frac{p_C C}{K_N} \tag{22}$$

In order to obtain the value of p_N, we can equate equations (8) and (11) and we obtain the static efficient allocation condition for labor between the sectors, $w_M = w_N$. This condition states that the value of the marginal product of labor in both sectors must be equal. Thus, the static efficient allocation condition is:

$$A_M(1-\alpha)n^{-\alpha} = p_N A_N z^{\beta}(1-\beta)(1-n)^{-\beta} \tag{23}$$

with equation (23), we can obtain p_N. In order to find $p_C C/K_N$, we use equations (7), (10) and (14), and we get:

$$\frac{p_C C}{K_N} = \hat{c}\left[\frac{A_M n^{1-\alpha}}{z} + \frac{p_N A_N (1-n)^{1-\beta}}{z^{1-\beta}}\right] \tag{24}$$

Likewise, considering the equilibrium condition for the manufacturing good market, (19), with the production function of the manufacturing sector, (7), the level of C_M, (16), and the identity $I_M = \dot{K}_M$, we obtain:

$$\frac{\dot{K}_M}{K_M} = A_M n^{1-\alpha} - \gamma z \frac{p_C C}{K_N} \tag{25}$$

Then the growth rate of z is given by equations (21), (22), (23), (24) and (25).

Next, we can obtain the second differential equation of (20). Taking logs and derivatives of both sides of the efficient allocation condition for labor, (23), we get the growth rate of n:

$$\frac{\dot{n}}{n} = \frac{(1-n)}{[\alpha(1-n)+\beta n]}\left[-\frac{\dot{p}_N}{p_N} - \beta\frac{\dot{z}}{z}\right] \tag{26}$$

In order to obtain the value of the growth rate of p_N, we can equate equations (9) and (12) and we obtain the dynamic arbitrage condition for the two capital goods, $r_M = r_N$. This dynamic condition states that the total returns for both capital goods must be the same, so the marginal product of K_M is equal to the marginal product of K_N plus capital gains on K_N. We can write the dynamic arbitrage condition for the two capital goods as:

$$\frac{\dot{p}_N}{p_N} = A_M \alpha \mathrm{n}^{1-\alpha} - A_N \beta \mathrm{z}^{\beta-1}(1-n)^{1-\beta} \tag{27}$$

Then the growth rate of n is given by equations (21), (22), (23), (24), (25), (26) and (27). We can see that the dynamic system (20) only depends on z, n, and parameters. In the next section, we will show that z is a sluggish variable and n is a jump variable.

Finally, it can be shown that the growth rate of the total output is:

$$\begin{aligned}\frac{\dot{Y}}{Y} &= \frac{Y_M}{Y}\left[\frac{\dot{K}_M}{K_M} + (1-\alpha)\frac{\dot{n}}{n}\right] \\ &\quad + \frac{p_N Y_N}{Y}\left[\beta\frac{\dot{z}}{z} + \frac{\dot{K}_M}{K_M} - (1-\beta)\frac{\dot{n}}{n}\frac{n}{(1-n)} + \frac{\dot{p}_N}{p_N}\right]\end{aligned} \tag{28}$$

where $Y_M/Y = A_M n^{1-\alpha}/[A_M n^{1-\alpha} + p_N A_N z^{\beta}(1-n)^{1-\beta}]$ is the share of Y_M in the value of total output and $p_N Y_N/Y = p_N A_N z^{\beta}(1-n)^{1-\beta}/[A_M n^{1-\alpha} + p_N A_N z^{\beta}(1-n)^{1-\beta}]$ is the share of $p_N Y_N$ in the value of total output. We note that the growth rate of the economy refers, in this model, to the total output growth rate.

4. The Steady State and Dynamics

In the steady state the values of z and n are constant, so the growth rates of the stationary variables are zero. With $\dot{z} = 0$, we have that $\dot{K}_N/K_N = \dot{K}_M/K_M$, so the two capital goods grow at the same rate in the steady state. Moreover, it is easy to show that Y_M, Y_N and Y grow at the same rate as K_M and K_N. Furthermore, given that p_N depends on z, n and parameters, see equation (23), we have that p_N is constant in the steady state. Therefore, given that the growth rates of K_M, K_N, Y_M, Y_N, and Y depend on z, n, p_N and parameters, we have that the common growth rate is constant and equals to g^*. We denote the steady state values of the variables with an *.

Now, we solve the dynamic system (20) in the steady state. With $\dot{z} = 0$, we have:

$$\begin{aligned}&\frac{A_N(1-n)^{1-\beta}}{z^{1-\beta}} - \frac{(1-\gamma)\hat{c}}{p_N}\left[\frac{A_M n^{1-\alpha}}{z} + \frac{p_N A_N(1-n)^{1-\beta}}{z^{1-\beta}}\right] \\ &\qquad = A_M n^{1-\alpha} - \gamma\hat{c}z\left[\frac{A_M n^{1-\alpha}}{z} + \frac{p_N A_N(1-n)^{1-\beta}}{z^{1-\beta}}\right]\end{aligned} \tag{29}$$

where p_N is given by equation (23). Moreover, with $\overset{\bullet}{n} = 0$ and $\overset{\bullet}{p}_N = 0$, we get:

$$A_M \alpha n^{1-\alpha} = A_N \beta z^{\beta-1}(1-n)^{1-\beta} \tag{30}$$

In order to find the steady state equilibrium, we need to solve equations (23), (29) and (30). First, we rewrite the previous equation (30) as:

$$\frac{(1-n)^{1-\beta}}{z^{1-\beta}} = \frac{A_M \alpha n^{1-\alpha}}{A_N \beta} \tag{31}$$

Substituting equation (31) in (29), we get:

$$\begin{aligned} &\frac{\alpha}{\beta} - (1-\gamma)\hat{c}\left[\frac{1}{p_N z} + \frac{\alpha}{\beta}\right] \\ &\quad = 1 - (\gamma\hat{c})\left[1 + p_N z \frac{\alpha}{\beta}\right] \end{aligned} \tag{32}$$

Manipulating the previous equation, we can obtain a polynomial:

$$v^2 + Dv + F = 0 \tag{33}$$

where $v = p_N z$. The coefficient D of the polynomial is:

$$D = \left[\frac{\beta}{\alpha} + \frac{(\alpha-\beta)}{\gamma\hat{c}\alpha} - \frac{(1-\gamma)}{\gamma}\right] \tag{34}$$

and the coefficient F of the polynomial is:

$$F = \frac{(\gamma-1)}{\gamma}\frac{\beta}{\alpha} \tag{35}$$

Thus, the polynomial has two roots: $v_1, v_2 = \lfloor -D \pm (D^2 - 4F)^{1/2} \rfloor / 2$. The coefficient D can be positive or negative and the coefficient F is always negative, given that $(\gamma - 1) < 0$. We know that $F = v_1 \cdot v_2$, so the two roots will be of opposite sign and there will be exactly one positive root. Therefore, the economy only has one positive solution.

Next, with the value of v, we can calculate the steady state value of n. Thus, using the static efficient allocation condition for labor $(w_M = w_N)$ and the dynamic arbitrage condition for the two capital goods ($r_M = r_N$), we can obtain that $v = (1-\alpha)\beta(1-n)/(1-\beta)\alpha n$. We can rewrite the previous equation as:

$$n^* = \frac{1}{v\frac{\alpha}{\beta}\frac{(1-\beta)}{(1-\alpha)} + 1} \tag{36}$$

so we have found the value of n in the steady state. With n^* and equation (30), we get the value of z in the steady state:

$$z^* = \left(\frac{A_N}{A_M}\right)^{1/(1-\beta)} \left(\frac{\beta}{\alpha}\right)^{1/(1-\beta)} \frac{(1-n^*)}{n^{*(1-\alpha)/(1-\beta)}} \tag{37}$$

With equations (36), (37) and (23), we obtain the steady state value of p_N:

$$p_N^* = \frac{A_M}{A_N}\frac{(1-\alpha)}{(1-\beta)}\frac{(1-n^*)^\beta}{n^{*\alpha} z^{*\beta}} \tag{38}$$

Using equations (25) and (23), we obtain the steady state growth rate:

$$g^* = A_M \left[n^{*1-\alpha}(1-\gamma\hat{c}) - \gamma\hat{c}\left(\frac{(1-\alpha)}{(1-\beta)}\frac{(1-n^*)}{n^{*\alpha}}\right)\right] \tag{39}$$

RESULT 1. The economy in the steady state has only one positive solution for the two stationary variables (n, z), for the relative price of the non-manufacturing good and for the aggregate growth rate.

We present an illustrative numerical case. We use the following parameter values: $\alpha = 0.5$, $\beta = 0.3$, $\gamma = 0.4$, $A_M = 0.35$ and $A_N = 0.35$. The steady state values of the variables are: $z^* = 0.6590$, $n^* = 0.3518$, $p_N^* = 1.1983$, $Y_M^* = 0.4318$ and $g^* = 0.0537$. Thus, the steady state growth rate of the economy is 5.37% per year.

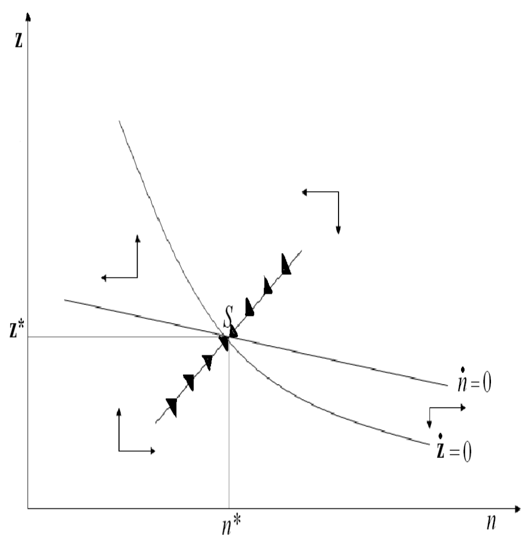
With respect to the transitional dynamics of the model, we calculate the Jacobian of the nonlinear system, equation (20), evaluated at the equilibrium, and we obtain that $\partial f_1/\partial z < 0$, $\partial f_1/\partial n < 0$, $\partial f_2/\partial z < 0$ and $\partial f_2/\partial n > 0$ (see appendix). Thus, we get that $|J| < 0$, so we have one negative characteristic root and one positive characteristic root. Given that z is the ratio of capitals, z is a sluggish (predetermined) variable. Moreover, given that there is no adjustment cost of labor, or migratory mechanism, we deduce that n is a jump variable. Thus, we have that the number of jump variables equals the number of the positive roots. Therefore, the model turns out to be locally saddle path stable.

In figure 1, we present the phase diagram in the region of the positive steady state equilibrium. From equation (A.1), see appendix, we obtain the $\dot{z} = 0$ schedule with a negative slope. From equation (A.2), we obtain the $\dot{n} = 0$ schedule with a negative slope. We can show that the slope of the $\dot{z} = 0$ curve is more negative than the slope of the $\dot{n} = 0$ curve (around the positive steady state). The value of z is decreasing at points above the $\dot{z} = 0$ locus and it is increasing at points below the $\dot{z} = 0$ locus. The value of n is decreasing at points above the $\dot{n} = 0$ locus and it is increasing at points below the $\dot{n} = 0$ locus. The slope of the stable arm is positive (the slope of the eigenvector is positive). Thus, the economy converges to the steady state, point S, if it starts in the stable saddle path.

RESULT 2. The economy is locally saddle path stable with a positive slope stable arm where n is a jump variable and z is a sluggish variable.

Figure 1

The Phase Diagram



5. Sector-Specific Productivity Shift in Manufacturing

We study how the variables of the model respond when the sector specific productivity in the manufacturing sector increases. First, notice that the coefficients D and F of the polynomial, (33), do not depend on the sector specific productivity parameters, A_M or A_N. Thus, an increase in A_M does not change the value of $v = p_N z$, so n^* does not change.

With respect to the ratio of capitals, with equation (37), we obtain that $\partial z^*/\partial A_M < 0$. Thus, when A_M increases, the rate of return in the manufacturing sector increases, so investment in K_M is stimulated and investment in K_N is discouraged and the level of z^* decreases. With regard to the relative price of non-manufacturing good, the decline of z^* produces excess demand in the non-manufacturing good market, so the value of p_N^* increases. Moreover, given that $v = p_N z$ is constant, the decline in z^* is compensated by the raise in p_N^*. With equation (39), we obtain that $\partial g^*/\partial A_M > 0$, so the steady state growth rate increases. Finally, we can show that the values of the shares Y_M^*/Y^* and $p_N^* Y_N^*/Y^*$ do not change.

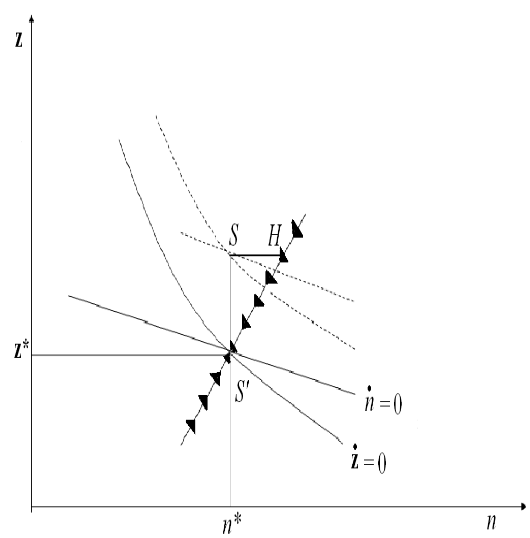
RESULT 3. In the steady state, when sector specific productivity is increased in the manufacturing sector, the value of n^* does not change and the value of z^* decreases. Moreover, the relative price of the non-manufacturing good increases and the aggregate growth rate of the economy increases.

Now, we present an illustrative numerical simulation when A_M increases from 0.35 to 0.4. The steady state values of the variables are: $z^* = 0.5445$, $n^* = 0.3518$, $p_N^* = 1.4502$, $Y_M^*/Y^* = 0.4318$ and $g^* = 0.0614$. We can see that the variables move in the predicted direction. The long run growth rate of the economy increases from 5.37% to 6.14% per year.

In figure 2, we present the transitional dynamics of the economy. When A_M increases, the $\dot{n} = 0$ and $\dot{z} = 0$ schedules move downward and the steady state moves from S to S'. Given that z is a sluggish variable and n is a jump variable, the path of adjustment is composed of a jump at time 0 from S to H, and a movement over time from H to S'. Thus, the value of the marginal productivity of labor in the manufacturing sector increases when A_M increases, so the value of n immediately jumps to H. Moreover, given that $(1-n)$ instantaneously decreases, the non-manufacturing good market is in excess demand, so p_N immediately increases. Likewise, the growth rate of the economy instantaneously increases. Meanwhile, $\dot{p}_N/p_N$ becomes positive (see equation 27). While the economy moves to the new steady state, p_N increases, n decreases, z decreases and the growth rate decreases. The level of n tends to the same steady state value.

Figure 2

The Effects of an Increase in Sector Specific Productivity in the Manufacturing Sector



RESULT 4. In the transition, when sector specific productivity is increased in the manufacturing sector, the fraction of labor employed in the manufacturing sector increases instantly, after that n decreases slowly and z decreases gradually to the new steady state. Thus, the fraction of labor employed in the manufacturing sector follows an inverted V curve.

Thus, the model captures approximately the pattern of development for the share of manufacturing employment, a bell shape over time.

6. Sector-Specific Productivity Shift in Non-Manufacturing

Now, we analyze how the variables of the model respond when the sector specific productivity in the non-manufacturing sector increases.

Thus, when A_N increases, the rate of return in the non-manufacturing sector increases, so the investment in is stimulated and the investment in K_N is discouraged and the level of z^* increases. With higher ratio of capitals, the non-manufacturing sector is in excess supply and the value of p_N^* decreases. Using equation (39), we can see that the growth rate does not change. The aggregate growth remains invariable because the manufacturing sector is the source of learning process. Thus, a shift in the sector specific productivity in the non-manufacturing sector does not affect the leading rate of return in the manufacturing sector, equation (9), and therefore A_N does not affect the aggregate growth rate. Finally, we can show that the values of the shares Y_M^*/Y^* and $p_N^* Y_N^*/Y^*$ do not change.

RESULT 5. In the steady state, when sector-specific productivity is increased in the non-manufacturing sector, the value of n^* does not change and the value of z^* increases. Moreover, the relative price of the non-manufacturing sector good decreases and the aggregate growth rate remains invariable.

We now present an illustrative numerical simulation when A_N increases from 0.35 to 0.4. The steady state values of the variables are: $z^* = 0.7975$, $n^* = 0.3518$, $p_N^* = 0.9902$, $Y_M^*/Y^* = 0.4318$ and $g^* = 0.0537$. Again, we can see that the variables move in the predicted direction. The long run growth rate of the economy has the same value.

The transition is opposite to the previous case. We know that $\dot{n} = 0$ and $\dot{z} = 0$ schedules move upward when A_N increases. The value of marginal productivity of labor in the non-manufacturing sector increase, so labor flows to this sector and the level of n immediately decreases. With higher $(1 - n)$, the non-manufacturing good market is in excess supply, so p_N decreases immediately. Likewise, the growth rate of the economy instantaneously decreases. The growth rate of p_N becomes negative. While the economy moves to the new steady state, p_N decreases, n increases, z increases and the growth rate increases. Thus, the value of n tends to the same steady state value.

RESULT 6. In the transition, when sector specific productivity is increased in the non-manufacturing sector, the fraction of labor employed in the manufacturing sector decreases instantly, after that n increases slowly and z increases gradually to the new steady state.

7. Conclusions

We have developed an endogenous growth model with manufacturing and non-manufacturing goods. We have assumed that the manufacturing sector is the learning sector. The non-manufacturing sector can use knowledge generated in the learning sector. In particular, we have studied how the relative price of the non-manufacturing good, the fraction of labor employed in the manufacturing sector and the aggregate growth rate respond to shifts in the sector-specific productivity parameters.

We have found results. First, we have concluded that when the sector-specific productivity in the manufacturing sector increases, in the transition, the fraction of labor employed in the manufacturing sector follows an inverted V curve. Therefore, the model captures approximately the pattern of development for the share of manufacturing employment, a bell shape over time. Thus, we have reproduced an import feature of the development process. When the sector-specific productivity in the non-manufacturing sector increases, the dynamics of the fraction of labor employed in the manufacturing sector is opposite to the previous case. In the steady state there is no structural change in employment.

Second, we have also concluded that when sector-specific productivity in the manufacturing sector increases, the aggregate growth rate increases in the steady state. When sector-specific productivity in the non-manufacturing sector increases, the aggregate growth rate does not change in the steady state. Thus, in order to maintain positive growth rates in the long run, it is necessary that the sources of the balanced productivity growth do not vanish.

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Appendix: Local Stability Analysis

The dynamic system is:

$$\overset{\bullet}{z} = A_N(1-n)^{1-\beta}z^{\beta}$$
$$-(1-\gamma)\hat{c}\left(n\frac{A_N(1-\beta)z^{\beta}}{(1-\alpha)(1-n)^{\beta}} + A_N(1-n)^{1-\beta}z^{\beta}\right) \quad \text{(A.1)}$$
$$-A_M n^{1-\alpha}z + \gamma\hat{c}\left(A_M n^{1-\alpha}z + \frac{A_M(1-\alpha)}{(1-\beta)n^{\alpha}}(1-n)z\right)$$

$$\overset{\bullet}{n} = \frac{(1-n)n}{[\alpha(1-n)+\beta n]}\left[A_N\beta z^{\beta-1}(1-n)^{1-\beta} - A_M\alpha n^{1-\alpha} - \beta\frac{\overset{\bullet}{z}}{z}\right] \quad \text{(A.2)}$$

We calculate the Jacobian of the nonlinear system evaluated at the equilibrium:

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial z} & \frac{\partial f_1}{\partial n} \\ \frac{\partial f_2}{\partial z} & \frac{\partial f_2}{\partial n} \end{bmatrix}$$

where we obtain:

$$\frac{\partial f_1}{\partial z} = g^*(\beta-1) < 0$$

$$\frac{\partial f_1}{\partial n} = -\frac{(1-\beta)}{(1-n)}(z^*g^*) - (1-\gamma)\hat{c}z^*\left(\frac{A_M}{z^*n^{*\alpha}}\frac{1}{p_N^*}\frac{1}{(1-n^*)}\right)$$
$$-\frac{(1-\alpha)}{n^*}(z^*g^*) - \gamma\hat{c}z^{*2}\frac{1}{n^*} < 0$$

$$\frac{\partial f_2}{\partial z} = \left[\frac{(1-n^*)n^*}{[\alpha(1-n^*)+\beta n^*]}\frac{\beta(\beta-1)}{z^*}\right]$$
$$\cdot\left[\frac{(1-\gamma)\hat{c}}{p_N^*}\left(\frac{A_M n^{*(1-\alpha)}}{z^*} + \frac{p_N^* A_N(1-n^*)^{1-\beta}}{z^{*(1-\beta)}}\right)\right] < 0$$

$$\frac{\partial f_2}{\partial n} = A_N \beta z^{*(\beta-1)} \left[\frac{[\beta(1-n^*)^{-\beta-1} n^* + (1-n^*)^{-\beta}]}{[\alpha(1-n^*) + \beta n^*]} \right] + A_M \alpha \left[\frac{[n^{*(-\alpha)} + \alpha n^{*(-\alpha-1)}(1-n^*)]}{[\alpha(1-n^*) + \beta n^*]} \right] - \beta \left[\frac{(\partial f_1/\partial n)(1/z^*)}{[\alpha(1-n^*) + \beta n^*]} \right] > 0$$

Thus, we get that the determinant of the Jacobian is negative. Therefore, we have one negative characteristic root and one positive characteristic root. Therefore, the equilibrium is locally a saddle point.